

## Section 20 PN Diode I-V Characteristics

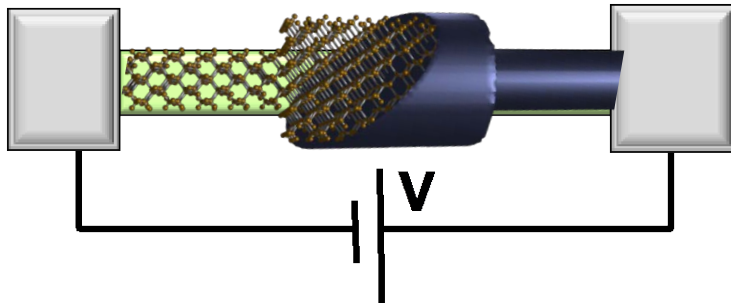
### 20.2 Derivation of the forward bias formula

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Computer Engineering

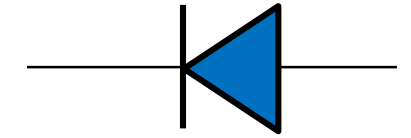
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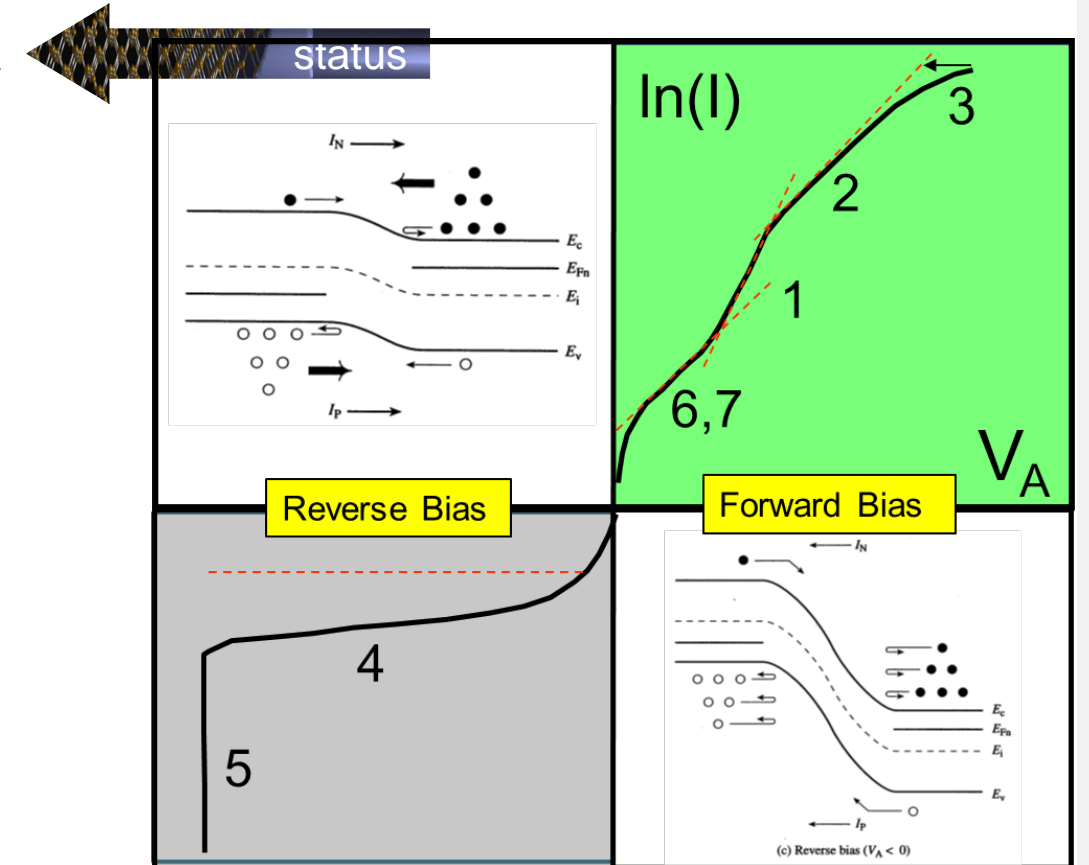
$$I = G \times V$$

$$= q \times n \times v \times A$$

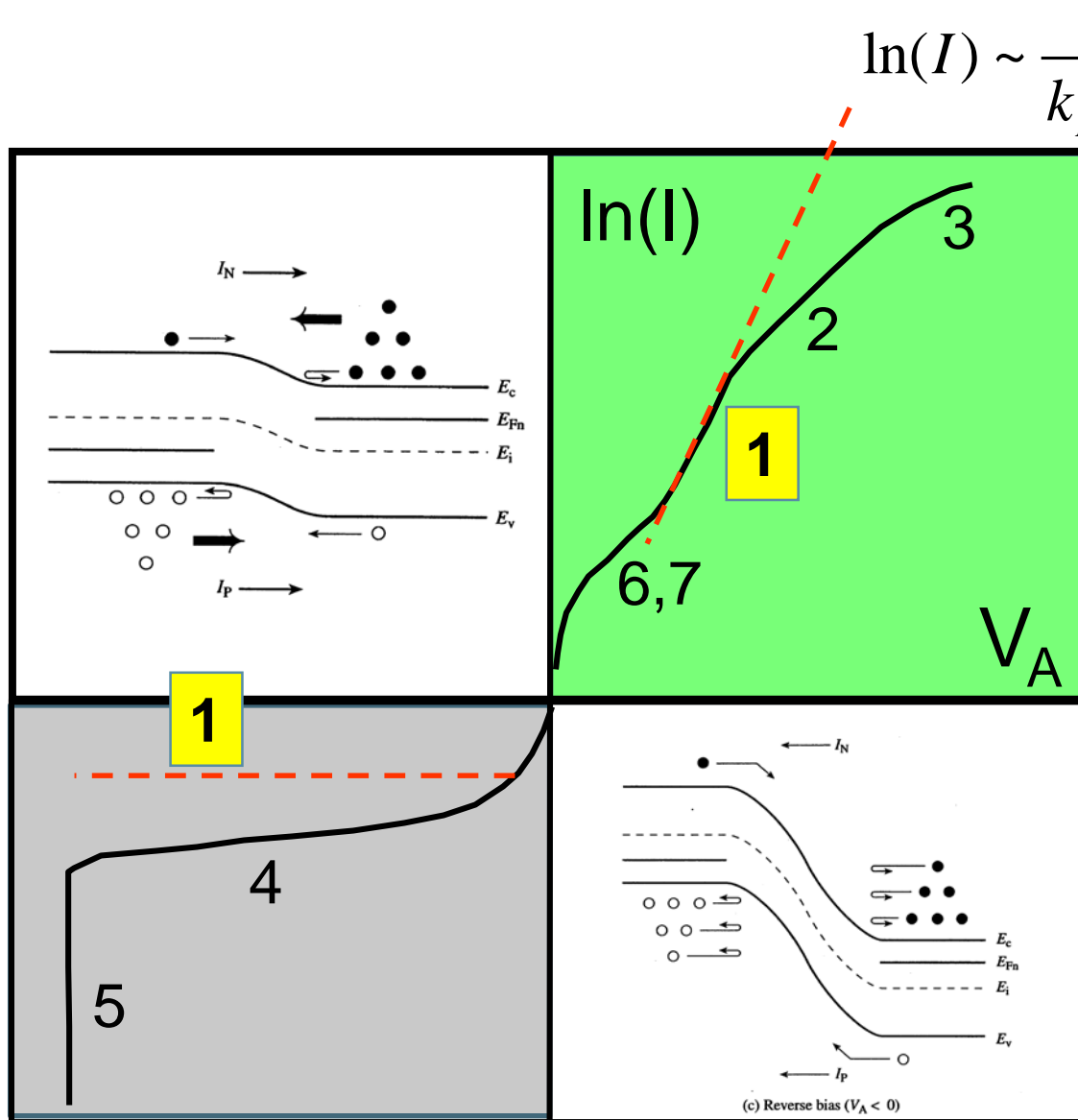
↑ charge density    ↑ velocity    ↑ area



- > • 20.1 Band diagram with applied bias
- > • 20.2 Derivation of the forward bias formula
- 20.3
- 20.4



# Various Regions of I-V Characteristics



**1. Diffusion limited**

2. Ambipolar transport

3. High injection

4. R-G in depletion

5. Breakdown

6. Trap-assisted R-G

7. Esaki Tunneling

# Recall: One Sided Minority Diffusion

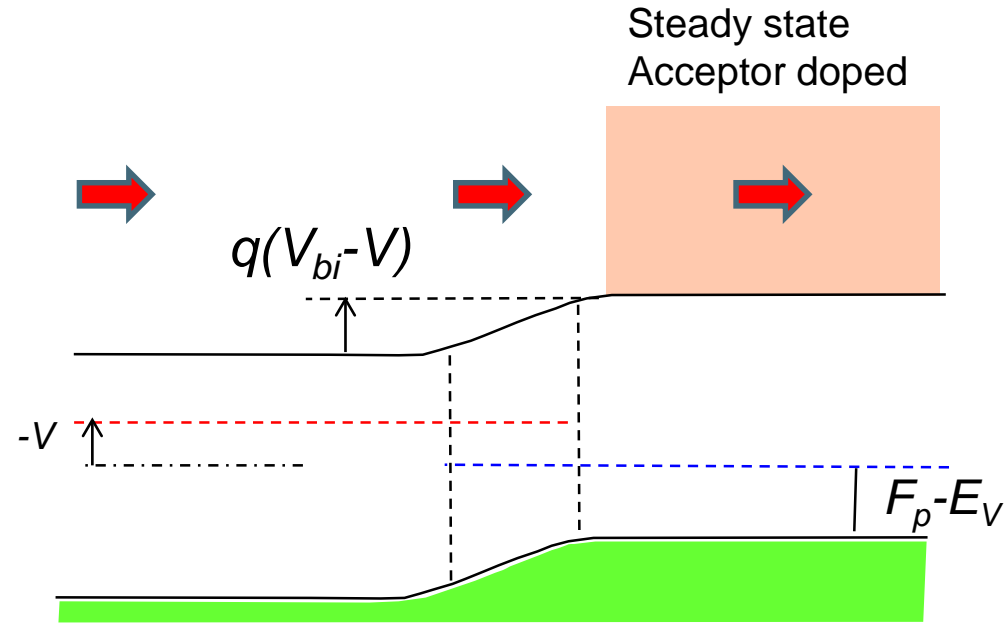
If current is continuous, one can calculate for the current anywhere. Position doesn't matter! Calculate at "easiest" position.

→ Minority carrier current on p-side.  
We know the solution from earlier  
Assume steady state, no  $r_n$ ,  $g_n$

$$\frac{\partial n}{\partial t} = \frac{1}{q} \frac{dJ_n}{dx} - r_n + g_n$$

$$J_n = qn\mu_n \mathcal{E} + qD_n \frac{dn}{dx}$$

$$0 = D_n \frac{d^2 n}{dx^2}$$



# Boundary Conditions

Boundary Conditions

$$n(x = 0^+) = n_i e^{(F_n - E_i)\beta}$$

$$p(x = 0^+) = n_i e^{-(F_p - E_i)\beta}$$

Difference of Quasi-Fermi-levels equals applied voltage

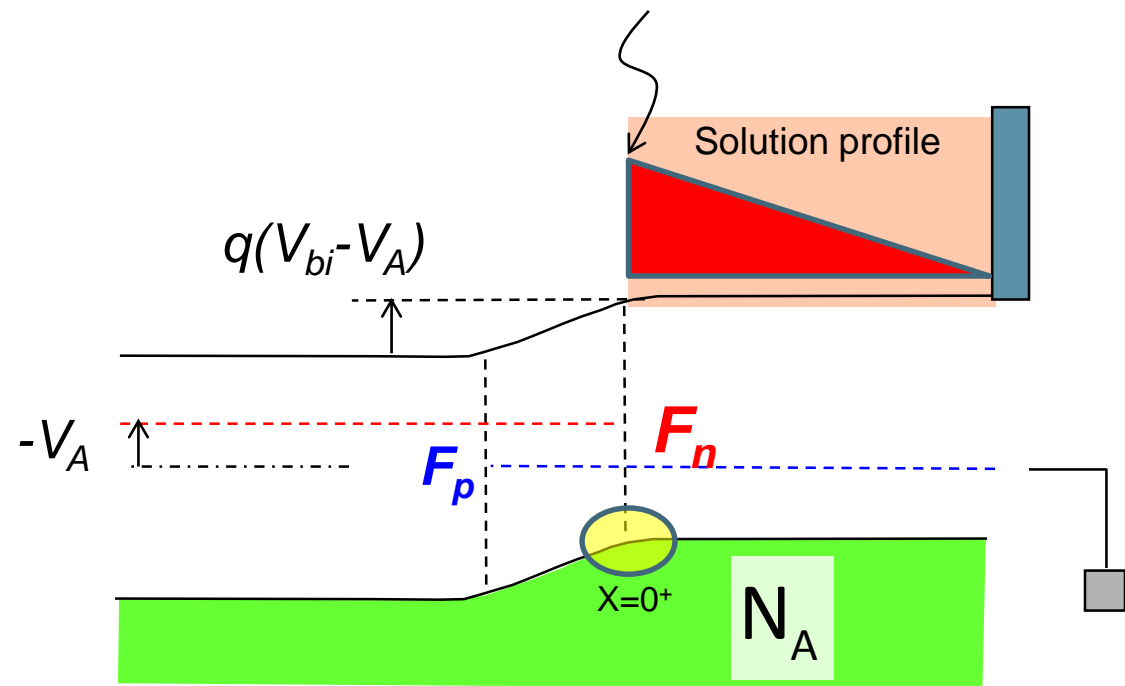
$$np = n_i^2 e^{(F_n - F_p)\beta} = n_i^2 e^{qV_A\beta}$$

$$p(0^+) = N_A$$

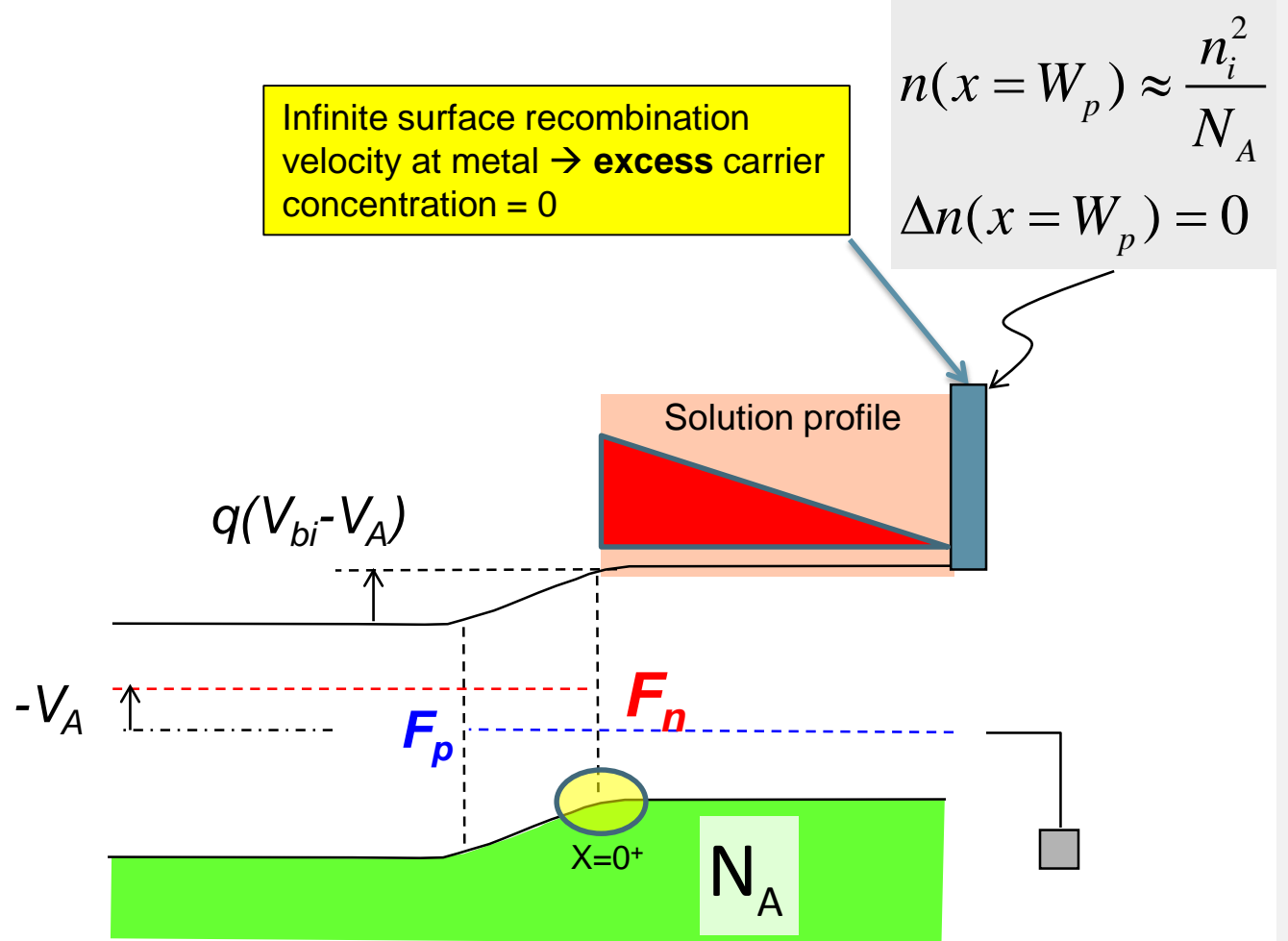
$$n(0^+) = \frac{n_i^2}{N_A} e^{qV_A\beta}$$

$$\Delta n(0^+) = n(0^+)_{V_G} - n(0^+)_{V_G=0}$$

$$= \frac{n_i^2}{N_A} (e^{qV_A\beta} - 1)$$



# Right Boundary Condition



# Example: One Sided Minority Diffusion

$$D_N \frac{d^2 n}{dx^2} = 0$$

Ansatz

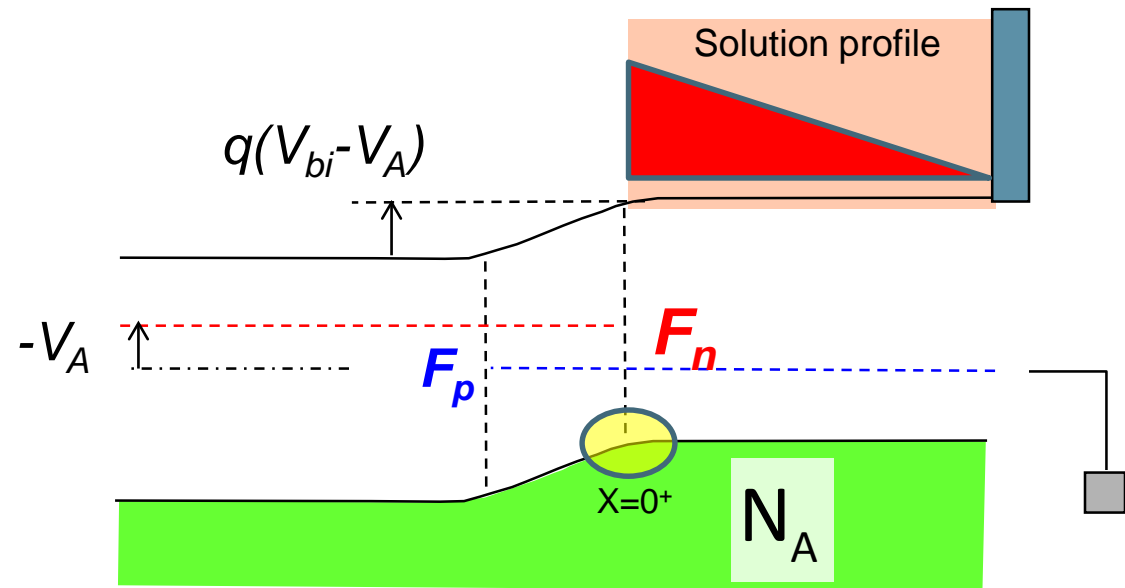
$$\Delta n(x, t) = C + Dx$$

Plug in B.C.

$$x = W_p, \quad \Delta n(x = W_p) = 0 \Rightarrow C = -DW_p$$

$$x = 0', \quad \Delta n(x = 0) = \frac{n_i^2}{N_A} \left( e^{qV_A \beta} - 1 \right) = C$$

$$\Delta n(x, t) = \frac{n_i^2}{N_A} \left( e^{qV_A \beta} - 1 \right) \left( 1 - \frac{x}{W_p} \right)$$



**Final result:** Excess electron carrier concentration (P-side) as function of position

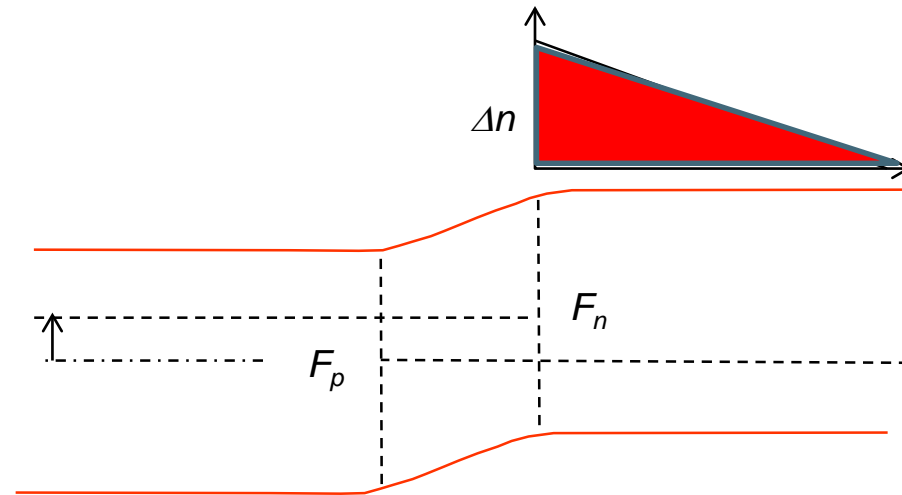
# Electron & Hole Fluxes

$$\Delta n(x) = \frac{n_i^2}{N_A} (e^{qV_{A\beta}} - 1) \left(1 - \frac{x}{W_p}\right)$$

$$\mathbf{J}_N = qn\mu_N \mathbf{E} + qD_N \nabla n$$

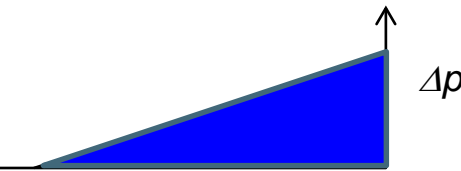
Current (electrons)

$$J_n = qD_n \left. \frac{dn}{dx} \right|_{x=0} = -\frac{qD_n n_i^2}{W_p N_A} (e^{qV_{A\beta}} - 1)$$



Current (holes)

$$J_p = -qD_p \left. \frac{dp}{dx} \right|_{x=0'} = -\frac{qD_p n_i^2}{W_n N_D} (e^{qV_{A\beta}} - 1)$$





# Total Current

## Current (electrons)

$$J_n = qD_n \left. \frac{dn}{dx} \right|_{x=0} = -\frac{qD_n n_i^2}{W_p N_A} (e^{qV_{A\beta}} - 1)$$

## Current (holes)

$$J_p = -qD_p \left. \frac{dp}{dx} \right|_{x=0'} = -\frac{qD_p n_i^2}{W_n N_D} (e^{qV_{A\beta}} - 1)$$

$$J_T = -q \left[ \frac{D_n n_i^2}{W_p N_A} + \frac{D_p n_i^2}{W_n N_D} \right] (e^{qV_{A\beta}} - 1)$$

## Forward Bias

$$\ln J_T \approx qV_A/k_B T + \ln(const.)$$

## Reverse Bias

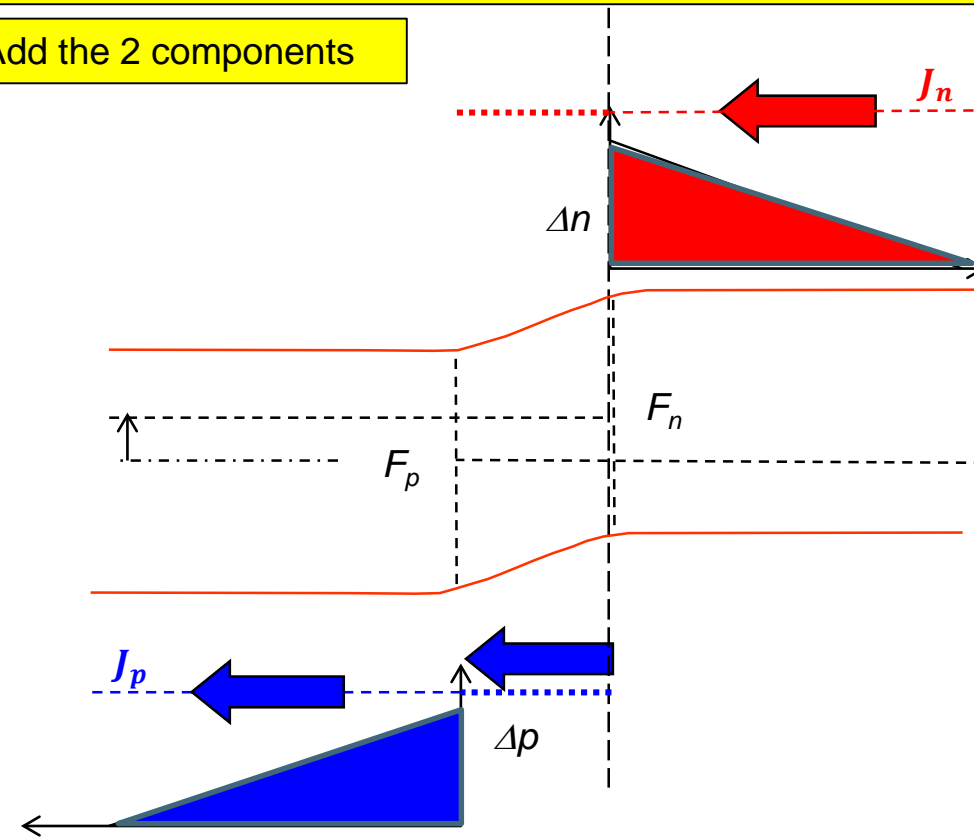
$$J_T \approx const. \quad \text{Diffusion current}$$

To get total current, add electron and hole current values at **identical** x-points.

But we have only expressions at different x-points

If there is no R-G in junction,  
=> then the diffusion currents will be continuous

=> Add the 2 components



# Ideal pn-junction current

$$J_T = -q \left[ \frac{D_n n_i^2}{W_p N_A} + \frac{D_p n_i^2}{W_n N_D} \right] (e^{qV_A/k_B T} - 1)$$

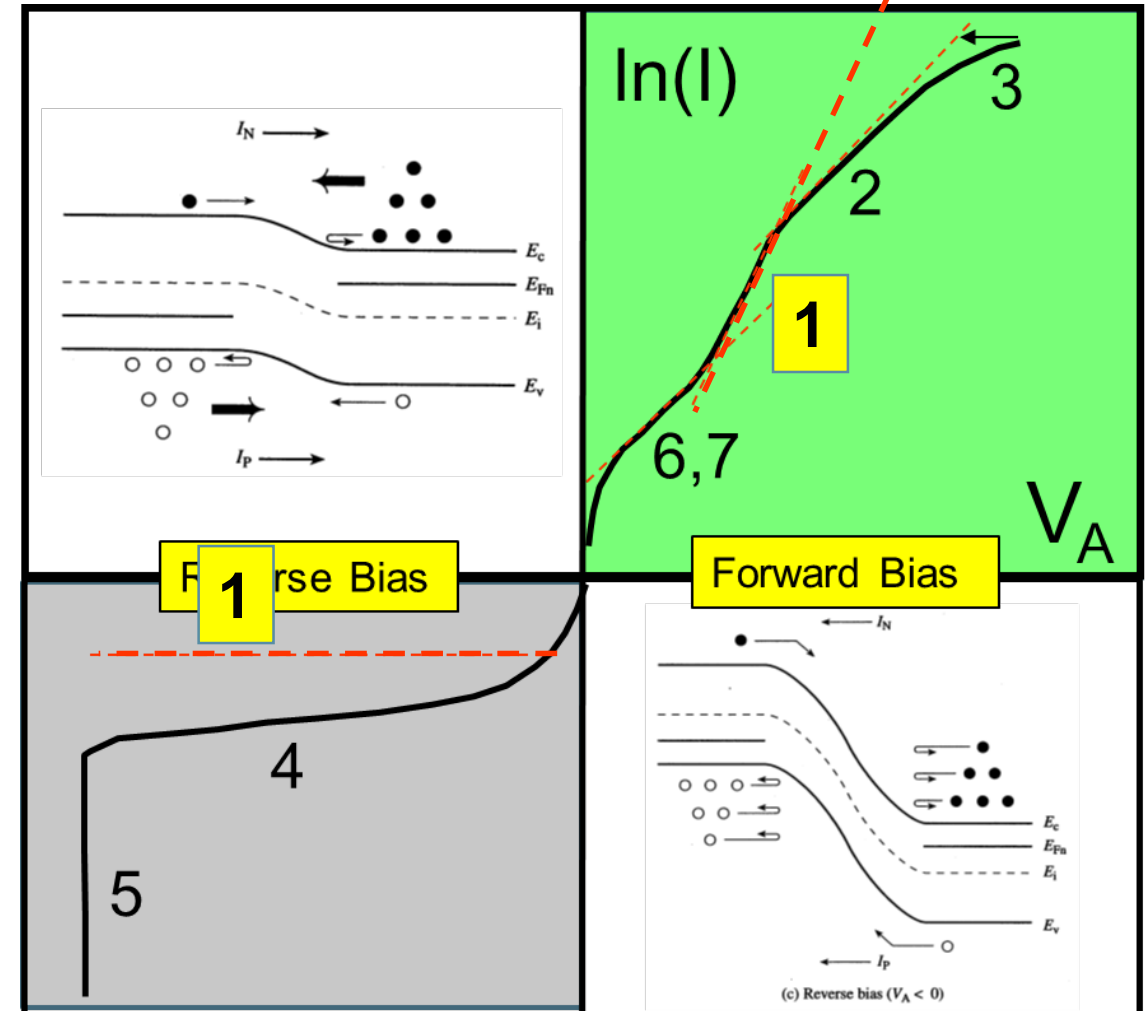
## Forward Bias

$$\ln J_T \approx qV_A/k_B T + \ln(const.)$$

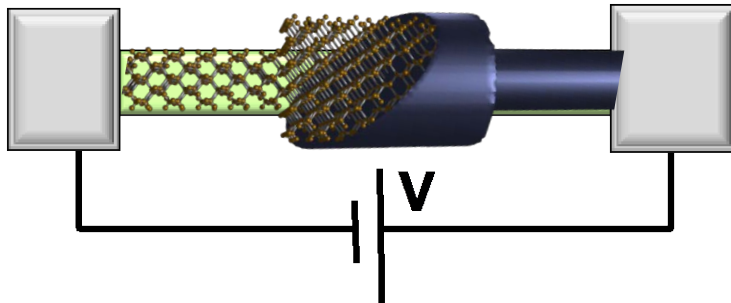
Slope = 1

## Reverse Bias

$$J_T \approx const. \quad \text{Diffusion current}$$



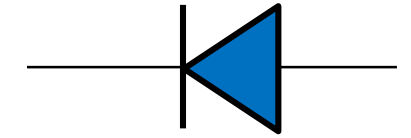
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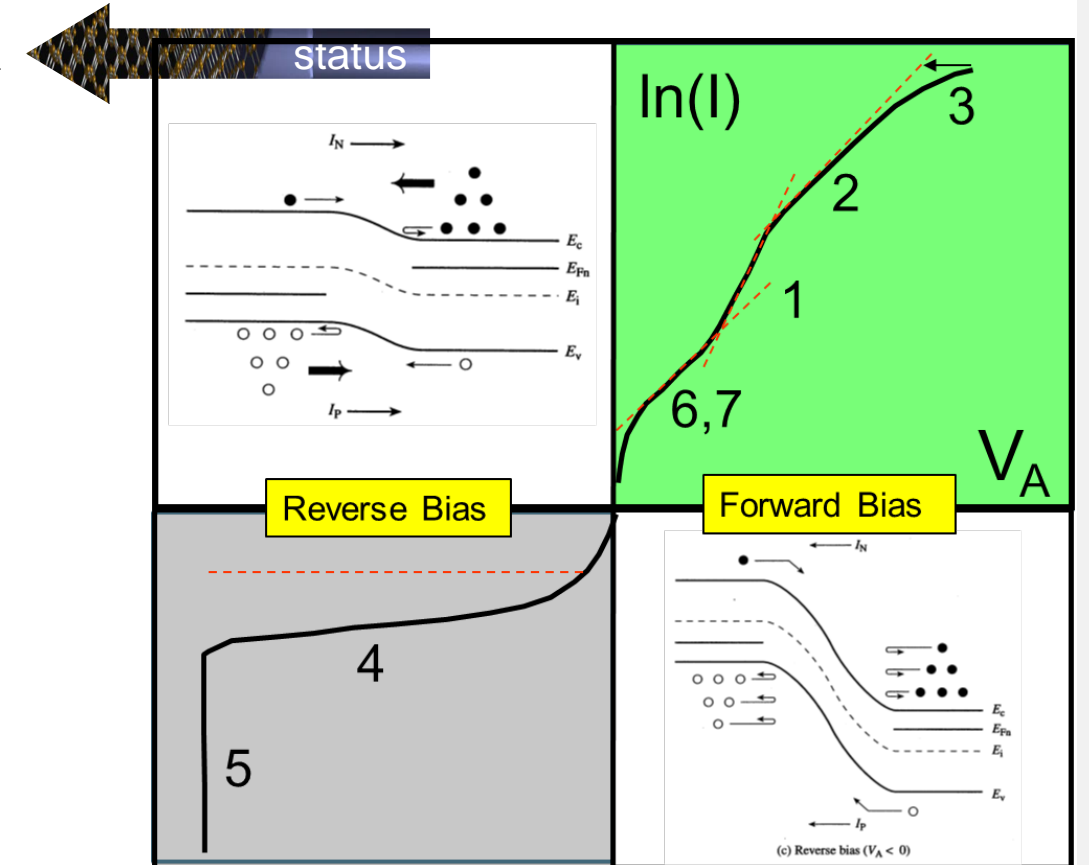
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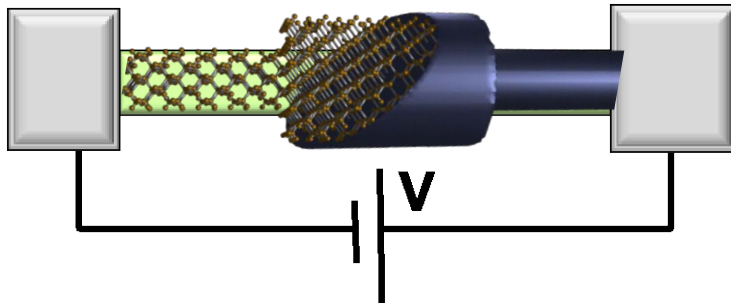
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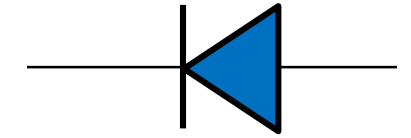
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- 20.3 Forward Bias - Non-linear Regime
  - » Resistive drop
  - » Ambipolar regime
- 20.4

