Section 20
PN Diode I-V Characteristics

20.4 Non-Ideal Effects

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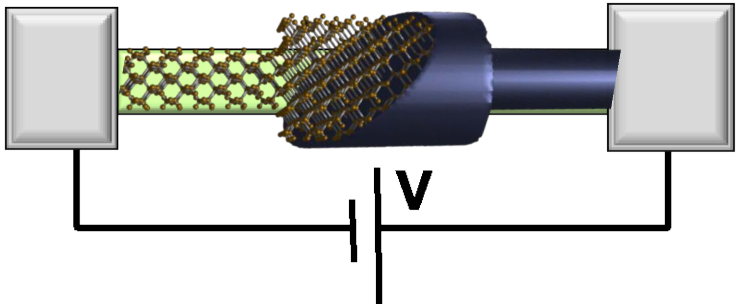
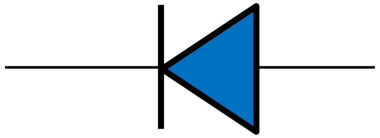
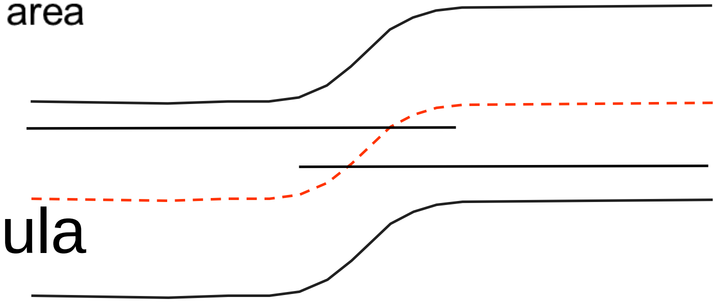


$I = G \times V$

$= q \times n \times v \times A$

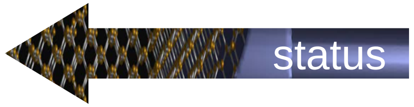
charge density velocity area



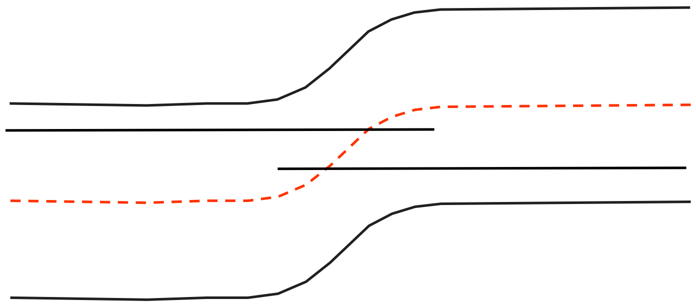


$$np = n_i^2 e^{(F_n - F_p)\beta} = n_i^2 e^{qV_A\beta}$$

- 20.1 Band diagram with applied bias
- 20.2 Derivation of the forward bias formula
- 20.3 Forward Bias - Non-linear Regime
  - »Resistive drop
  - »Ambipolar regime
- 20.4 Non-ideal effects:
  - »Junction recombination
  - »Tunneling
  - »Impact ionization



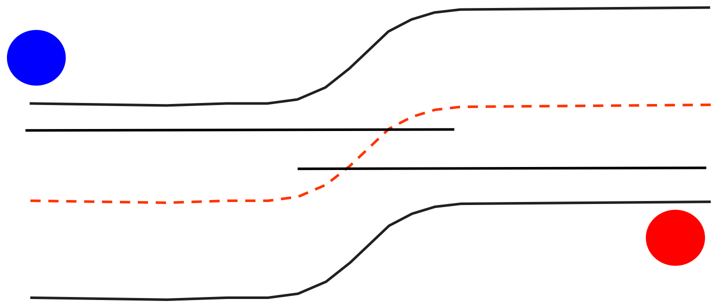
Mass action in non-equilibrium



$$np = n_i^2 e^{(F_n - F_p)\beta} = n_i^2 e^{qV_A\beta}$$

What is the recombination current?

$$I_R = -qA\int_0^W \frac{\partial n}{\partial t} dx$$



Shockley-Reed Hall

$$\frac{\partial n}{\partial t} = -\frac{[n(x)p(x) - n_i^{\ 2}]}{\tau_p[n(x) + n_1] + \tau_n[p(x) + p_1]}$$

$$np = n_i^2 e^{(F_n - F_p)\beta} = n_i^2 e^{qV_A\beta}$$

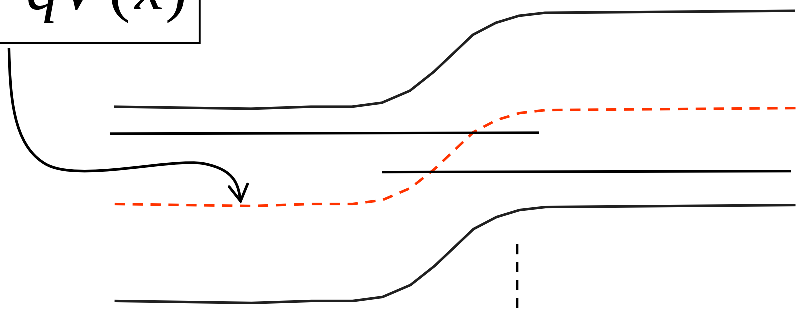
Follows from assuming midgap traps

Assume $\tau_n = \tau_p$ $\quad E_i = E_T$ $\quad n_1 = p_1 = n_i$

$$\frac{\partial n}{\partial t} = -\frac{n_i^{\ 2}(e^{qV_A/kT} - 1)}{\tau[n(x) + p(x) + 2n_i]}$$

Electron/Hole Concentrations at Junction

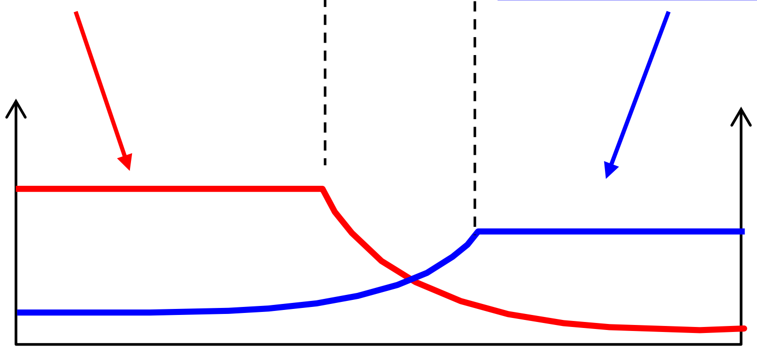
$$E_i(x) = E_{iL} - qV(x)$$



$$n(x) = n_i e^{(F_N - E_i(x))/kT} = n_i e^{[F_N - E_{iL} + qV(x)]/kT}$$

$$p(x) = \frac{n_i^2 e^{qV_A/kT}}{n_i e^{[F_N - E_{iL} + qV(x)]/kT}} = n_i e^{-[F_N - E_{iL} + qV(x)]/kT + qV_A/kT}$$

$$\frac{\partial n}{\partial t} = -\frac{n_i^2 (e^{qV_A/kT} - 1)}{\tau[n(x) + p(x) + 2n_i]}$$



position

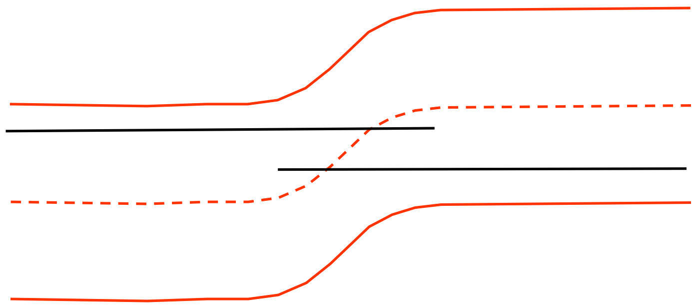
Junction Recombination

$$n(x) = n_i e^{(F_N - E_i(x))/kT}$$
$$= n_i e^{[F_N - E_{iL} + qV(x)]/kT}$$

$$p(x) = \frac{n_i^2 e^{qV_A/kT}}{n_i e^{[F_N - E_{iL} + qV(x)]/kT}}$$
$$= n_i e^{-[F_N - E_{iL} + qV(x)]/kT + qV_A/kT}$$

$$U_{FN} = \frac{F_N - E_{iL}}{kT} \qquad U_A = \frac{V_A}{kT/q}$$

$$\frac{\partial n}{\partial t} = -\frac{n_i^2 (e^{qV_A/kT} - 1)}{\tau[n(x) + p(x) + 2n_i]} \qquad \frac{\partial n}{\partial t} = -\frac{n_i (e^{U_A} - 1)}{\tau[e^{U_{FN}+U} + e^{-U_{FN}-U+U_A}]}$$



$$\Rightarrow \frac{\partial n}{\partial t} = -\frac{n_i}{\tau} \frac{e^{U_A/2}(e^{U_A/2} - e^{-U_A/2})}{e^{U_A/2}[e^{U_{FN}+U-U_A/2} + e^{-U_{FN}-U+U_A/2}]}$$

$$\Rightarrow \frac{\partial n}{\partial t} = -\frac{n_i}{\tau} \frac{sinh(U_A/2)}{cosh[U_{FN} + U - U_A/2]}$$

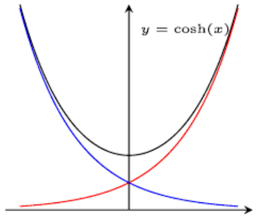
$$I_R = -qA \int_0^W \frac{\partial n}{\partial t} dx$$

$$I_R = -qA\left(\frac{n_i}{\tau}\right) \times sinh\left(\frac{U_A}{2}\right) \times \int_0^W \frac{dx}{cosh[U_{FN} + U - U_A/2]}$$

Junction Recombination in Forward Bias

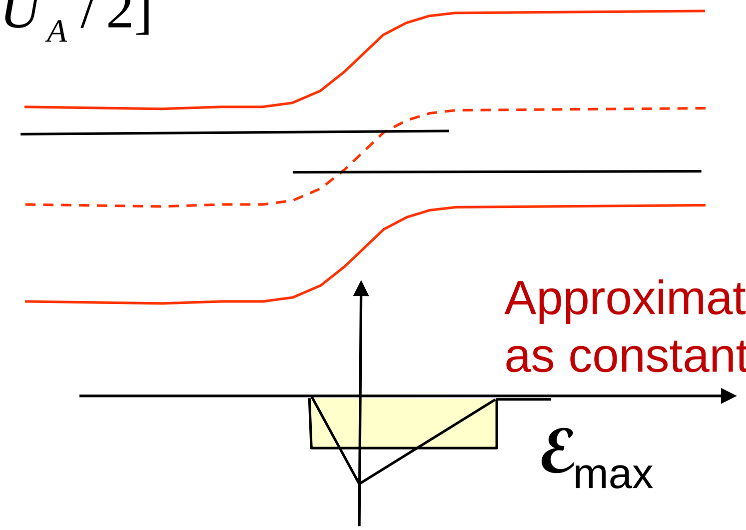
$$I_R = -qA\left(\frac{n_i}{\tau}\right)\times \sinh\left(\frac{U_A}{2}\right)\times\int_0^W \frac{dx}{\cosh[U_{FN}+U-U_A/2]}$$

Keep positive branch



$$\Rightarrow I_R \approx -qA(\frac{n_i}{\tau})\sinh(\frac{U_A}{2})\int_0^W \frac{dx}{e^{(U_{FN}+U-U_A/2)}}$$

$$\Rightarrow I_R \approx -qA(\frac{n_i}{\tau})\times \sinh(\frac{U_A}{2})\int_0^W \frac{dx}{e^{-(\mathcal{E}_{max}x)/(kT/2q)}}$$



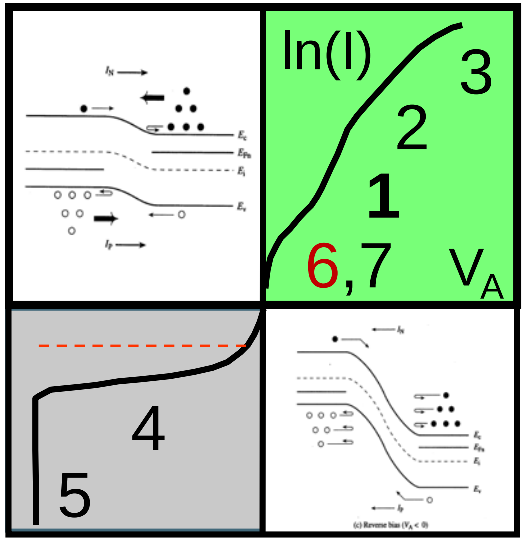
Approximate electric field as constant

$$\Rightarrow I_{Dep} = -qA\left[\frac{kT}{2q\mathcal{E}_{max}}\right]\left[\frac{n_i}{\tau}e^{qV_A/2kT}\right]$$

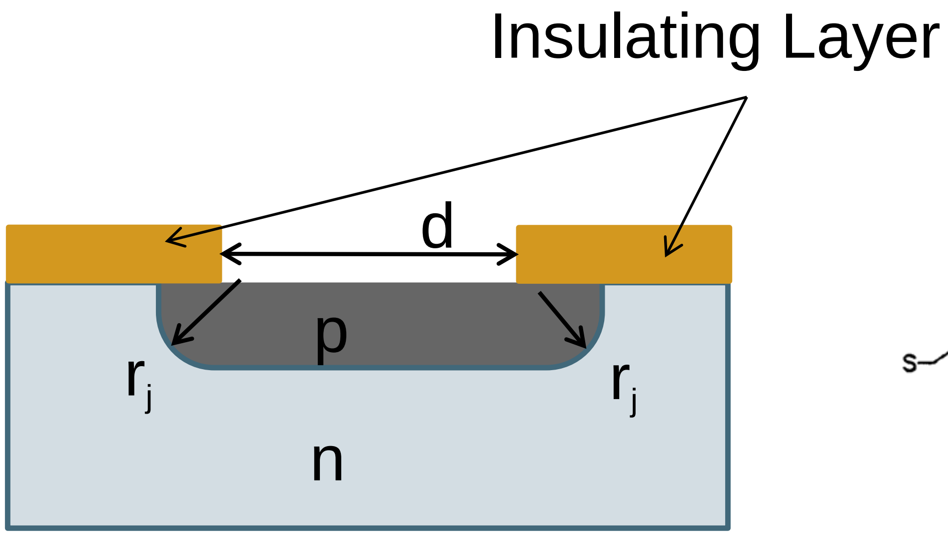
Effective spatial width of recombination

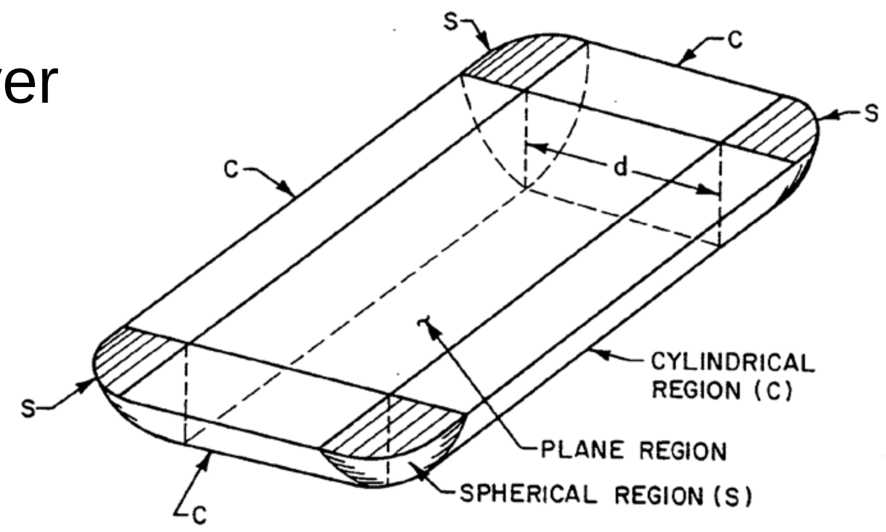
Excess Carrier at mid-junction
Factor ½ indicates recombination / traps in junction

Traps hidden in τ



Junction Leakage in Practice





Junction Design Considerations
Electric field stronger at corners, sharp edges. → increased recombination!

Junction Recombination in Reverse Bias

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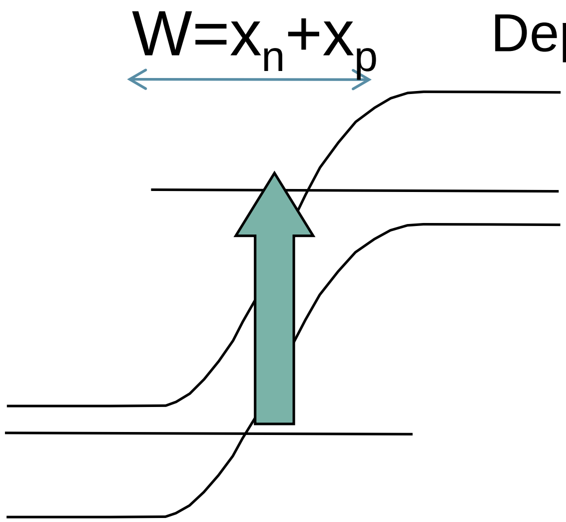


Depletion width

Shockley-Reed Hall

$$\frac{\partial n}{\partial t} = -\frac{[n(x)p(x) - n_i^2]}{\tau_p[n(x)+n_1] + \tau_n[p(x)+p_1]}$$

Assume $\tau_n = \tau_p$ $\quad E_i = E_T$ $\quad n_1 = p_1 = n_i$

Follows from assuming midgap traps

$$\frac{\partial n}{\partial t} = -\frac{n_i^2 (e^{qV_A/kT} - 1)}{\tau[n(x)+p(x)+2n_i]}$$

$$\boxed{\frac{\partial n}{\partial t} = -\frac{n_i}{2\tau}}$$

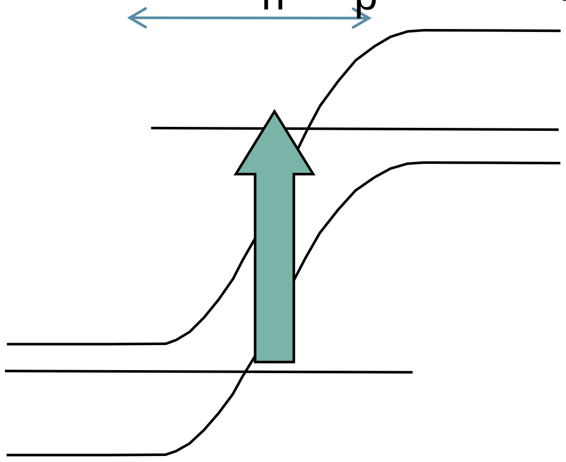
(Recombination in depletion region n=p=0)

Junction Recombination in Reverse Bias

$$\frac{\partial n}{\partial t} = -\frac{n_i}{2\tau}$$

(Recombination in depletion region n=p=0)

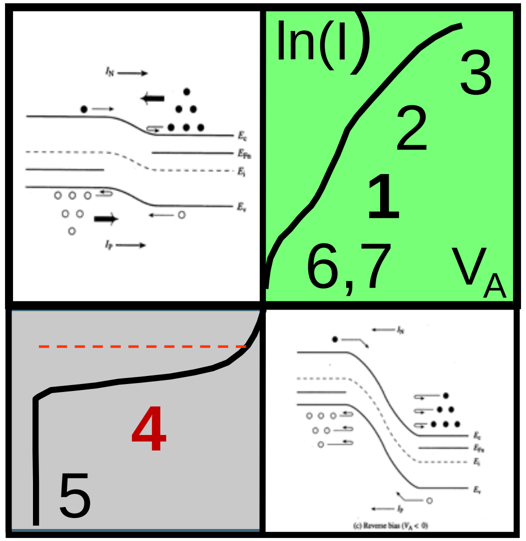
$W=x_n+x_p$ Depletion width



Integrate…

$$I_R \approx -qA\int_0^{W}\left(\frac{n_i}{2\tau}\right)dx$$

$$= -qA\frac{n_i W}{2\tau} \propto \sqrt{V_{bi} - V_A}$$



Ideally the flat – now it depends as a square root of voltage
=> another probe mechanism for traps in junction

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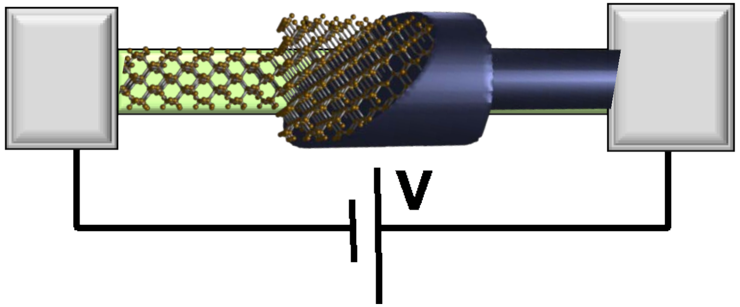

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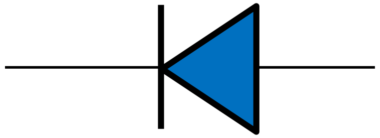
charge density velocity area



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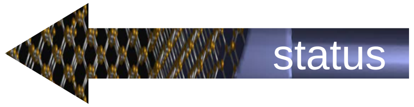

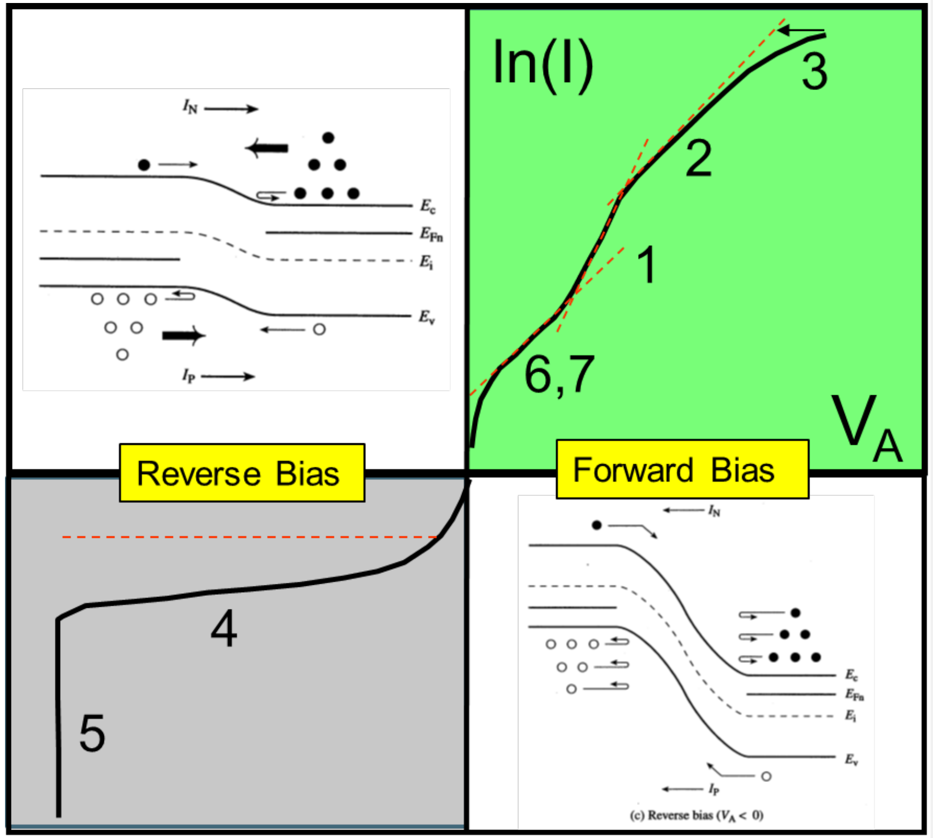

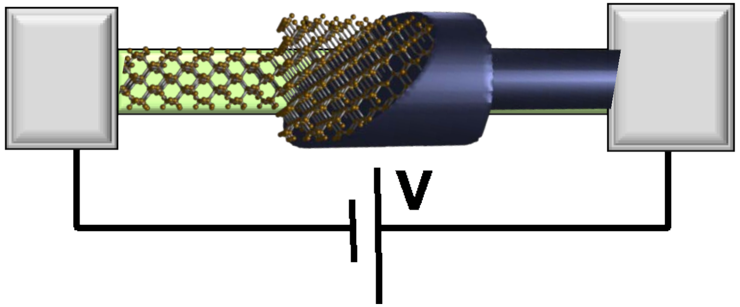

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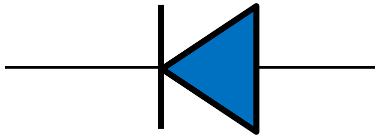

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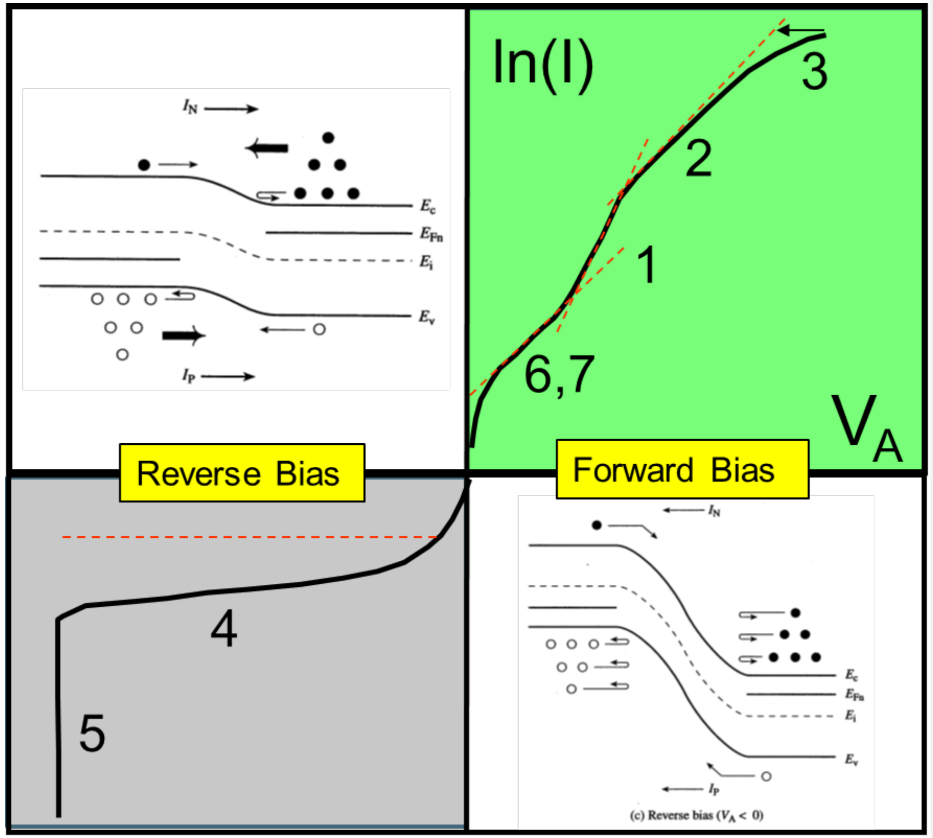
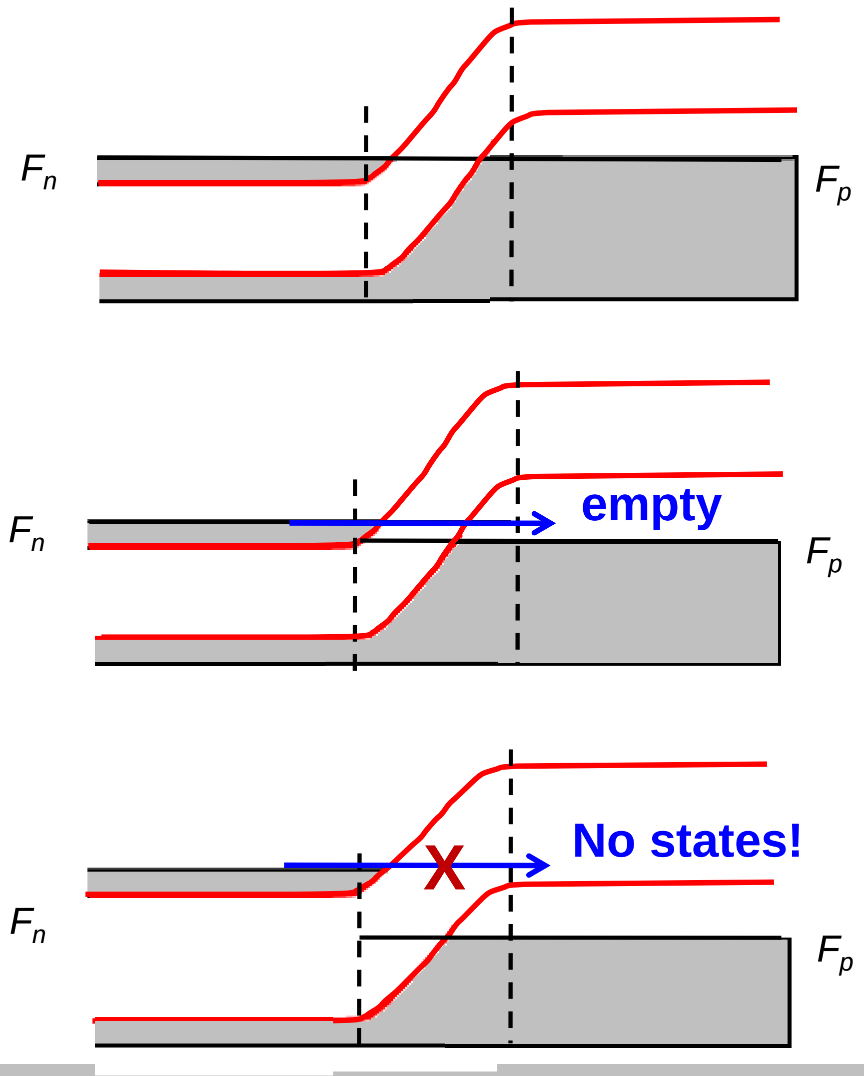
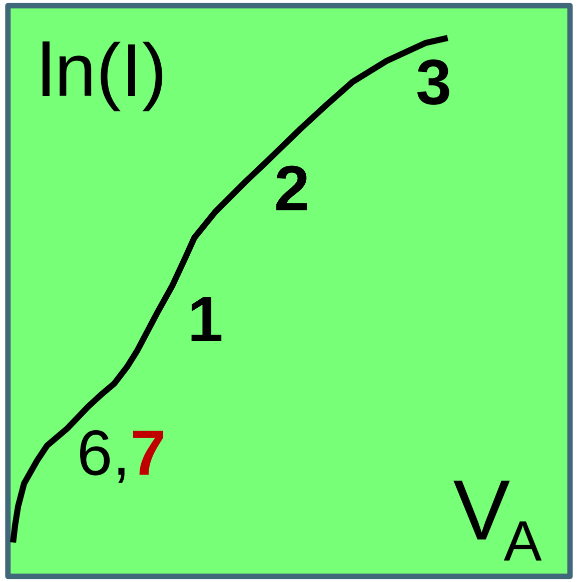
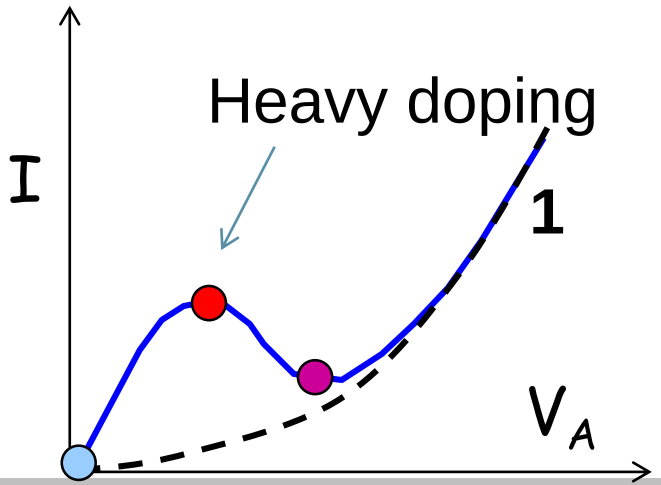
status



Esaki-Diode: **Heavily** doped diode

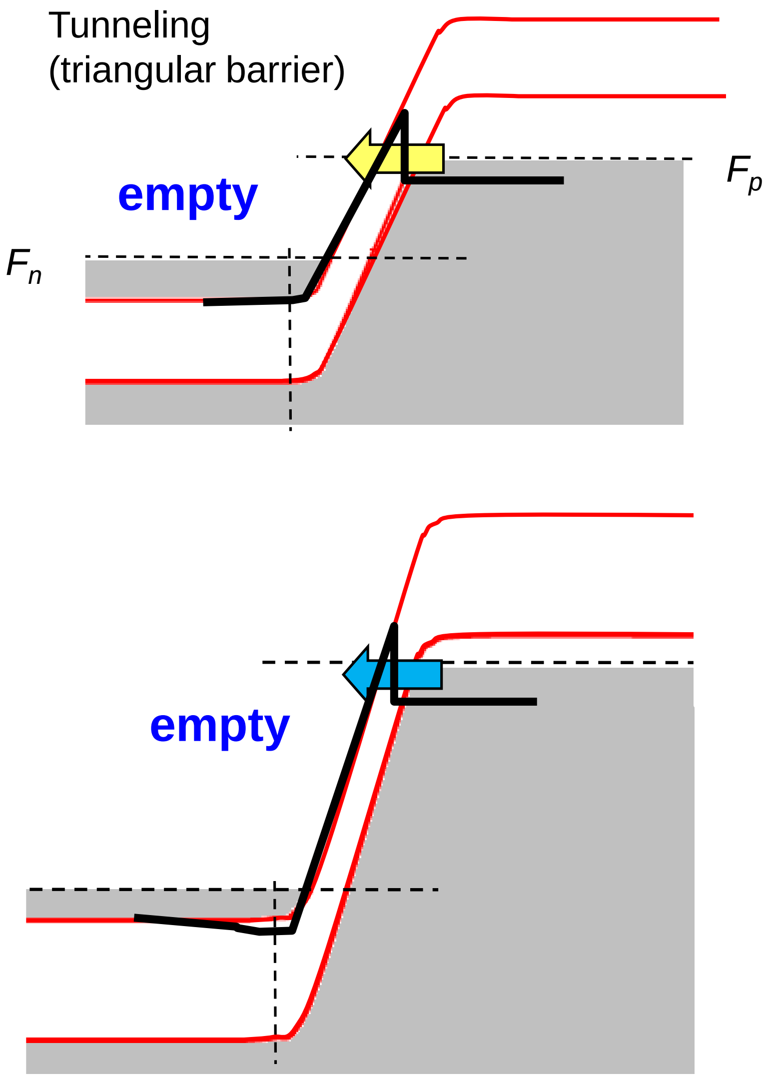


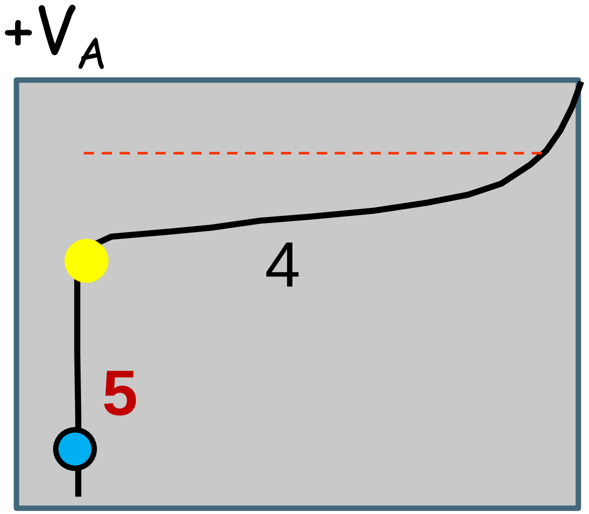




Tunneling in diodes.
Nobel Prize (Esaki)

Reverse Bias (5): Zener Tunneling





Zener tunneling occurs in every diode. (reverse bias)

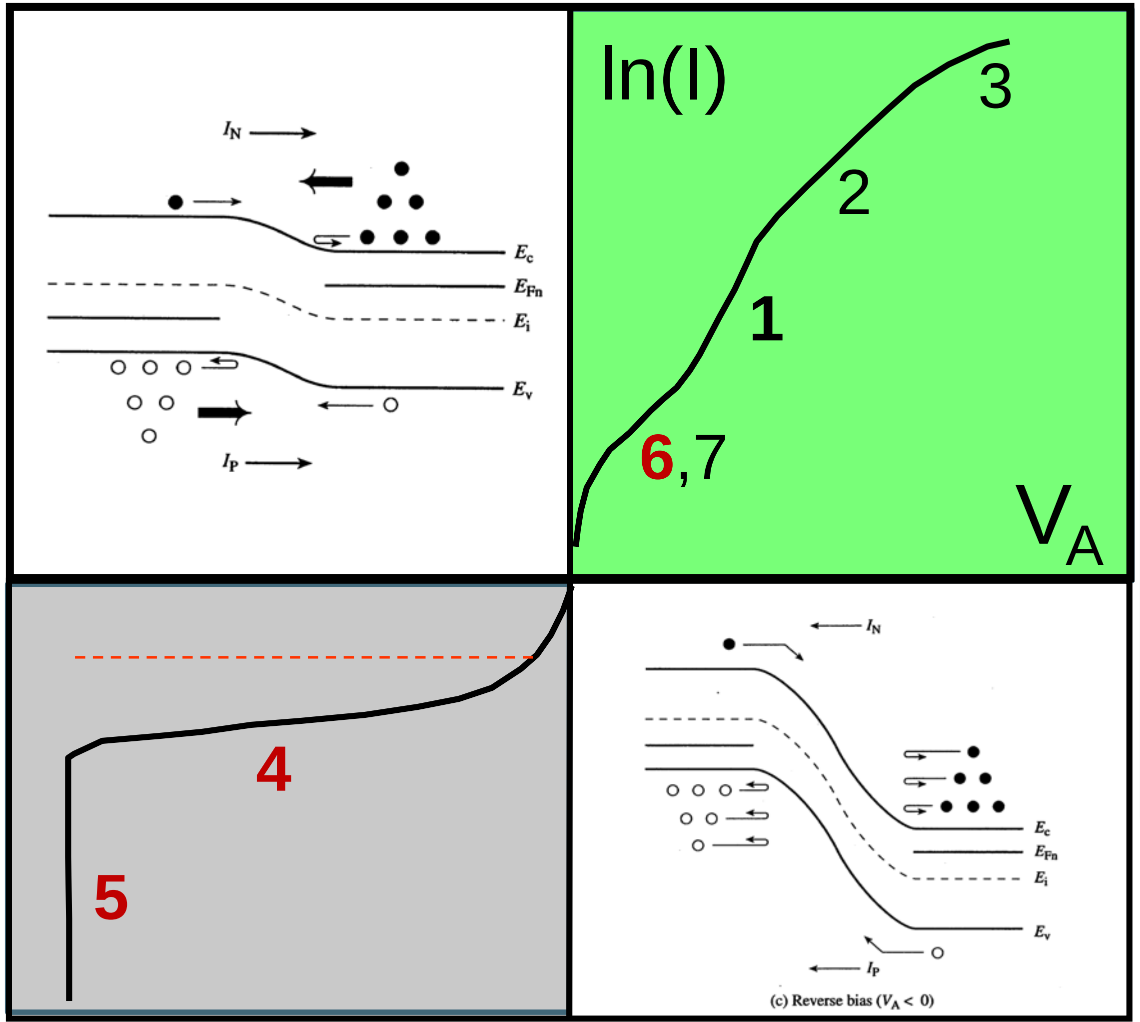
Remember: Tunneling through a triangular barrier

$$I = qpT\upsilon$$

$$T = \frac{4}{4\cosh^2 \alpha d + \left(\frac{\alpha}{k} - \frac{k}{\alpha}\right)\sinh^2 \alpha d}$$

(p.49 ADF)

Various Regions of I-V Characteristics



1. *Diffusion limited*
2. *Ambipolar transport*
3. *High injection*
4. ***R-G in depletion***
5. ***Breakdown***
6. ***Trap-assisted R-G***
7. *Esaki Tunneling*

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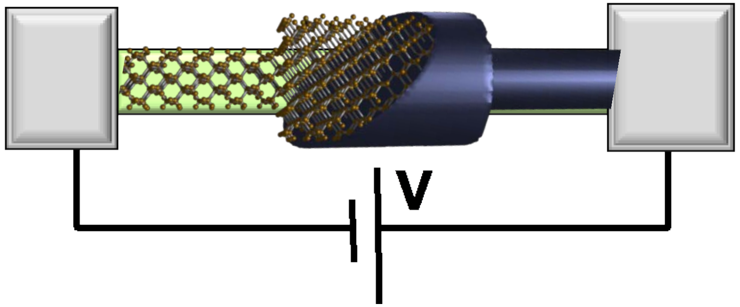

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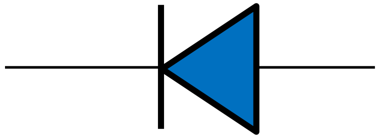

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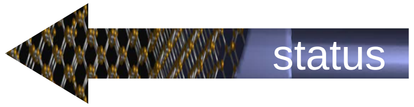
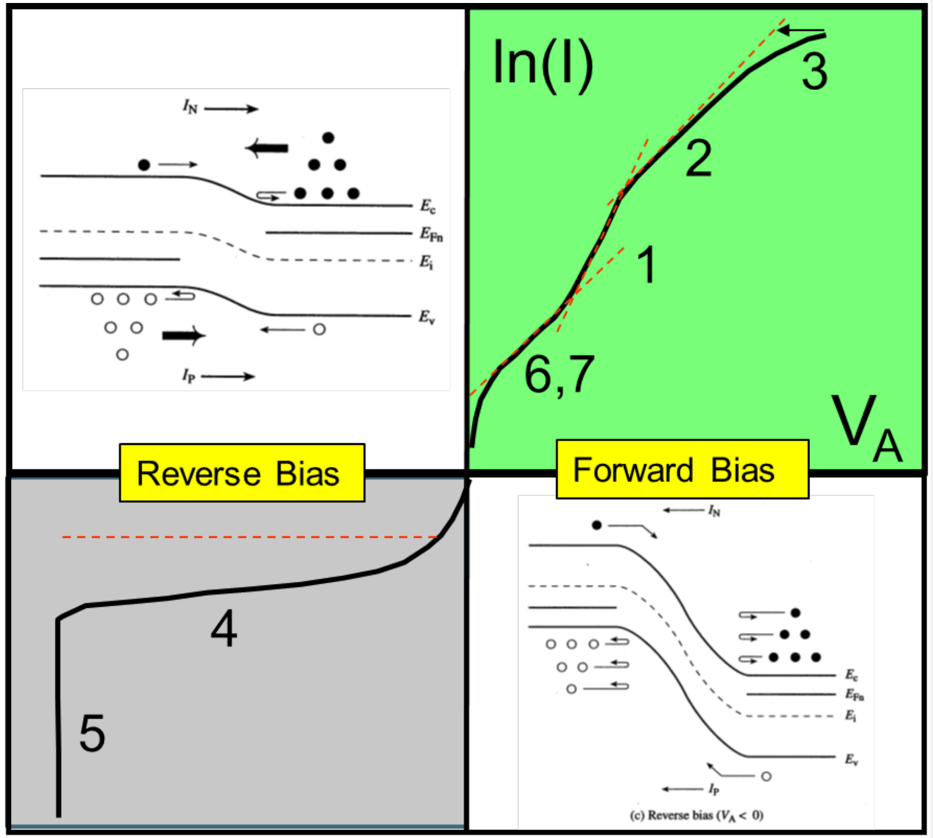
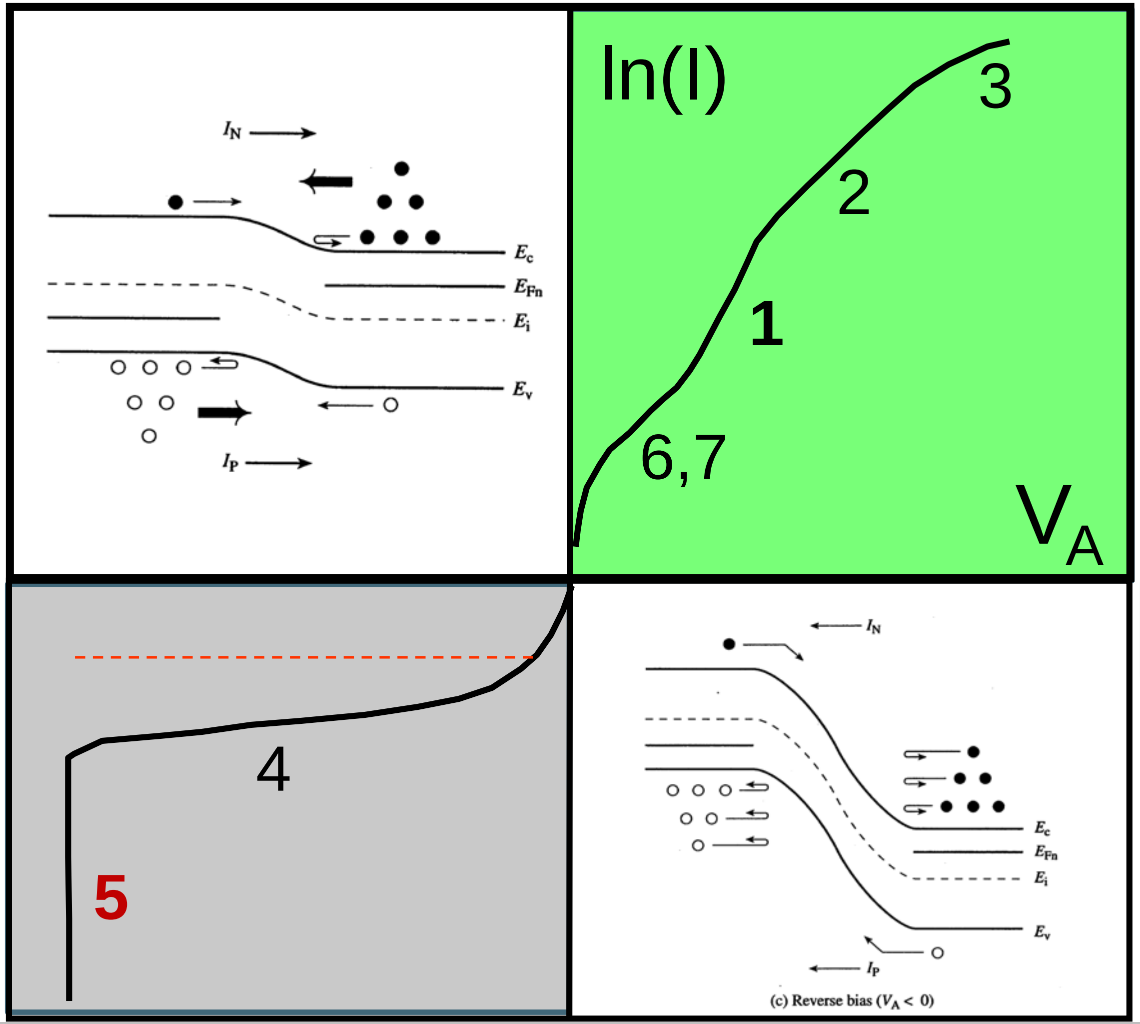




Avalanche Breakdown



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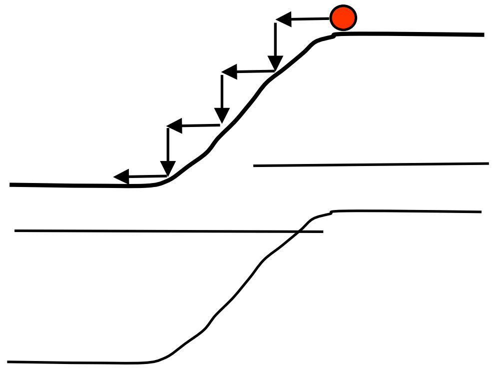
Nonlinearity due to Impact-Ionization

Reverse Bias



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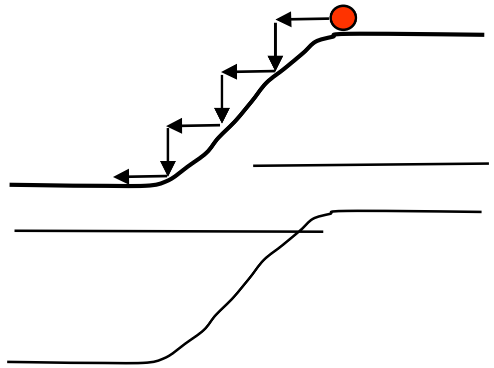
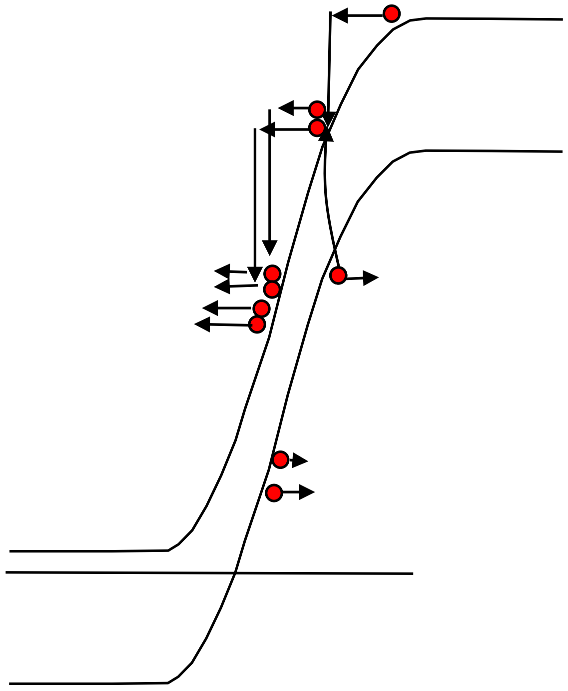
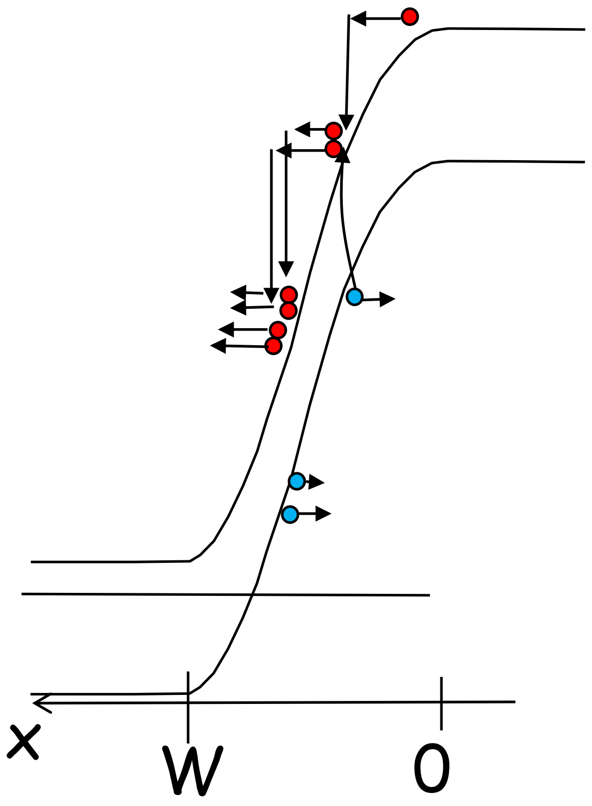


High Reverse Bias > bandgap (typically 3/2 bandgap)



Exponential current growth
(Impact Ionization or Inverse Auger process)

Impact-ionization and Flux Conservation



$$I_n(x+dx) = I_n(x) + \alpha_n I_n(x)dx + \alpha_p I_p(x)dx$$

Impact Ionization probabilities

$$\frac{I_n(x+dx) - I_n(x)}{dx} = \alpha_n I_n(x) + \alpha_p I_p(x)$$

$$\Rightarrow \frac{dI_n(x)}{dx} = \alpha_n I_n(x) + \alpha_p I_p(x)$$

Steady state: Define
$I_T = I_N + I_P$ (total current)
Constant in steady state

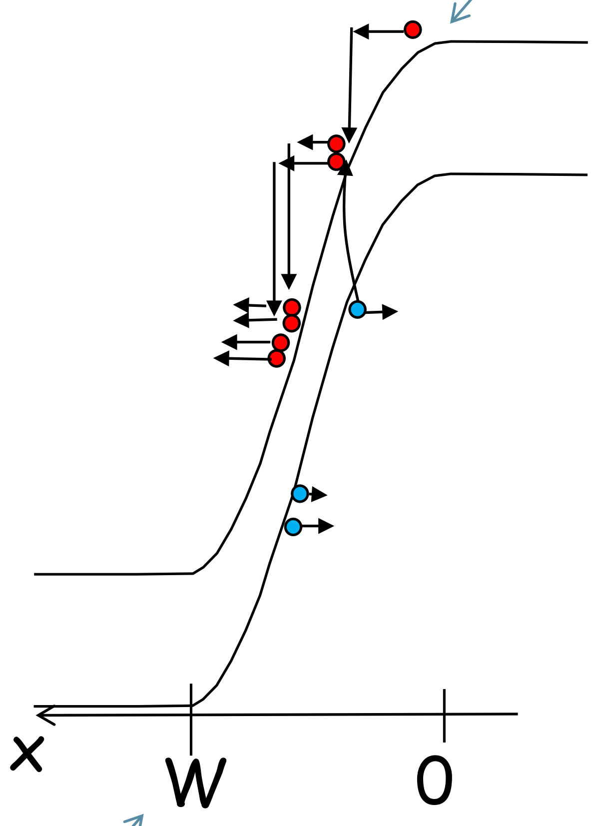
$$I_p = I_T - I_n(x)$$

$$\frac{dI_n(x)}{dx} = \alpha_p[I_T - I_n(x)] + \alpha_n I_n(x)$$

$$\frac{dI_n(x)}{dx} - (\alpha_n - \alpha_p)I_n(x) = \alpha_p I_T$$

Differential equation

Impact-ionization and Flux Conservation



$$\frac{dI_n(x)}{dx} - (\alpha_n - \alpha_p)I_n(x) = \alpha_p I_T$$

Differential equation

$$y' - ay = b$$

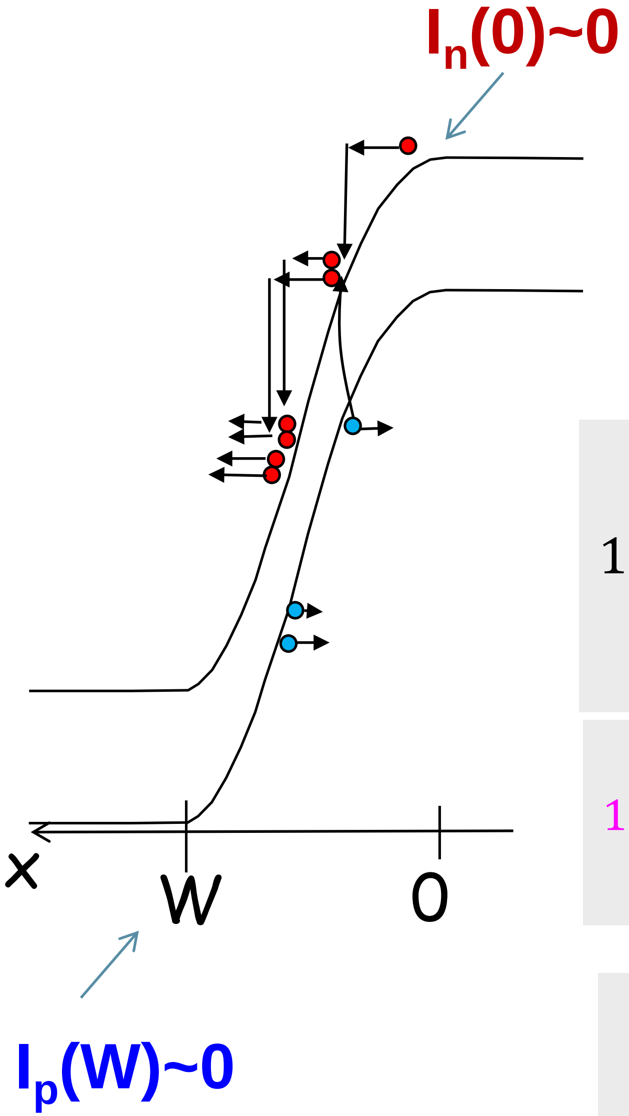
$$y = \frac{1}{a}e^{ax+ca} - \frac{b}{a}$$

Solution form of differential equation

$$\frac{I_n(W)}{I_T} = \frac{\int_0^W \alpha_p e^{-\int_0^x (\alpha_n - \alpha_p)dx'} dx + \frac{I_n(0)}{I_T}}{1 + \int_0^W (\alpha_p - \alpha_n) e^{-\int_0^x (\alpha_n - \alpha_p)dx'} dx}$$

Reverse diffusion current

Impact-ionization



Solution form of differential equation

$$\frac{I_n(W)}{I_T} = \frac{\int_0^W \alpha_p e^{-\int_0^x (\alpha_n-\alpha_p)dx'} dx + \frac{I_n(0)}{I_T}}{1+\int_0^W (\alpha_p-\alpha_n) e^{-\int_0^x(\alpha_n-\alpha_p)dx'} dx}$$

Reverse diffusion current

At x=W, I_N has grown exponentially, and I_P is now negligible.

$$\not{I_p}(W)+I_n(W)=I_T \Rightarrow I_n(W)\approx I_T$$

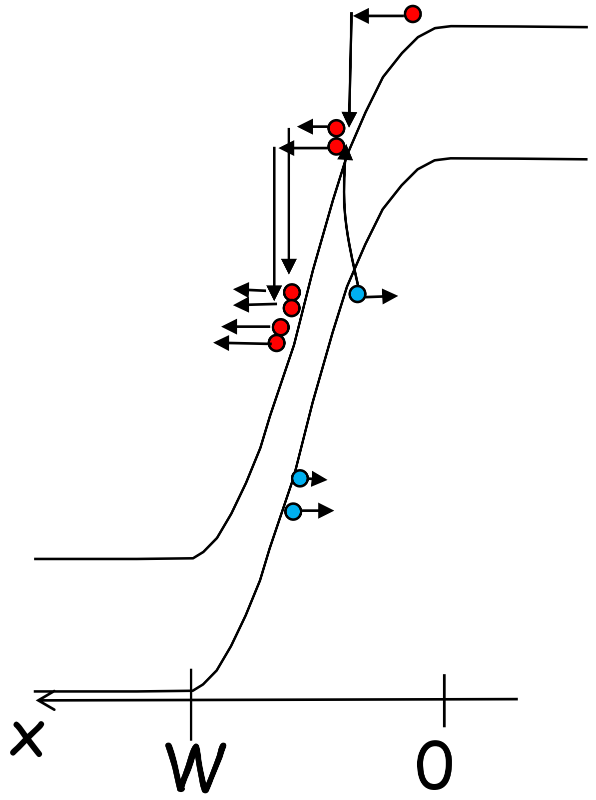
$$\frac{I_n(0)}{I_T} \equiv \frac{1}{M_p}$$ Multiplication Factor

$$1 = \frac{\int_0^W \alpha_p e^{-\int_0^x(\alpha_n-\alpha_p)dx'} dx + \frac{1}{M_p}}{1+\int_0^W(\alpha_p-\alpha_n)e^{-\int_0^x(\alpha_n-\alpha_p)dx'}dx}$$

$$1+\int_0^W(\not{\alpha_p}-\alpha_n)e^{-\int_0^x(\alpha_n-\alpha_p)dx'}dx = \int_0^W \not{\alpha_p} e^{-\int_0^x(\alpha_n-\alpha_p)dx'}dx + \frac{1}{M_p}$$

$$\int_0^W \alpha_n e^{-\int_0^x(\alpha_n-\alpha_p)dx'}dx = \left(1-\frac{1}{M_p}\right) \approx 1$$

Impact-ionization



$$\int_0^W \alpha_n e^{-\int_0^x (\alpha_n - \alpha_p)dx'} dx = \left(1 - \frac{1}{M_p}\right) \approx 1$$

Assume: $\alpha_n = \alpha_p$

this cannot be quite right… n, p different masses etc

$$\int_0^W \alpha_n(x)dx = 1$$

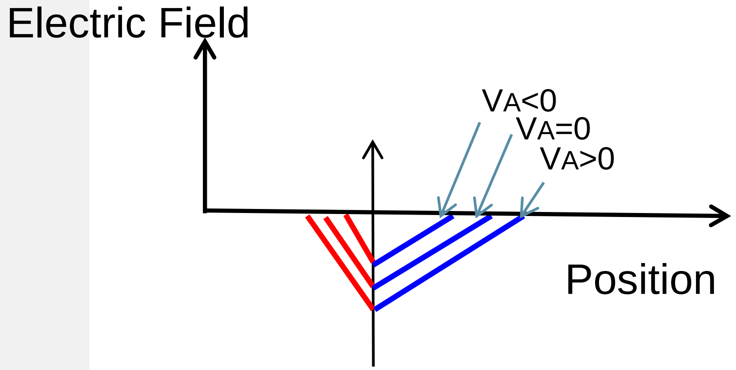
Assume: $\alpha_n(x) = \alpha_n$

Most avalanche happens spatially at maximum electric field

$$\alpha_n W = 1$$

$$\alpha_p = A_0 e^{-B/\mathcal{E}}$$

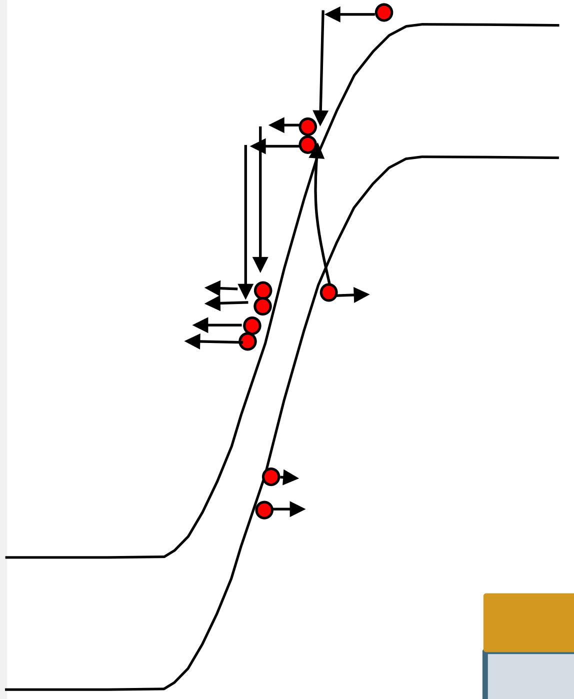
from experiment and theory
A and B material coeffients



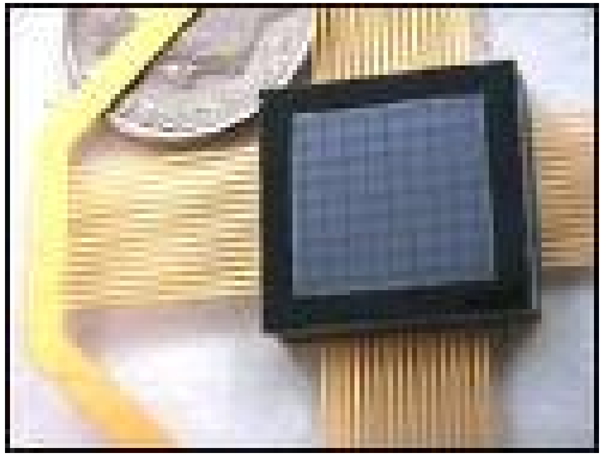
Breakdown-Field

$$\mathcal{E}\left(0^-\right) = \frac{qN_D x_n}{k_s \varepsilon_0} = \left[\frac{2q}{k_s \varepsilon_0} \frac{N_D N_A}{N_D + N_A}\left(V_{bi} - V_A\right)\right]^{1/2}$$

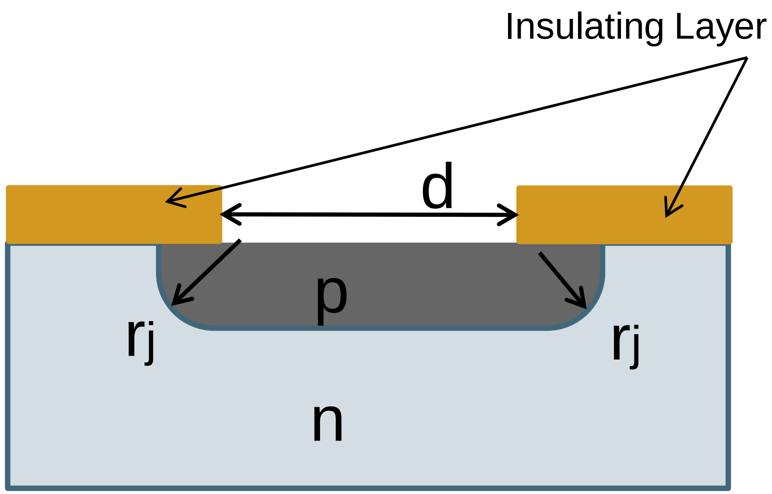
Impact-ionization: In Practice



Photon Detector



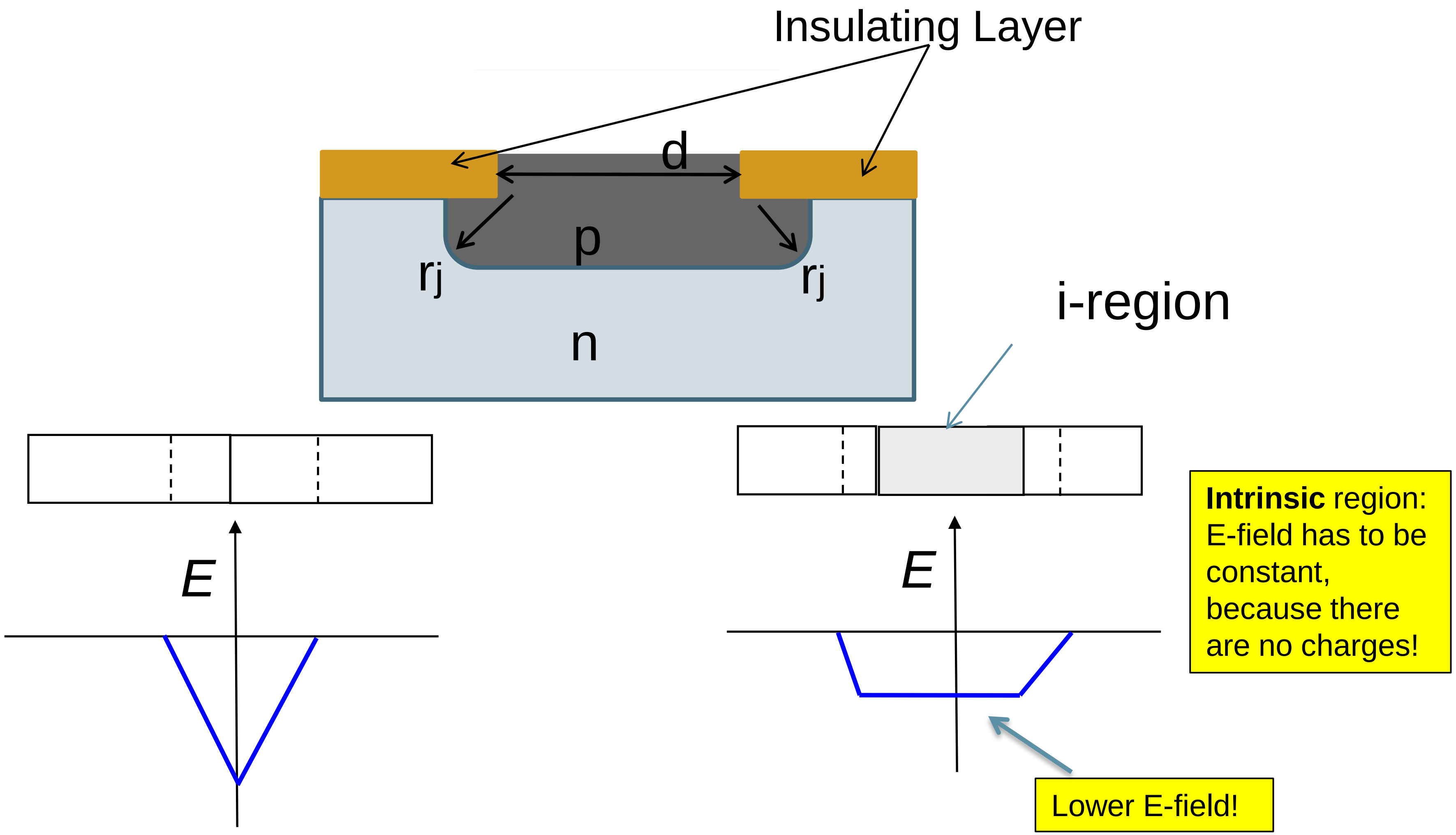
Good ….
Imagers
single photon can create an avalanche



High E-fields at junction corners → Breakdown

Bad….

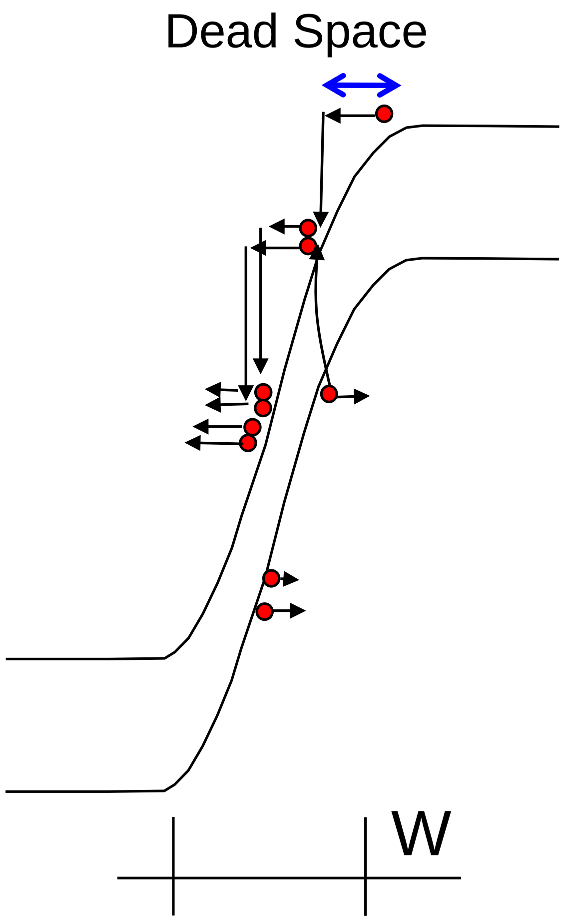
Junction Engineering



Reduced field for p-i-n junction, because V_{bi} (area under the curve) must be the same.

High practical relevance for MOSFET scaling
Needed to reduce fields on the drain side….

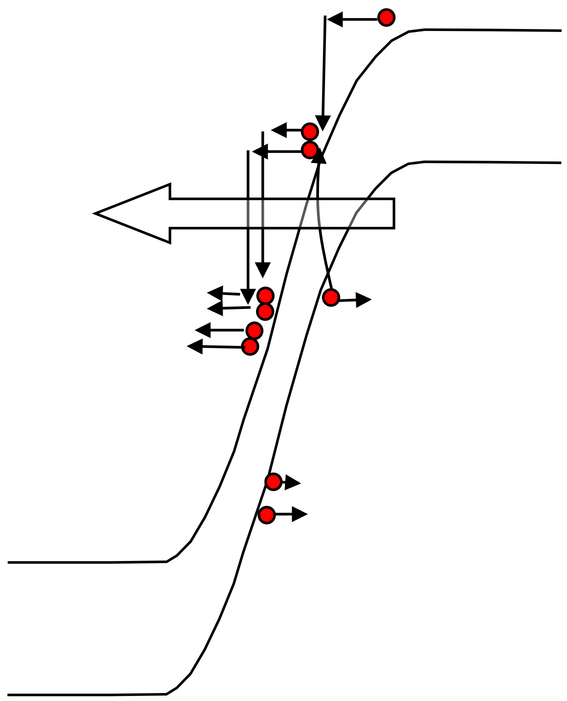
Modern Considerations: Dead Space

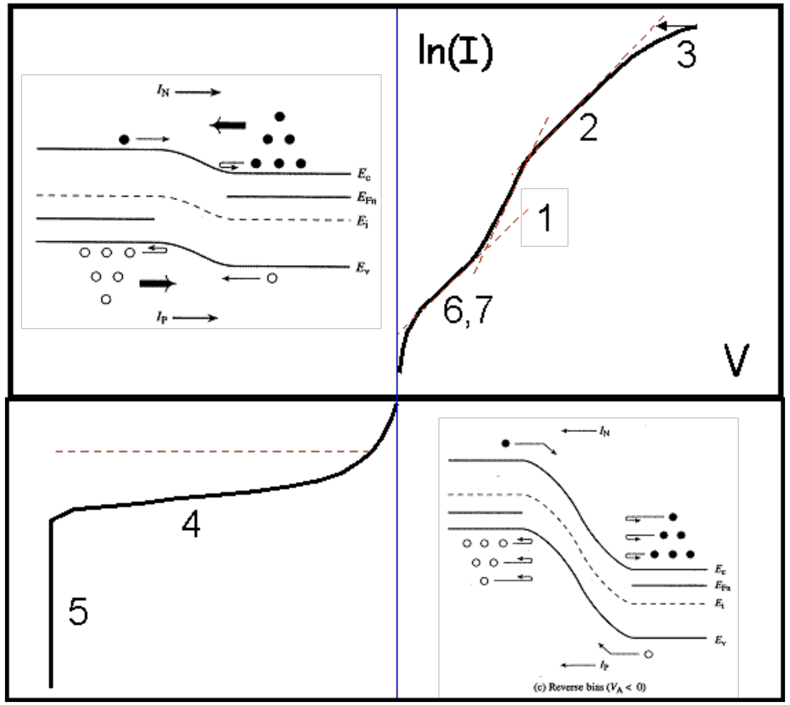


Dead Space:
Space you need before an electron can impact ionize.

For very small (ballistic) junctions, electrons can cross the junction without inducing impact Ionization.
(Dead space too small)

Zener Breakdown vs. Impact Ionization





How do you differentiate between Zener tunneling and impact-ionization?

Can happen for smaller reverse bias ~1V

Need bias > bandgap

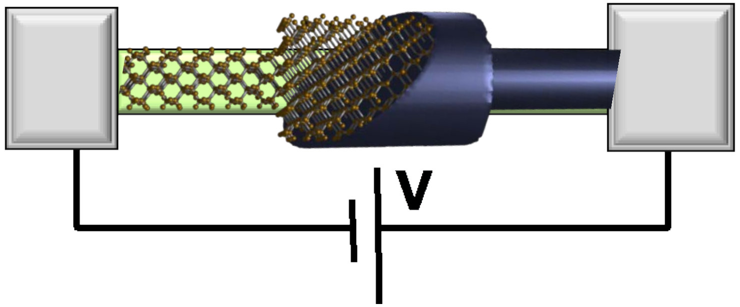
Very noisy – can be measured

Non-Ideal Effects: Conclusion

1) Junction recombination is often used as a diagnostic tool for process maturity. Defects in junction arises from misplaced donor impurities, not necessary from deep-trap impurities.

2) Impact ionization plays an important role in wide variety of devices (e.g. avalanche photo-diodes).

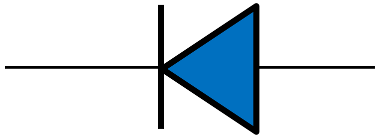
Section 20
PN Diode I-V Characteristics



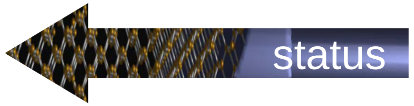
$I = G \times V$

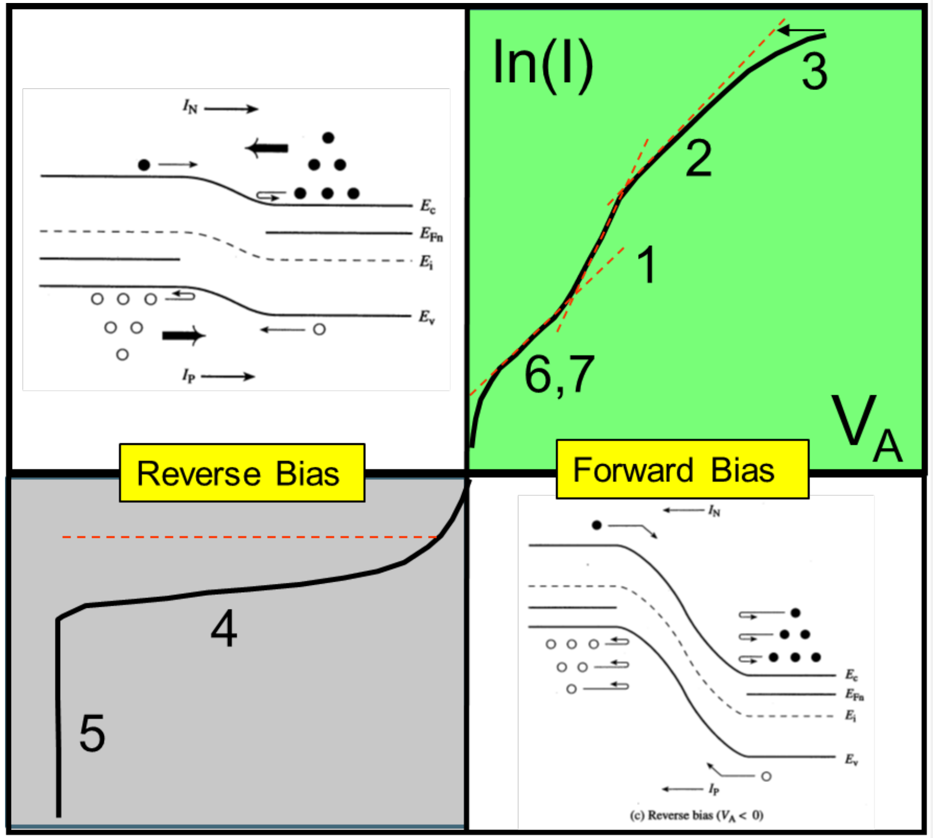
$= q \times n \times v \times A$

charge density velocity area



- 20.1 Band diagram with applied bias
- 20.2 Derivation of the forward bias formula
- 20.3 Forward Bias - Non-linear Regime
  - »Resistive drop
  - »Ambipolar regime
- 20.4 Non-ideal effects:
  - »Junction recombination
  - »Tunneling
  - »Impact ionization





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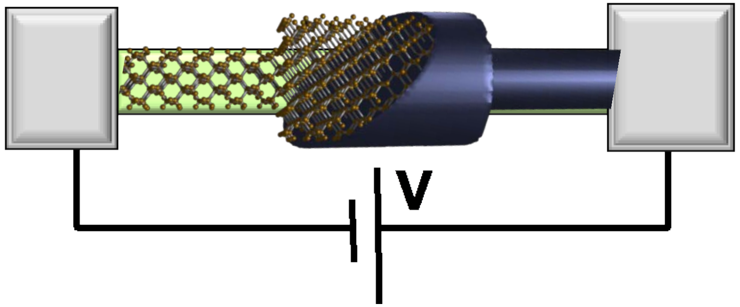

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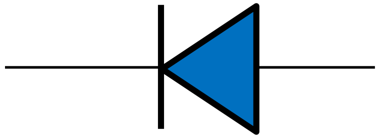

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