Section 19
Introduction to PN Junctions

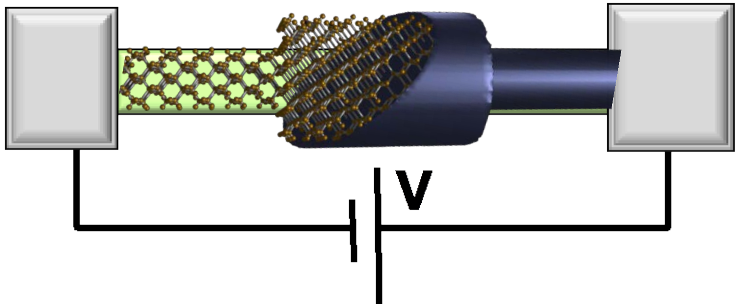
19.2 Drawing band-diagrams in equilibrium

Gerhard Klimeck
gekco@purdue.edu

PURDUE UNIVERSITY

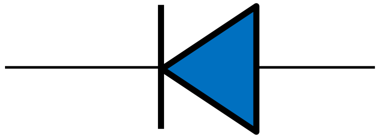
School of Electrical and Computer Engineering

Section 19
Introduction to PN Junctions



$$I = G \times V$$
$$= q \times n \times v \times A$$

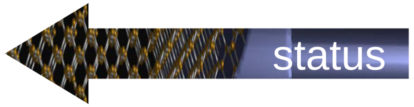
charge density, velocity, area

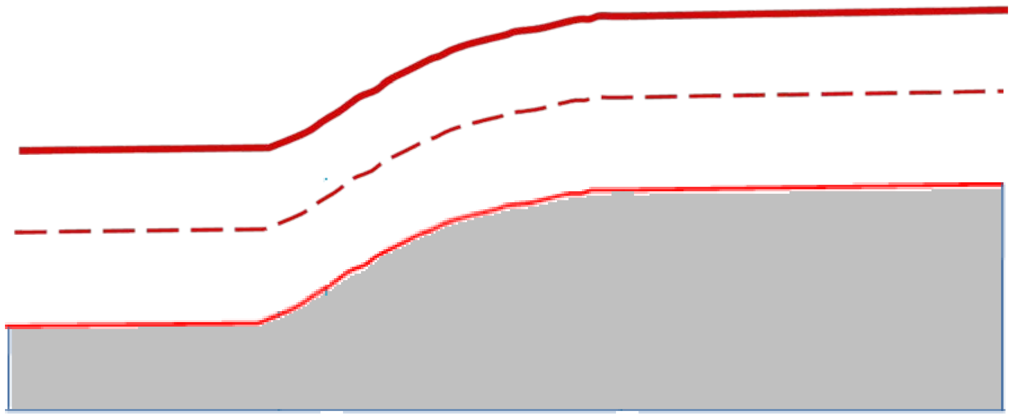


Vid

Vid

- 19.1 Structure and Depletion Region
- 19.2 Drawing band-diagrams in equilibrium





$$\nabla \bullet D = q\left(p - n + N_D^+ - N_A^-\right)$$

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$

$$\mathbf{J}_N = qn\mu_N E + qD_N \nabla n$$

$$\frac{\partial p}{\partial t} = \frac{-1}{q}\nabla \bullet \mathbf{J}_P - r_P + g_P$$

$$\mathbf{J}_P = qp\mu_P E - qD_P \nabla p$$

Drawing Band Diagram in Equilibrium...

Previously constant in homogeneous semiconductors. But for pn diode: f(x) !!

$$\nabla \bullet D = q\left(p - n + N_D^+ - N_A^-\right)$$

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$

$$\mathbf{J}_N = qn\mu_N E + qD_N \nabla n$$

$$\frac{\partial p}{\partial t} = \frac{-1}{q}\nabla \bullet \mathbf{J}_P - r_P + g_P$$

$$\mathbf{J}_P = qp\mu_P E - qD_P \nabla p$$

← **Equilibrium (Start here)**

In equilibrium J=0 (no current flow).
But, Electric fields or diffusion might still be present. → Detailed balance

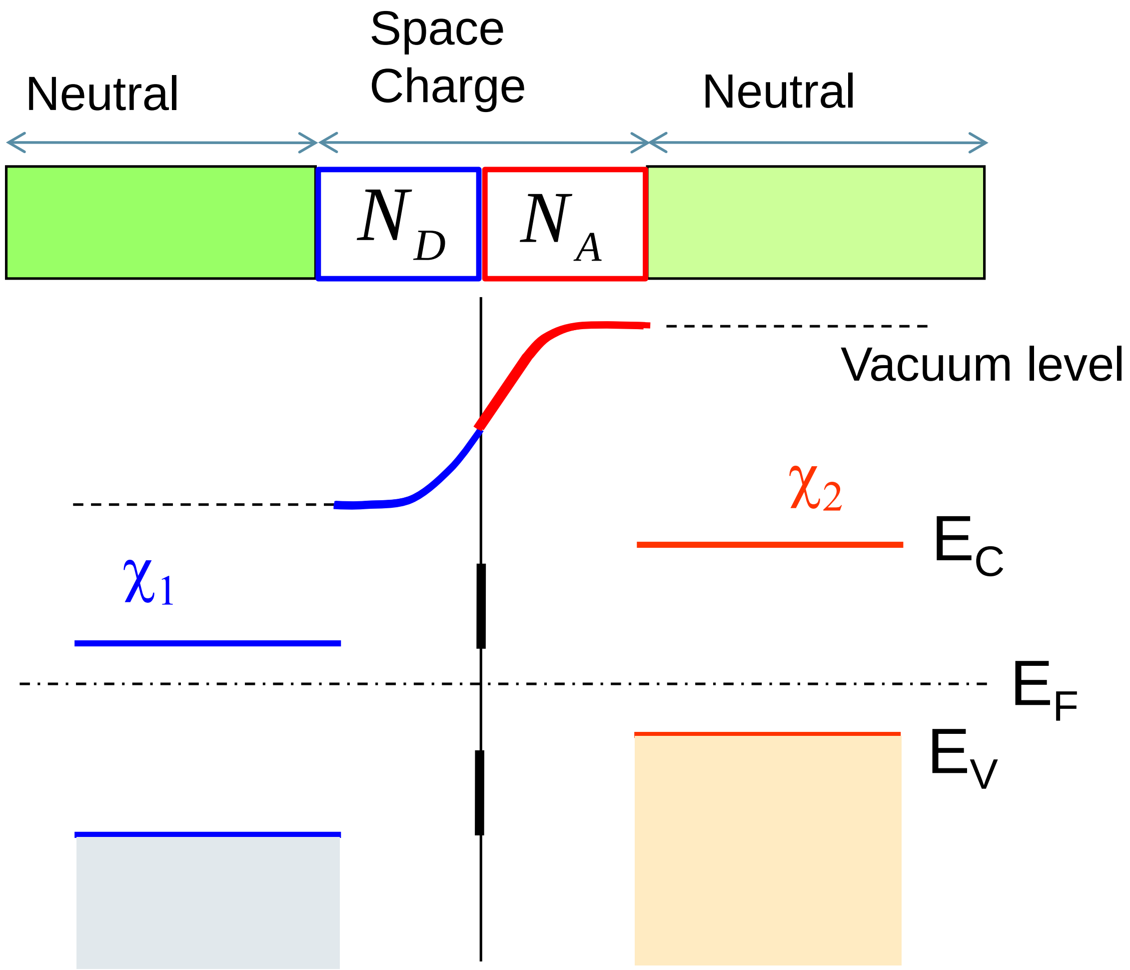
Non-Equilibrium (refine later)

DC dn/dt=0

Small signal dn/dt ~ jωt x n

Transient --- full solution

Short-cut to Band-diagram

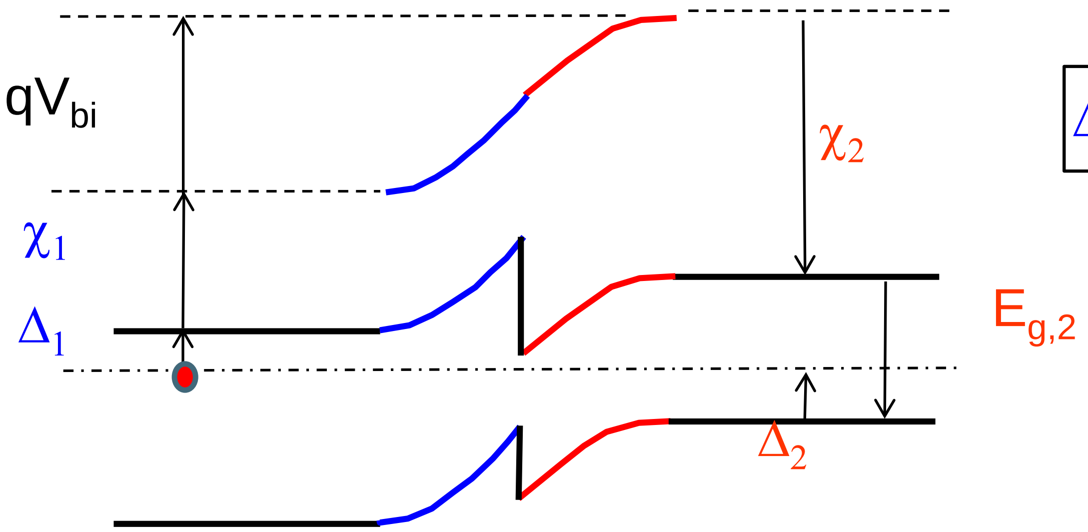


Drawing Recipe

1) Start with EF
2) Ec/Ev in bulk n-side
3) Ec/Ev in bulk p-side
4) Vacuum level in N
5) Vacuum level in P
6) Join vacuum levels
7) "Transfer" vacuum slopes to join Ec/Ev

… is equivalent to solving the Poisson equation

Built-in Potential: boundary conditions @infinity



Always true in equilibrium

$$\Delta_1 + \chi_1 + qV_{bi} = \chi_2 + E_{g,2} - \Delta_2$$

$\Delta_{1,2}$ determined via doping concentrations

$X_{1,2}$ material parameters

Built-in potential V_{bi} unknown!

$$qV_{bi} = E_{g,2} - \Delta_2 - \Delta_1 + \chi_2 - \chi_1$$

$$= \left(E_{g,2} + k_B T \ln \frac{N_A}{N_{V,2}} \right) + k_B T \ln \frac{N_D}{N_{C,1}} + \left(\chi_2 - \chi_1 \right)$$

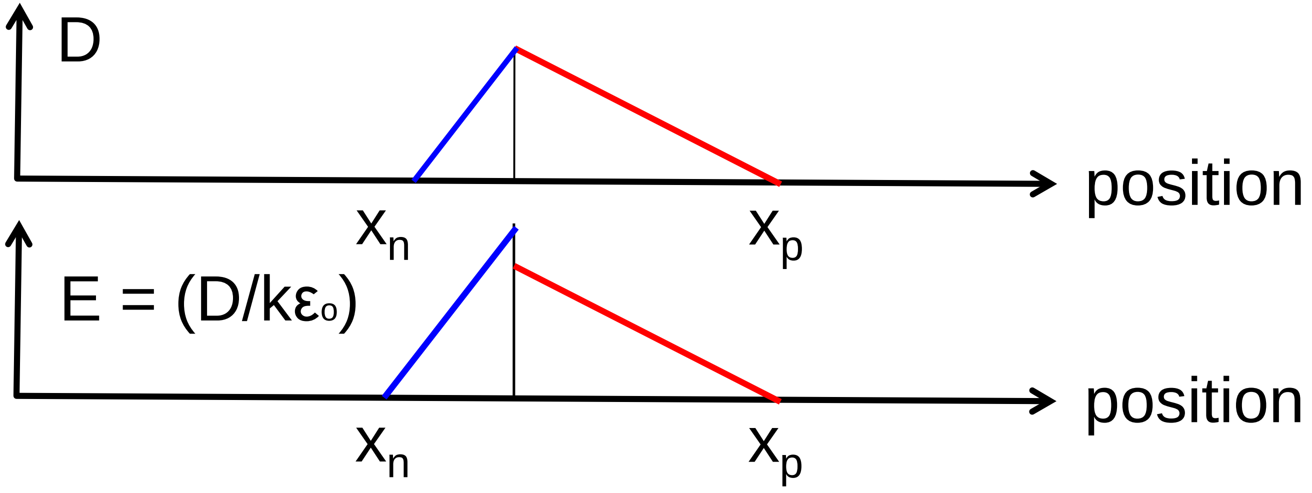
$$= k_B T \ln \frac{N_A N_D}{N_{V,2} N_{C,1} e^{-E_{g,2}/k_B T}} + \left(\chi_2 - \chi_1 \right)$$

Interface Boundary Conditions

Homo - Junction

Hetero - Junction

Field not continuous across junction

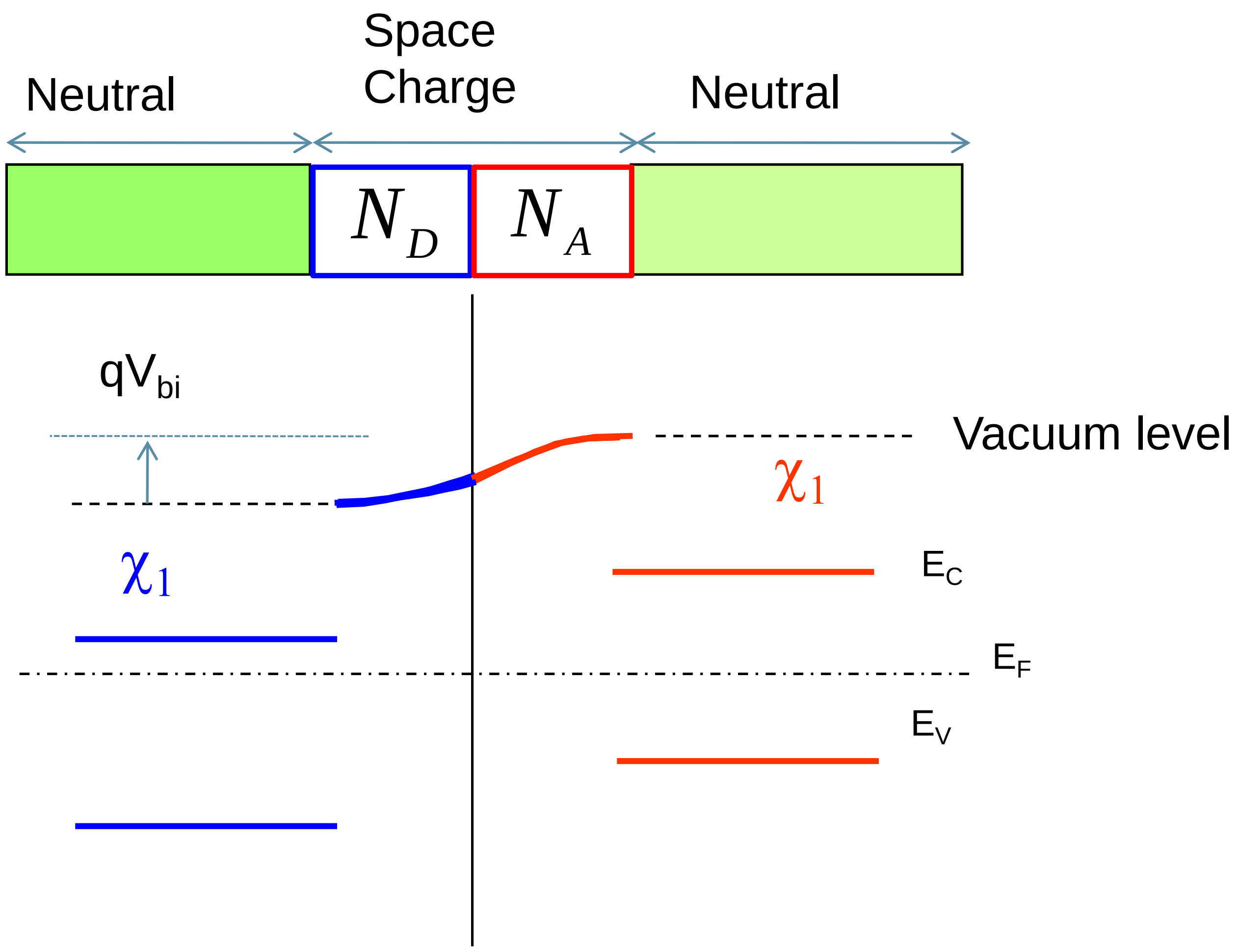


$$D_1 = K_1 \varepsilon_0 E(0^-) = K_2 \varepsilon_0 E(0^-) = D_2$$

$$E(0^-) = \frac{K_2}{K_1} E(0^+)$$

Displacement is continuous across the interface, field need not be ..

Built-in voltage for Homo-junctions



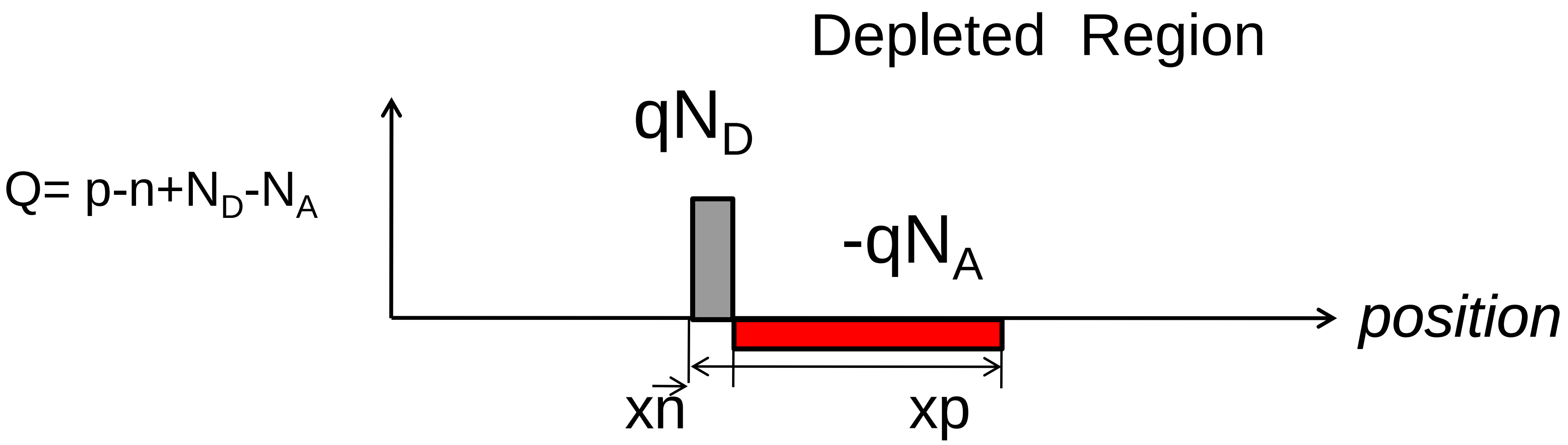
Drawing Recipe

1) Start with EF
2) Ec/Ev in bulk n-side
3) Ec/Ev in bulk p-side
4) Vacuum level in N
5) Vacuum level in P
6) Join vacuum levels
7) "Transfer" vacuum slopes to join Ec/Ev

Zero for homo-junctions

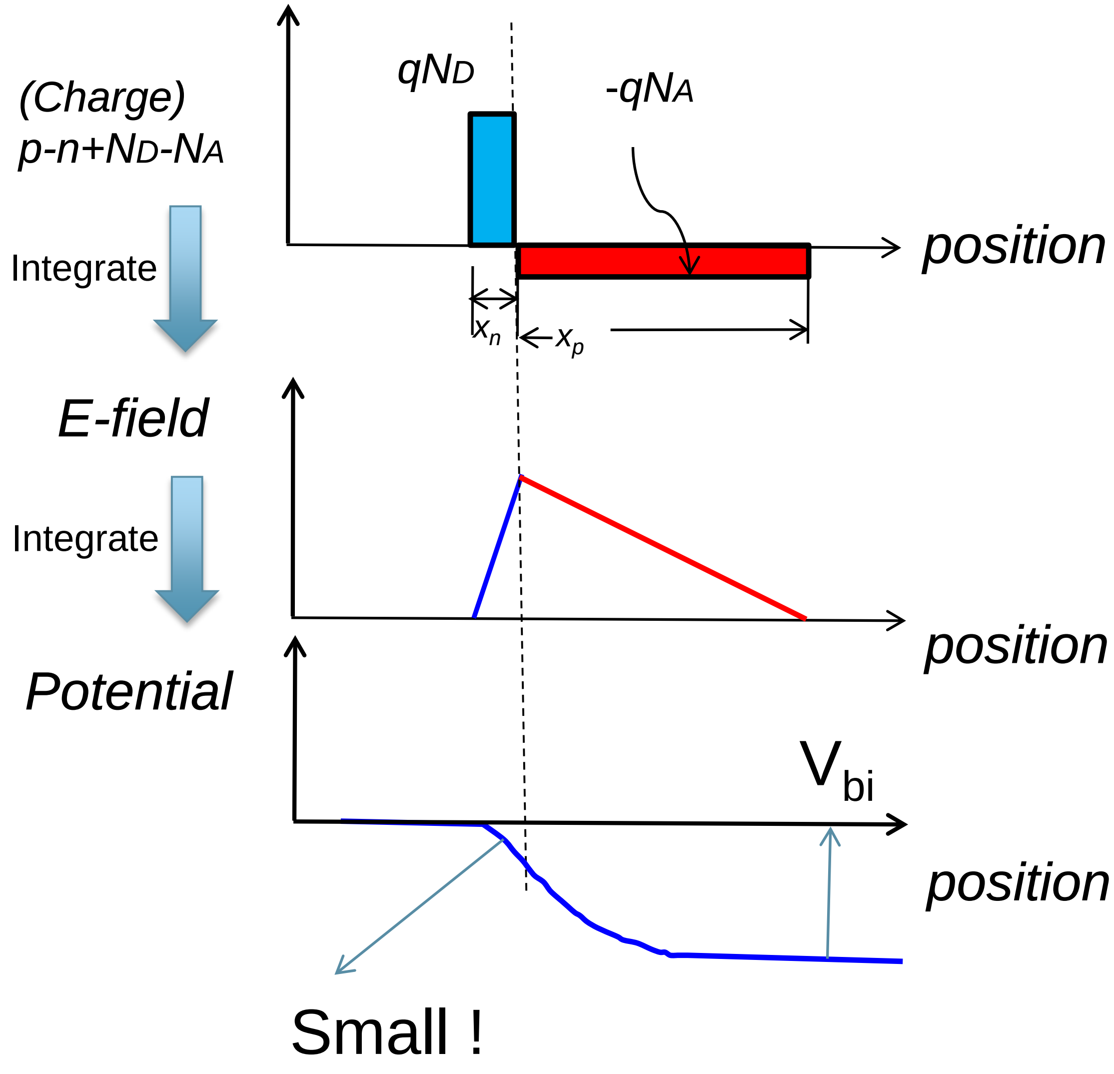
$$qV_{bi} = k_B T \ln \frac{N_A N_D}{N_{V,2} N_{C,1} e^{-E_{g,2}/k_B T}} + (\chi_2 - \chi_1) = k_B T \ln \frac{N_A N_D}{N_V N_C e^{-E_g/k_B T}} = k_B T \ln \frac{N_A N_D}{n_i^2}$$

Analytical Solution of Poisson Equation



$$K_S \varepsilon_0 \frac{d^2V}{dx^2} = -q\left(p - n + N_D^+ - N_A^-\right)$$

Analytical Solution for Homojunctions



E-field

$$E\left(0^-\right)=\frac{qN_D x_n}{k_s\varepsilon_0}$$

$$E\left(0^+\right)=\frac{qN_A x_p}{k_s\varepsilon_0}$$

$$\Rightarrow N_D x_n = N_A x_p$$

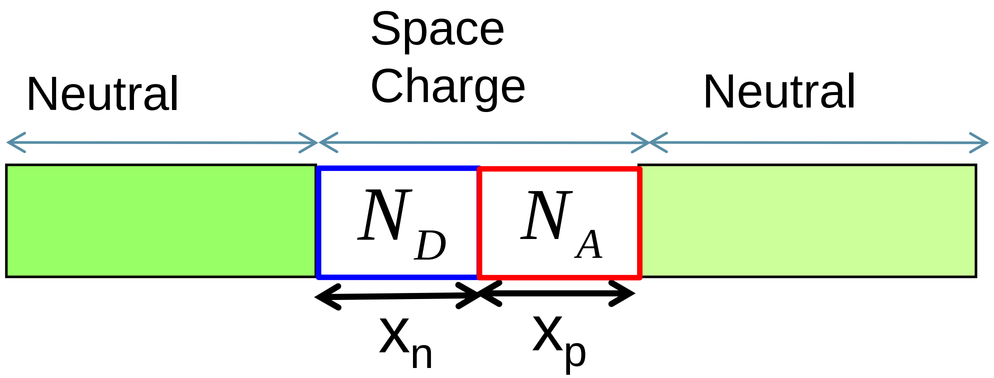
Integrate

Potential

$$qV_{bi}=\frac{E\left(0^-\right)x_n}{2}+\frac{E\left(0^+\right)x_p}{2}$$

$$=\frac{qN_D x_n^{\ 2}}{2k_s\varepsilon_0}+\frac{qN_A x_p^{\ 2}}{2k_s\varepsilon_0}$$

Depletion Regions in Homojunctions



Solve for x_n, x_p

$$N_D x_n = N_A x_p$$

$$qV_{bi} = \frac{qN_D x_n^2}{2k_s\varepsilon_0} + \frac{qN_A x_p^2}{2k_s\varepsilon_0}$$

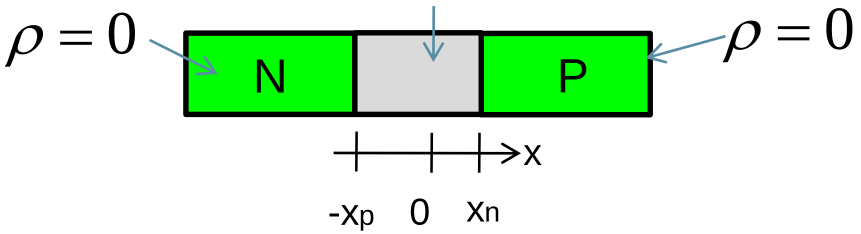
$$x_n = \sqrt{\frac{2k_s\varepsilon_0}{q}\frac{N_A}{N_D(N_A+N_D)}V_{bi}}$$

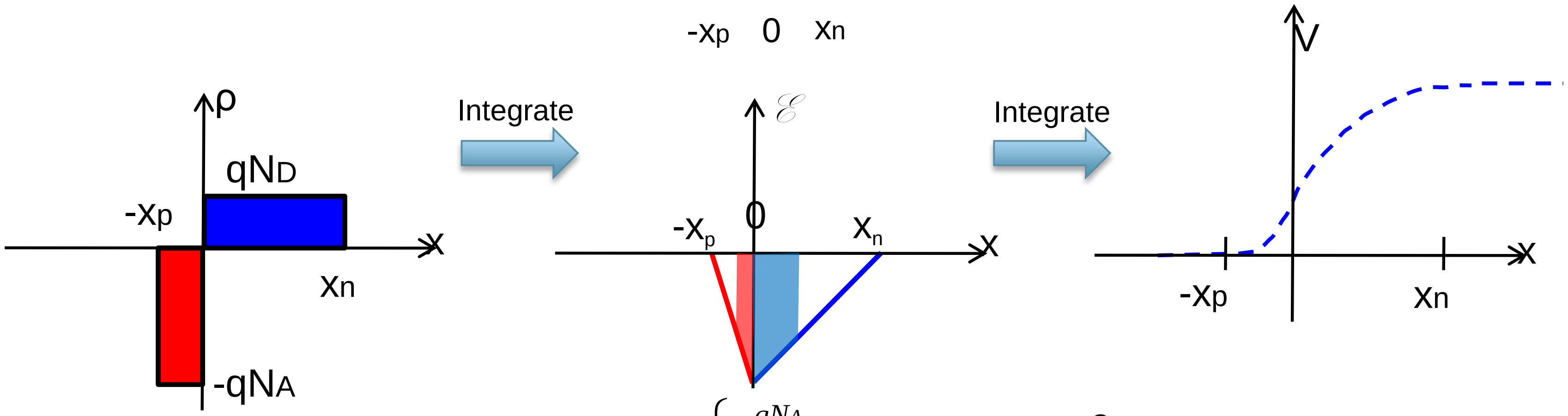
$$x_p = \sqrt{\frac{2k_s\varepsilon_0}{q}\frac{N_D}{N_A(N_A+N_D)}V_{bi}}$$

Small Project: Solve the same problem for a hetero-junction

Complete Analytical Solution

$$\rho = q(N_D - N_A)$$





If you need to calculate electric field at specific points…

$$\frac{d\mathcal{E}}{dx} = \begin{cases} \dfrac{-qN_A}{K_s\varepsilon_o} \ldots\ldots\ldots -x_p \le x \le 0 \\ \dfrac{qN_D}{K_s\varepsilon_o} \ldots\ldots\ldots 0 \le x \le x_n \\ 0 \ldots\ldots\ldots x \le -x_p,\ x \ge x_n \end{cases}$$

$$\int_0^{\mathcal{E}(x)} d\varepsilon' = -\int_{-x_p}^{x} \frac{qN_A}{K_S\varepsilon_0} dx'$$

$$\mathcal{E}(x) = -\frac{qN_A}{K_S\varepsilon_0}(x_p + x) \ldots\ldots\ldots -x_p \le x \le 0$$

$$\int_{\varepsilon(x)}^{0} d\varepsilon' = \int_{x}^{x_n} \frac{qN_D}{K_S\varepsilon_0} dx'$$

$$\mathcal{E}(x) = -\frac{qN_D}{K_S\varepsilon_0}(x_n - x) \ldots\ldots\ldots 0 \le x \le x_n$$

Summary
PN-Junction Electrostatics

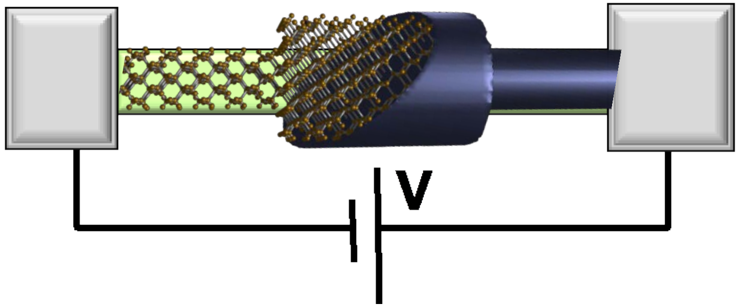
1) Learning to draw **band-diagrams** is one of the most important topics you learn in this course. Band-diagrams are a graphical way of quickly solving the Poisson equation.

2) If you consistently follow the rules of drawing band-diagrams, you will always get correct results. Try to follow the rules, **not guess** the final result.

Drawing Recipe

1) Start with EF
2) Ec/Ev in bulk n-side
3) Ec/Ev in bulk p-side
4) Vacuum level in N
5) Vacuum level in P
6) Join vacuum levels
7) "Transfer" vacuum slopes to join Ec/Ev

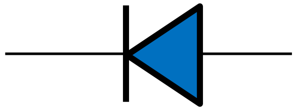
Section 19
Introduction to PN Junctions



I = G x V

= q x n x v x A

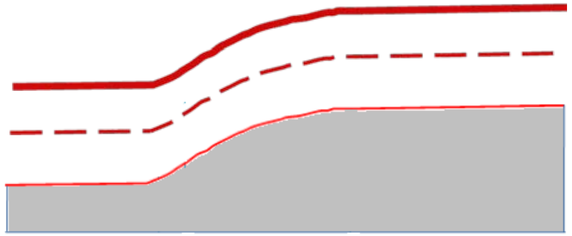
charge density velocity area



Vid

Vid

- 19.1 Structure and Depletion Region
- 19.2 Drawing band-diagrams in equilibrium



$$\nabla \bullet D = q\left(p - n + N_D^+ - N_A^-\right)$$

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$

$$\mathbf{J}_N = qn\mu_N E + qD_N \nabla n$$

$$\frac{\partial p}{\partial t} = \frac{-1}{q}\nabla \bullet \mathbf{J}_P - r_P + g_P$$

$$\mathbf{J}_P = qp\mu_P E - qD_P \nabla p$$

1) Learning to draw **band-diagrams** is one of the most important topics you learn in this course.

   Band-diagrams are a graphical way of quickly solving the Poisson equation.

2) If you consistently follow the rules of drawing band-diagrams, you will always get correct results.

   Try to follow the rules, **not guess** the final result.

Drawing Recipe

1) Start with EF
2) Ec/Ev in bulk n-side
3) Ec/Ev in bulk p-side
4) Vacuum level in N
5) Vacuum level in P
6) Join vacuum levels
7) “Transfer” vacuum slopes to join Ec/Ev