Section 18
Semiconductor Equations

18.2 Analytical Solutions (Strategy & Examples)

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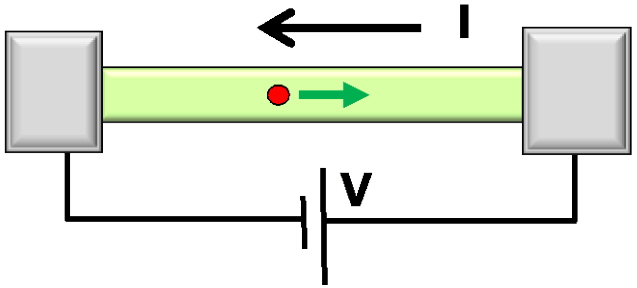
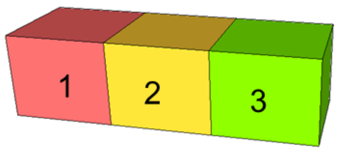


$I = G \times V$

$= q \times n \times v \times A$

charge density velocity area



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- 18.1 Continuity Equations
- 18.2 Analytical Solutions (Strategy & Examples)
- 18.3

status

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$

$$\mathbf{J}_N = qn\mu_N E + qD_N \nabla n$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla \bullet J_P + g_P - r_P$$

$$\mathbf{J}_P = qp\mu_P E - qD_P \nabla p$$

$$\nabla \bullet D = q(p - n + N_D^+ - N_A^-)$$

Analytical Solutions

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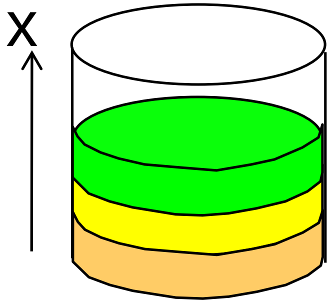
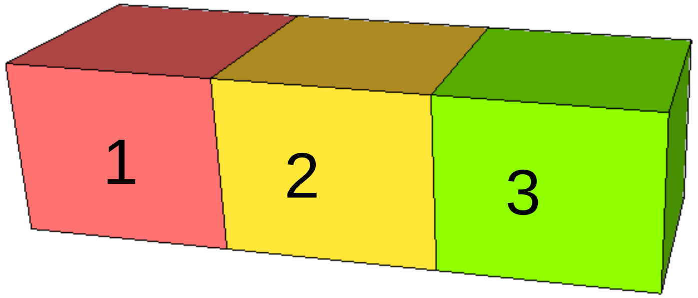
← Diffusion approximation, Minority carrier transport, Ambipolar transport

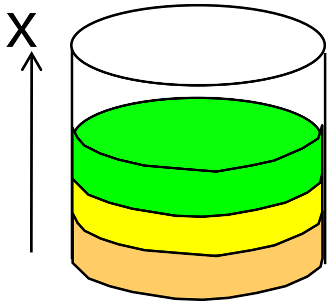
$$\nabla \bullet D = q(p - n + N_D^+ - N_A^-)$$

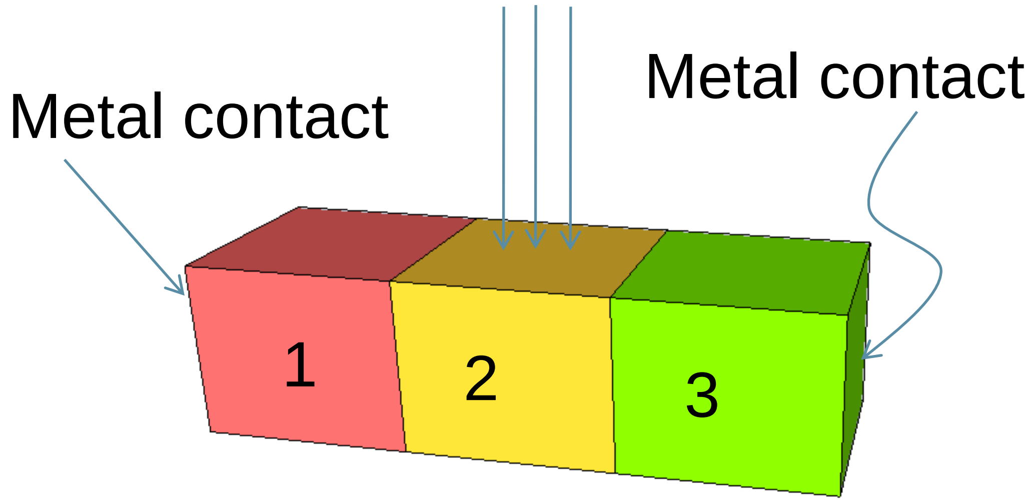
← Band-diagram





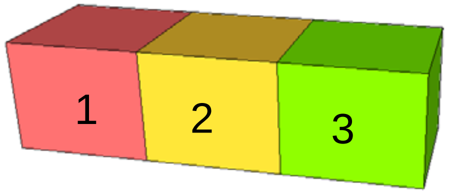
Consider a complicated real device example





- Acceptor doped
- Light turned on in the middle section.
- The right region is full of mid-gap traps
  because of dangling bonds due to unpassivated surface.
- Interface traps at the end of the right region
  (That’s where the dangling bonds are…)
- The left region is trap free.
- The left/right regions contacted by metal electrode.

Recall: Analytical Solution of Schrodinger Equation



1) $$\frac{d^2\psi}{dx^2}+k^2\psi=0$$ → 2N unknowns for N regions

2) $$\psi(x=-\infty)=0$$ $$\psi(x=+\infty)=0$$ → Reduces 2 unknowns

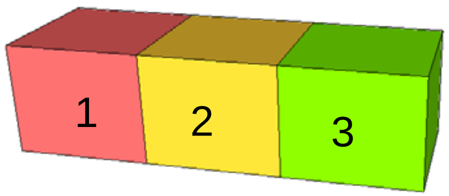
3) $$\psi\Big|_{x=x_B^-}=\psi\Big|_{x=x_B^+}$$ $$\frac{d\psi}{dx}\Big|_{x=x_B^-}=\frac{d\psi}{dx}\Big|_{x=x_B^+}$$ → Set 2N-2 equations for 2N-2 unknowns (for continuous U)

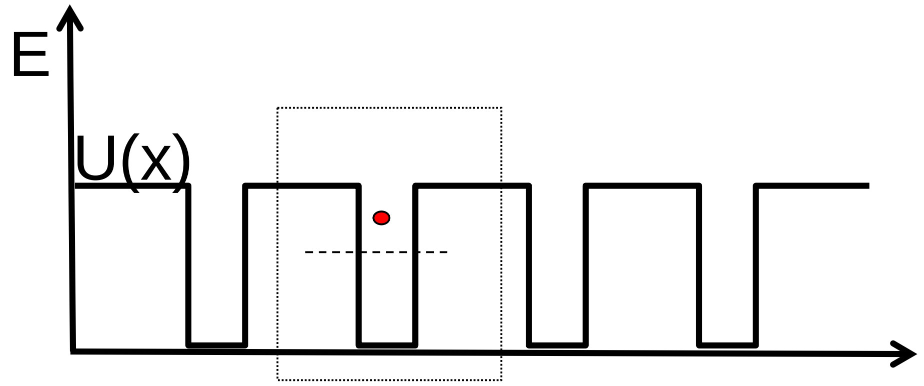
4) Det(coefficient matix)=0
And find E by graphical or numerical solution

5) $$\int_{-\infty}^{\infty}\left|\psi(x,E)\right|^2 dx=1$$

for wave function

Recall: Bound-levels in Finite well



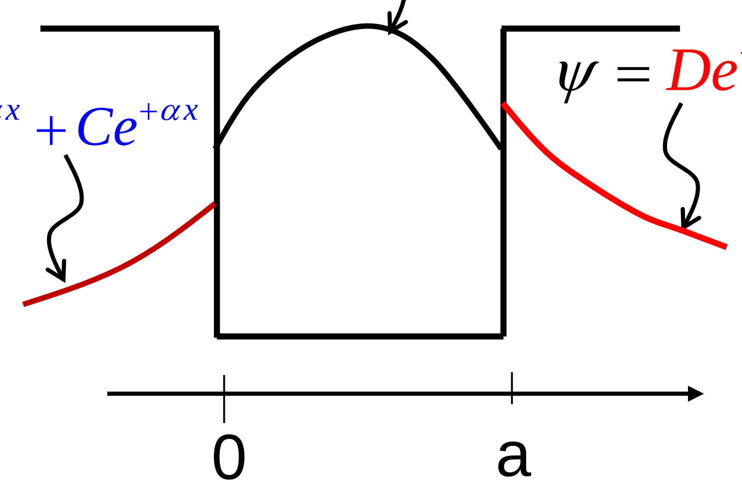


1)

$$\psi = A\sin kx + B\cos kx$$

$$\psi = Me^{-\alpha x} + Ce^{+\alpha x}$$

$$\psi = De^{-\alpha x} + Ne^{+\alpha x}$$



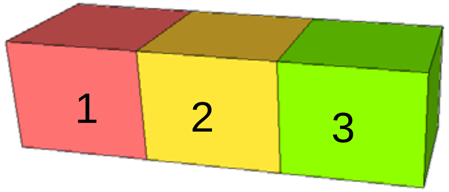
2) Boundary Conditions …

$$\psi(x = -\infty) = 0$$

$$\psi(x = +\infty) = 0$$

Analogously, we solve for our device

Solve the equations in different regions independently.



Bring them together by applying boundary conditions.

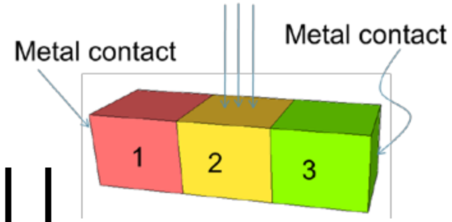
Region 2: Transient, Uniform Illumination, Uniform doping

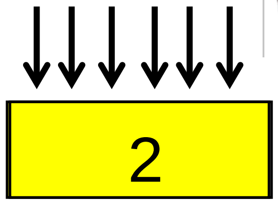
(uniform)

$$\frac{\partial n}{\partial t} = \frac{1}{q}\cancel{\nabla} \bullet \mathbf{J}_N - r_N + g_N \qquad \mathbf{J}_N = qn\mu_N E + qD_N \nabla n$$

$$\frac{\partial(\cancel{n_0} + \Delta n)}{\partial t} = -\frac{\Delta n}{\tau_n} + G$$

Recall Shockley-Read-Hall





Acceptor doped

(uniform)

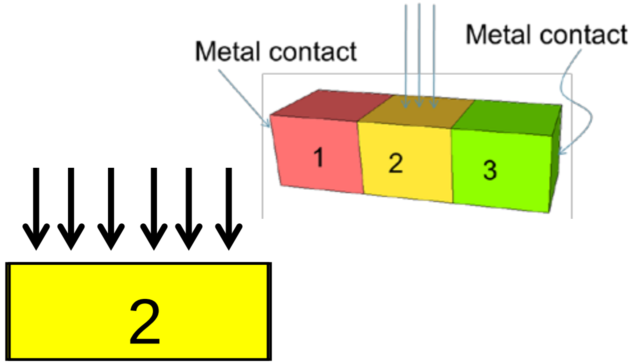
$$\frac{\partial p}{\partial t} = \frac{-1}{q}\cancel{\nabla} \bullet \mathbf{J}_p - r_P + g_p \qquad \mathbf{J}_p = qp\mu_p E - qD_P \nabla p$$

Electric field still zero because new carriers balance

$$\nabla \bullet D = q(p - n + N_D^+ - N_A^-) = q(p_0 + \Delta p - n_0 - \Delta n + N_D^+ - N_A^-) = 0$$

$$\frac{\partial(\not{n}_0+\Delta n)}{\partial t}=-\frac{\Delta n}{\tau_n}+G$$

$$\frac{\partial(\Delta n)}{\partial t}=-\frac{\Delta n}{\tau_n}+G$$



Acceptor doped

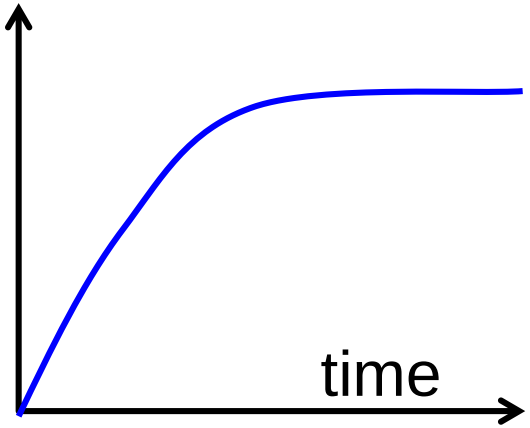
$$\Delta n(x,t)=A+Be^{-t/\tau_n}$$

No carriers yet generated…

$$t=0,\quad \Delta n(x,0)=0\Rightarrow A=-B$$

$$t\to\infty,\quad \Delta n(x,\infty)=G\tau_n=A$$

Steady state, no change in carriers with time…



$$\Delta n(x,t)=G\tau_n\left(1-e^{-t/\tau_n}\right)$$

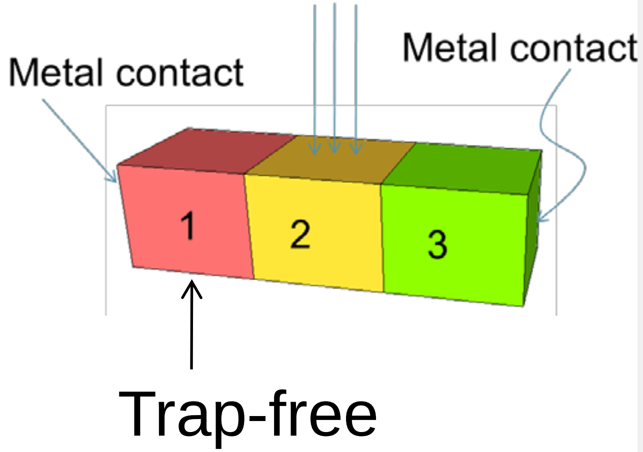
Region 1:
One sided Minority Diffusion at steady state

$$\frac{\partial n}{\partial t} = 0(\text{steady-state})$$

$$r_N = 0(\text{trap free})$$

$$g_N = 0(\text{nogeneration})$$

Steady state
Acceptor doped

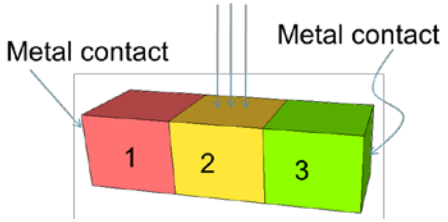


$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{dJ_n}{dx} - r_N + g_N$$

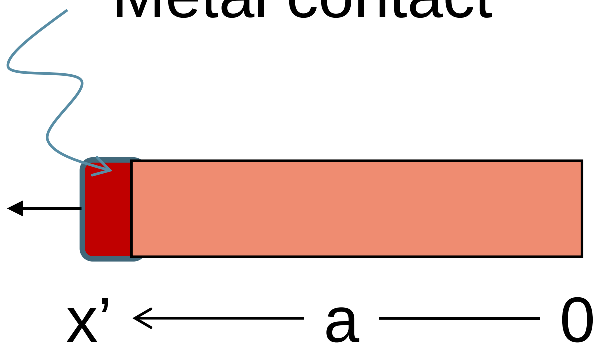
$$E = 0 \qquad \mathbf{J}_N = qn\mu_N E + qD_N \frac{dn}{dx}$$

$$0 = D_N \frac{d^2 n}{dx^2}$$

$$D_N \frac{dn}{dx} \neq 0 \text{ (due to insertion of electrons from central region)}$$



$$0 = D_N \frac{d^2 n}{dx^2}$$



$$\Delta n(x,t) = C + Dx'$$

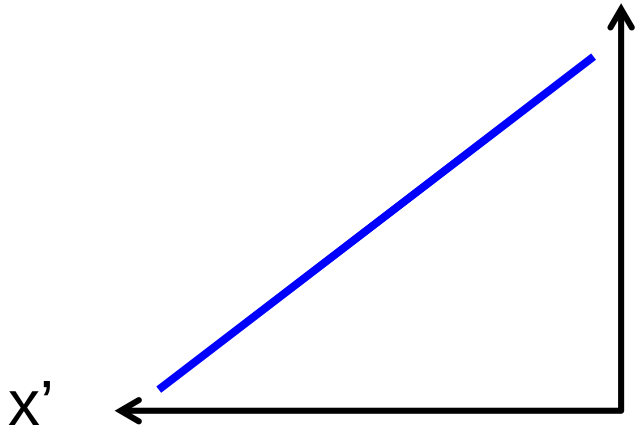
$$x = a, \quad \Delta n(x' = a) = 0 \Rightarrow C = -Da$$

(Metal has high electron density as compared to semiconductor)

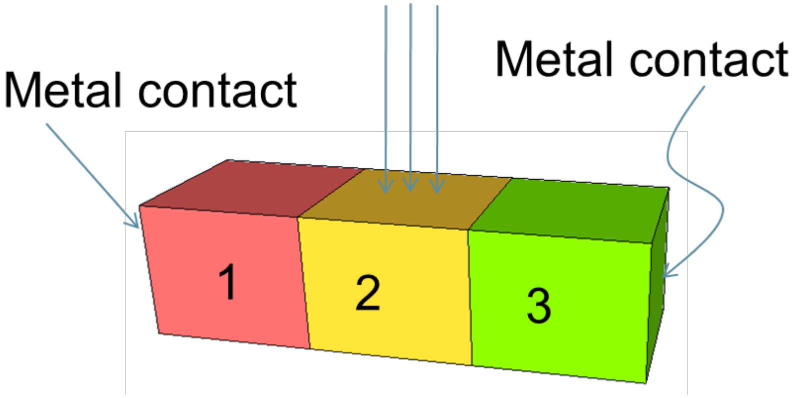
$$x = 0', \quad \Delta n(x' = 0') = C$$

Just substitute x=0 in above eqn.

$$\Delta n(x,t) = \Delta n(x = 0')\left(1 - \frac{x'}{a}\right)$$



Region 3: Steady state Minority Diffusion with recombination



$$\frac{\partial n}{\partial t} = 0 (\text{steady-state})$$

$$r_N \neq 0 (\text{not trap free})$$

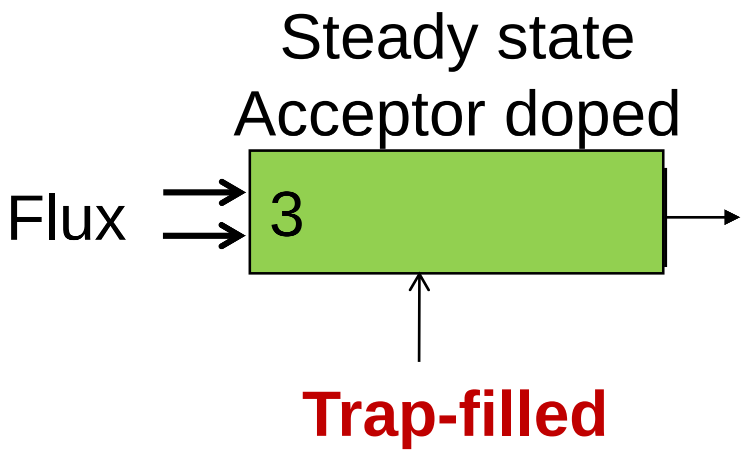
$$g_N = 0 (\text{nogeneration})$$

$$E = 0$$

$$D_N \frac{dn}{dx} \neq 0 \text{ (due to insertion of electrons from central region)}$$

$$0 = D_N \frac{d^2(n_0 + \Delta n)}{dx^2} - \frac{\Delta n}{\tau_n}$$

$$0 = D_N \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n}$$



$$D_N \frac{d^2\Delta n}{dx^2} - \frac{\Delta n}{\tau_n} = 0$$

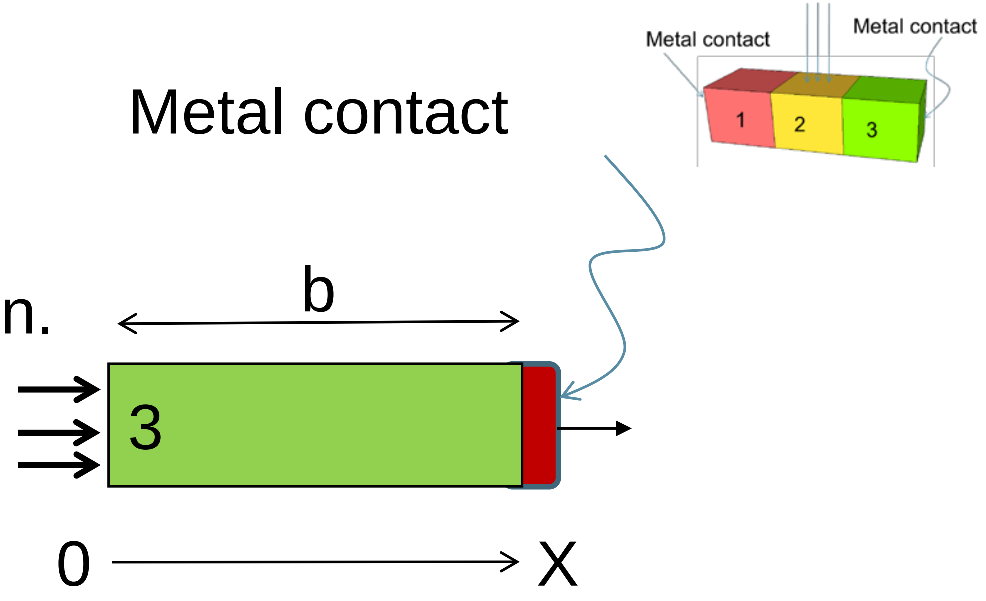
Functionally similar to Schrodinger eqn.

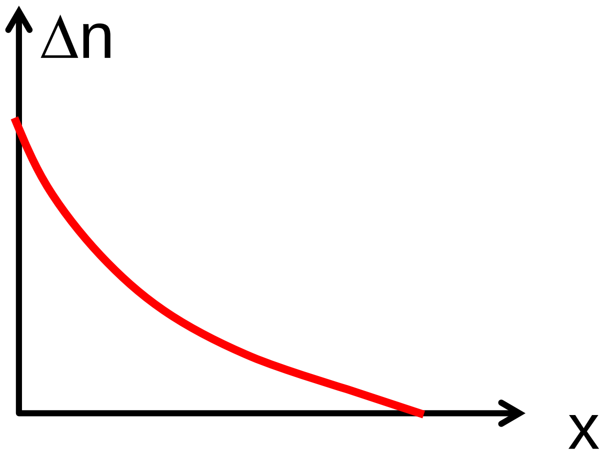
$$\Delta n(x,t) = E e^{x/L_n} + F e^{-x/L_n}$$

$$x = b, \quad \Delta n(x=b) = 0 \quad \Rightarrow F = -E e^{2b/L_n}$$

$$x = 0, \quad \Delta n(x=0) = E + F = \Delta n(x=0)$$

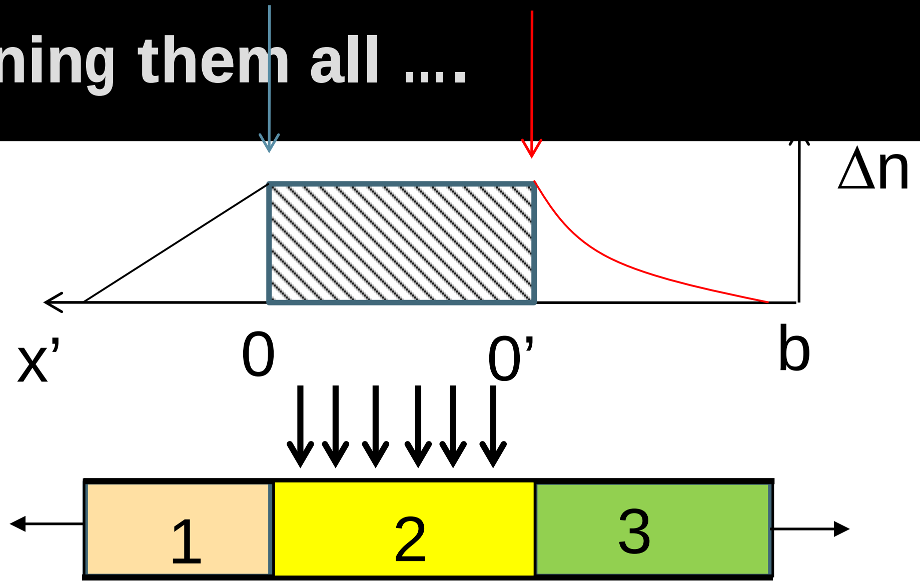
$$\Delta n(x,t) = \frac{\Delta n(0)}{(1-e^{2b/L_n})}\left(e^{x/L_n} - e^{2b/L_n} e^{-x/L_n}\right)$$

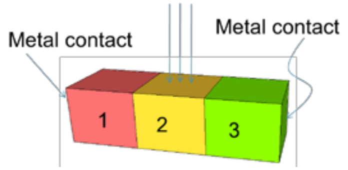




$$\Delta n_2(x) = G\tau_n =$$

$$\Delta n_2(0) = \Delta n_2(0')$$





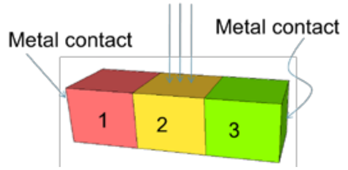
Match boundary condition

$$\Delta n_1(x') = \Delta n(x=0)\left(1-\frac{x'}{a}\right) = G\tau_n\left(1-\frac{x'}{a}\right)$$

$$\Delta n(x) = \frac{\Delta n(0')}{(1-e^{2b/L_n})}\left(e^{x/L_n} - e^{2b/L_n}e^{-x/L_n}\right) = \frac{G\tau_n\left(e^{x/L_n} - e^{2b/L_n}e^{-x/L_n}\right)}{(1-e^{2b/L_n})}$$

Calculating current $\mathbf{J}_N = qn\mu_N E + qD_N \frac{dn}{dx}$

Analytical Solutions Summary



1) Continuity Equations form the basis of analysis of all the devices we will study in this course.

2) Full numerical solution of the equations are possible and many commercial software are available to do so.

3) Analytical solutions however provide a great deal of insight into the key physical mechanism involved in the operation of a device.

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$

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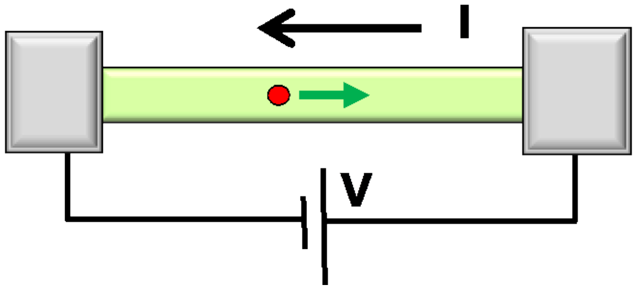
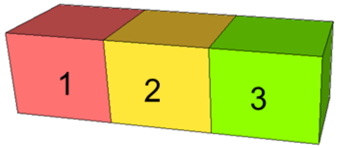


I = G x V

= q x n x v x A

charge density velocity area



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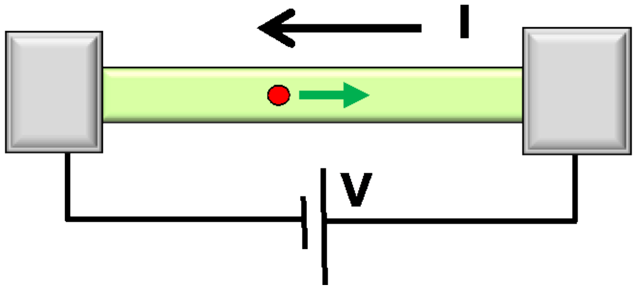

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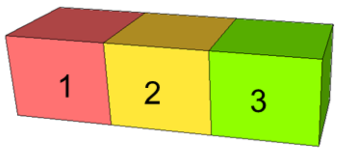
$$= q \times n \times v \times A$$

charge density velocity area



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- 18.1 Continuity Equations
- 18.2 Analytical Solutions (Strategy & Examples)
- 18.3 Numerical Solutions
  - » Basic Transport Equations ← status
  - » Gridding and finite differences
  - » Discretizing equations and boundary conditions
  - » Iterative Solution Approach

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$$\mathbf{J}_N = qn\mu_N E + qD_N \nabla n$$

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