Section 21
PN Diode AC Response

21.3 Minority carrier diffusion capacitance

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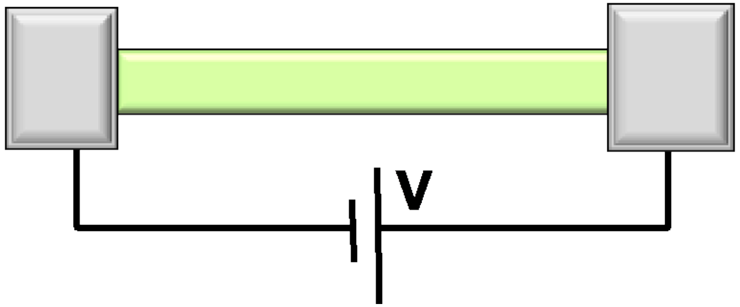
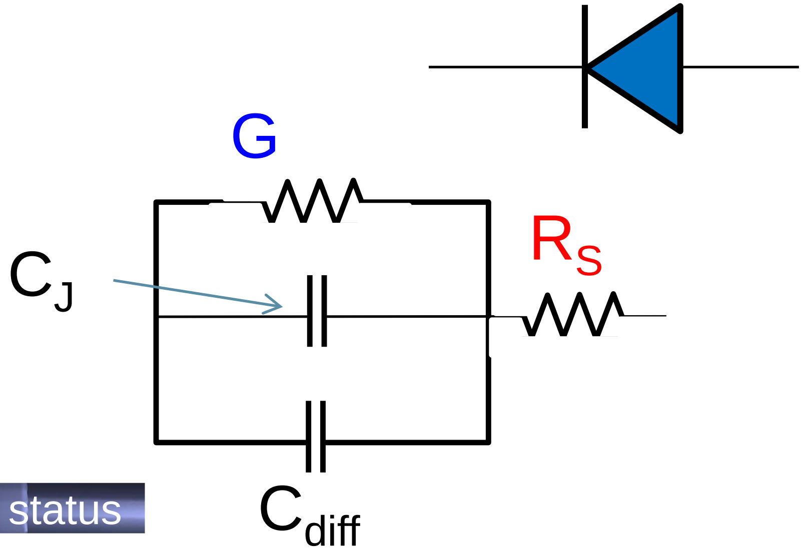


$I = G \times V$

$= q \times n \times v \times A$

charge density velocity area



- > • 21.1 Conductance and series resistance
- > • 21.2 Majority carrier junction capacitance
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status

$$\boxed{\frac{1}{C_J{}^2} \approx \frac{2}{qN_D(x)K_s\varepsilon_0 A^2}\left(V_{bi} - V_A\right)}$$

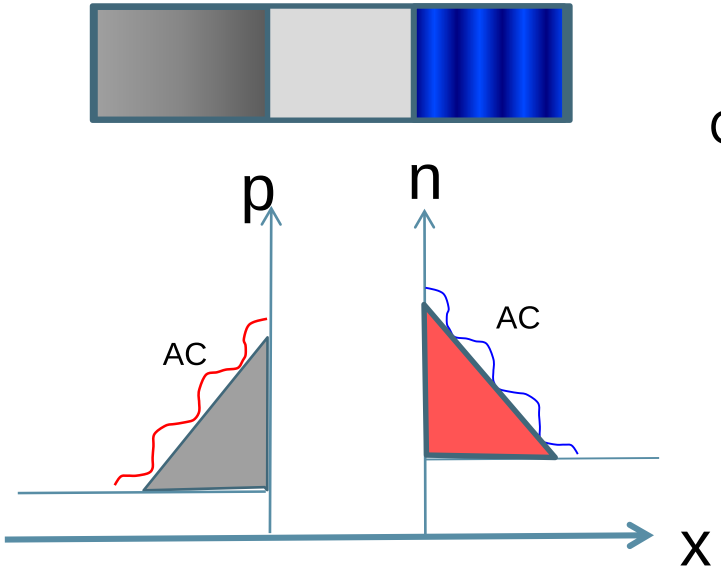
Diffusion Capacitance for Minority Carriers

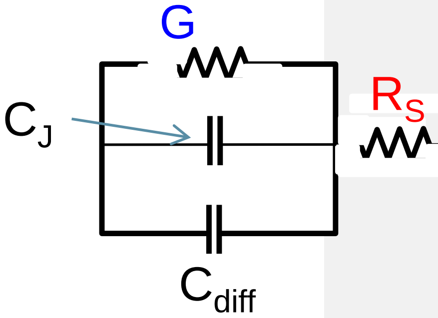
Minority Carrier side

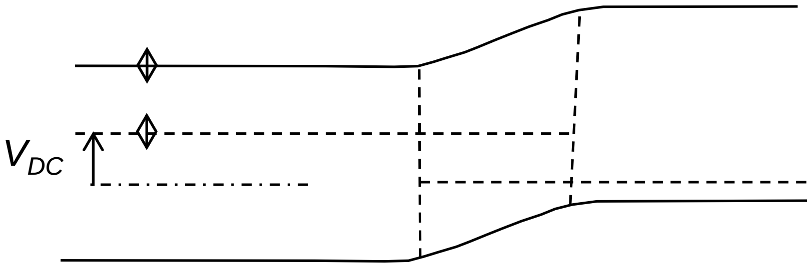
$$\mathbf{J}_N = qn\cancel{\mu_N}\mathcal{E} + qD_N \frac{dn}{dx}$$

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{dJ_n}{dx} - r_N + g_N$$

$$n = n_0 + \underbrace{\Delta n_{dc}}_{\text{DC}} + \underbrace{\Delta n_{ac} e^{j\omega t}}_{\text{AC}}$$

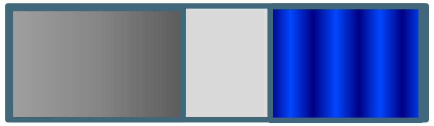




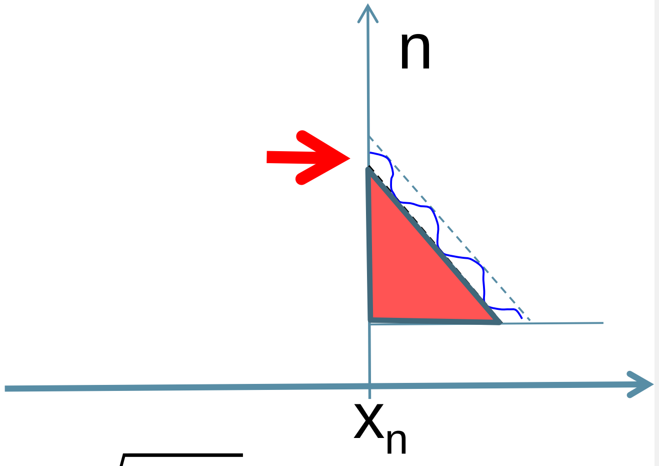


$$\frac{\partial\left(n_0 + \Delta n_{dc} + \Delta n_{ac} e^{j\omega t}\right)}{\partial t} = D_N \frac{d^2\left(n_0 + \Delta n_{dc} + \Delta n_{ac} e^{j\omega t}\right)}{dx^2} - \frac{\Delta n_{dc} + \Delta n_{ac} e^{j\omega t}}{\tau_n}$$

Diffusion Capacitance for Minority Carriers



$$\frac{\partial\left(n_0+\Delta n_{dc}+\Delta n_{ac}e^{j\omega t}\right)}{\partial t} = D_N \frac{d^2\left(n_0+\Delta n_{dc}+\Delta n_{ac}e^{j\omega t}\right)}{dx^2} - \frac{\Delta n_{dc}+\Delta n_{ac}e^{j\omega t}}{\tau_n}$$



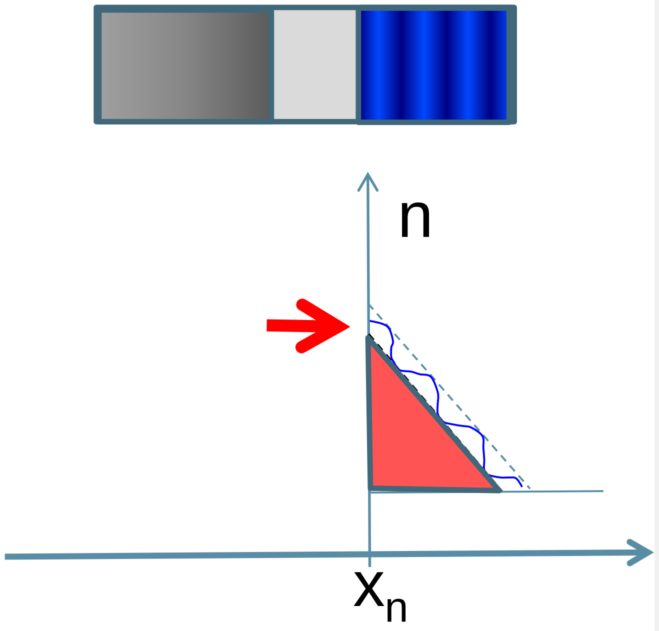
$$\boxed{j\omega\Delta n_{ac}e^{j\omega t} = D_N\frac{d^2\Delta n_{dc}}{dx^2} + e^{j\omega t}\frac{d^2\Delta n_{ac}}{dx^2} - \frac{\Delta n_{dc}}{\tau_n} - e^{j\omega t}\frac{\Delta n_{ac}}{\tau_n}}$$

$$\boxed{\text{DC:}\quad 0 = D_N\frac{d^2\Delta n_{dc}}{dx^2} - \frac{\Delta n_{dc}}{\tau_n} \Rightarrow \Delta n_{dc} = Ae^{-\frac{x}{L_n}} + Be^{+\frac{x}{L_n}}} \Rightarrow Ae^{-\frac{x}{L_n}} \text{ with } L_n = \sqrt{D_n\tau_n}$$

$$\boxed{\text{AC}:\quad 0 = D_N\frac{d^2\Delta n_{ac}}{dx^2} - \left(j\omega\tau_n+1\right)\frac{\Delta n_{ac}}{\tau_n} \Rightarrow \Delta n_{ac} = Ce^{-\frac{x}{L_n^*}} + De^{+\frac{x}{L_n^*}} \rightarrow Ce^{-\frac{x}{L_n^*}}}$$

$$L_n^* = \sqrt{D_n\tau_n/\left(1+j\omega\tau_n\right)} \qquad \tau_n^* = \tau_n/\left(1+j\omega\tau_n\right)$$

AC Boundary Conditions



$$\Delta n_{dc}(x=0) = \frac{n_i^2}{N_A}\left(e^{\frac{qV_{dc}}{kT}} - 1\right)$$

$$\left(\Delta n_{dc} + \Delta n_{ac} e^{j\omega t}\right) = \frac{n_i^2}{N_A + \Delta p_{ac} e^{j\omega t}}\left(e^{q\frac{V_{dc}+V_{ac}e^{j\omega t}}{kT}} - 1\right)$$

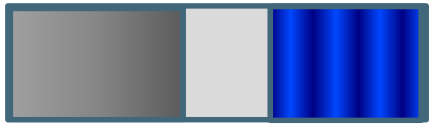
$$\left(\Delta n_{dc} + \Delta n_{ac} e^{j\omega t}\right) \approx \frac{n_i^2}{N_A}\left(e^{\frac{qV_{dc}}{kT}} e^{\frac{qV_{ac}e^{j\omega t}}{kT}} - 1\right)$$

Taylor expansion

$$\approx \frac{n_i^2}{N_A}\left\{e^{\frac{qV_{dc}}{kT}}\left(1 + \frac{qV_{ac}e^{j\omega t}}{kT}\right) - 1\right\}$$

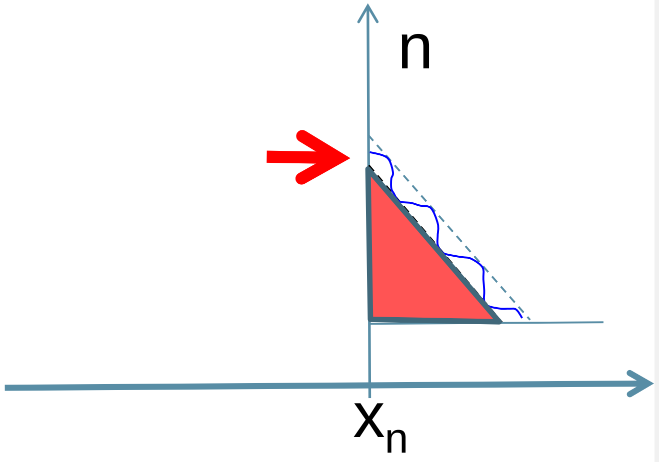
$$\Delta n_{ac}(x=0) = \frac{qV_{ac}}{kT}\frac{n_i^2}{N_A}e^{\frac{qV_{dc}}{kT}} = C$$

AC Current and Impedance



$$\Delta n_{ac}(x=0) = \frac{qV_{ac}}{kT}\frac{n_i^2}{N_A}e^{\frac{qV_{dc}}{kT}} = C$$

$$\Delta n_{ac}(x) = Ce^{-\frac{x}{L_n^*}} + De^{+\frac{x}{L_n^*}} \rightarrow Ce^{-\frac{x}{L_n^*}}$$



Finally…

$$\boxed{J_{ac} = -qD_n \left.\frac{d\Delta n_{ac}}{dx}\right|_{x=0} = \frac{qD_n}{L_n^*}\frac{qV_{ac}}{kT}\frac{n_i^2}{N_A}e^{\frac{qV_{dc}}{kT}}}$$

AC Current

$$\boxed{Y_{ac} = \frac{J_{ac}}{V_{ac}} = \frac{q^2 D_n}{L_n^* kT}\frac{n_i^2}{N_A}e^{\frac{qV_{dc}}{kT}} \equiv G_0\sqrt{1+j\omega\tau_n}}$$

AC Admittance

$$L_n^* = \sqrt{D_n\tau_n/(1+j\omega\tau_n)} \qquad \tau_n^* = \tau_n/(1+j\omega\tau_n)$$

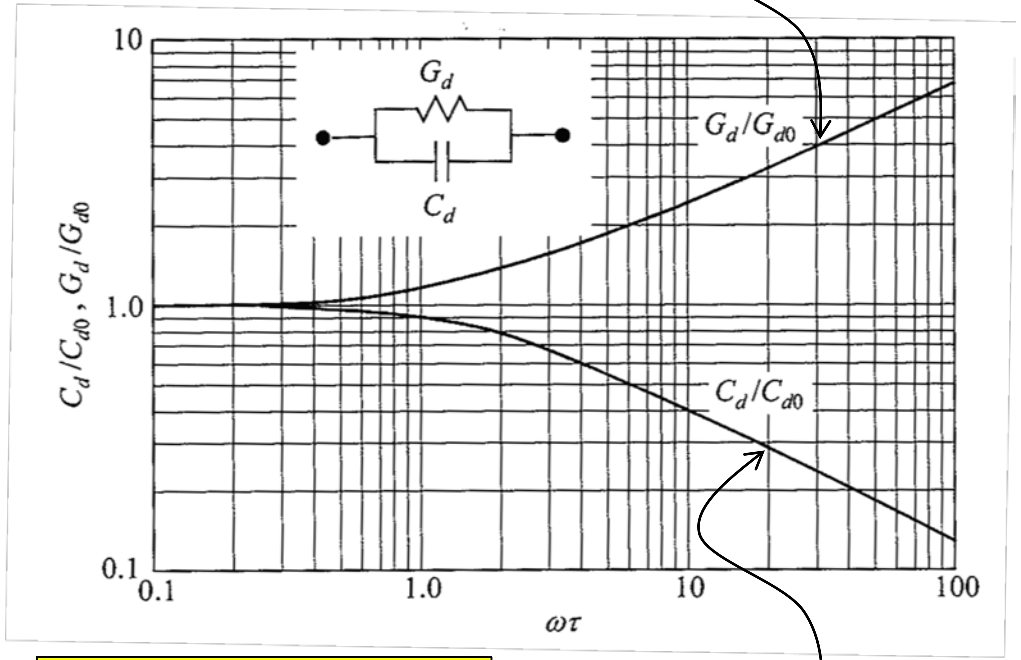
$$G_D \propto \sqrt{\omega}$$

$$Y_{ac} = G_D + j\omega C_D \equiv G_0\sqrt{1 + j\omega\tau_n}$$

Separate in real & imaginary parts …

$$G_D = \frac{G_0}{\sqrt{2}}\left[\sqrt{1+\omega^2\tau_n^{\ 2}}+1\right]^{1/2}$$

$$\omega C_D = \frac{G_0}{\sqrt{2}}\left[\sqrt{1+\omega^2\tau_n^{\ 2}}-1\right]^{1/2}$$



Product of G_D and C_D frequency-independent

$$C_D \propto 1/\sqrt{\omega}$$

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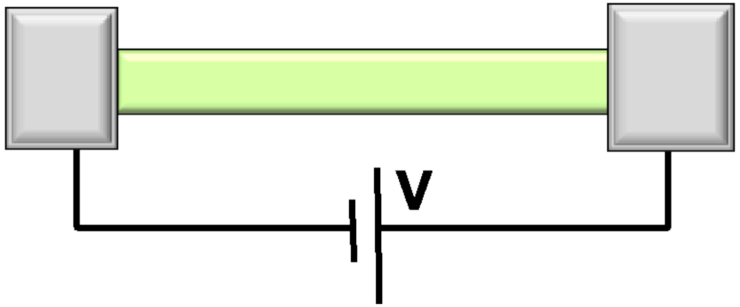

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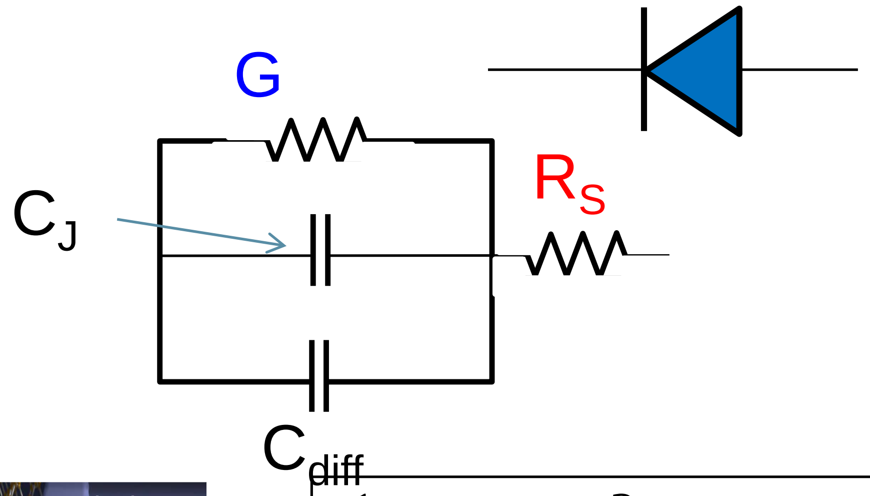

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$$\boxed{\frac{1}{C_J^{\,2}} \approx \frac{2}{qN_D(x)K_s\varepsilon_0 A^2}(V_{bi} - V_A)}$$

$$Y_{ac} = G_D + j\omega C_D \equiv G_0\sqrt{1 + j\omega\tau_n}$$

1) Small signal response relevant for many analog applications.
2) Small signal parameters always refer to the DC operating conditions, as such the parameter changes with bias condition.
3) Important to distinguish between majority and minority carrier capacitance. Their relative importance depends on specific applications.