Section 18
Semiconductor Equations

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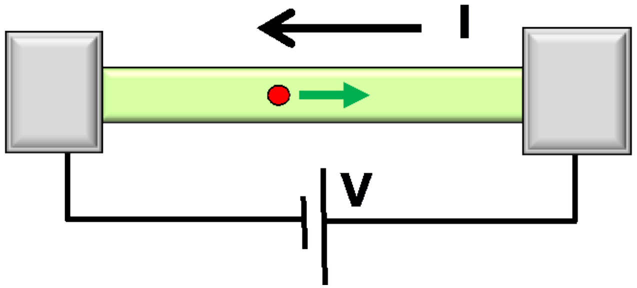
School of Electrical and Computer Engineering

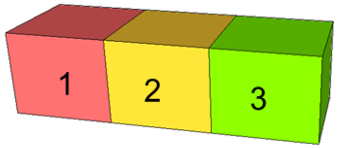
Section 18
Continuity Equations



$I = G \times V$

$= q \times n \times v \times A$

charge density velocity area



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- 18.1 Continuity Equations
- 18.2 Analytical Solutions (Strategy & Examples)
- 18.3 Numerical Solutions
  - » Basic Transport Equations
  - » Gridding and finite differences
  - » Discretizing equations and boundary conditions
  - » Iterative Solution Approach

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$

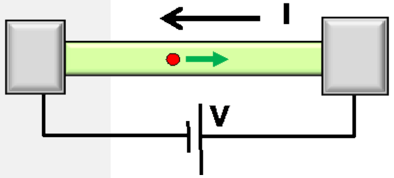
$$\mathbf{J}_N = qn\mu_N E + qD_N \nabla n$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla \bullet \mathbf{J}_P - r_P + g_P$$

$$\mathbf{J}_P = qp\mu_P E - qD_P \nabla p$$

$$\nabla \bullet D = q\left(p - n + N_D^+ - N_A^-\right)$$

Now the Continuity Equations …



$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$

→ Continuity eqn. for electrons

$$\mathbf{J}_N = qn\mu_N E + qD_N \nabla n$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla \bullet \mathbf{J}_P - r_P + g_P$$

→ Continuity eqn. for holes

$$\mathbf{J}_P = qp\mu_P E - qD_P \nabla p$$

$$\nabla \bullet D = q\left(p - n + N_D^+ - N_A^-\right)$$

→ Poisson equation

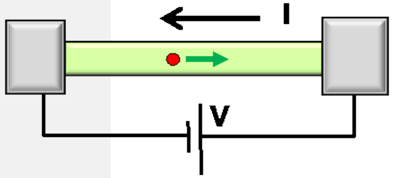
These equations have been state of the art in device modeling until ‘recently’ (20 years ago…)

Nano-scaled Devices – Non-Equilibrium Green Functions (NEGF)

Theory for Electrical Engineers: Supriyo Datta (Purdue)

NEGF Simulation Tools for atomistic devices: NEMO - Klimeck

Some critical remarks about these equations



- Continuity equations are *always* valid (regardless of the detailed physical model describing the device) because they describe a conservation law.

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N \qquad \frac{\partial p}{\partial t} = -\frac{1}{q}\nabla \bullet \mathbf{J}_P - r_P + g_P$$

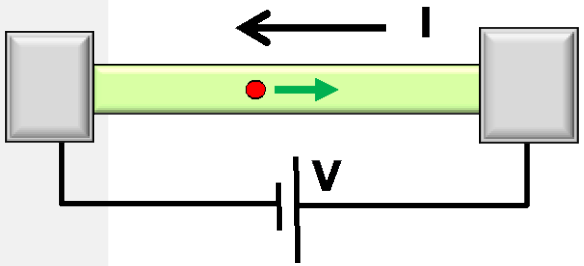
- Poisson equation as given does not account for explicit electron-electron repulsion. Might need to be modified for strongly correlated systems.

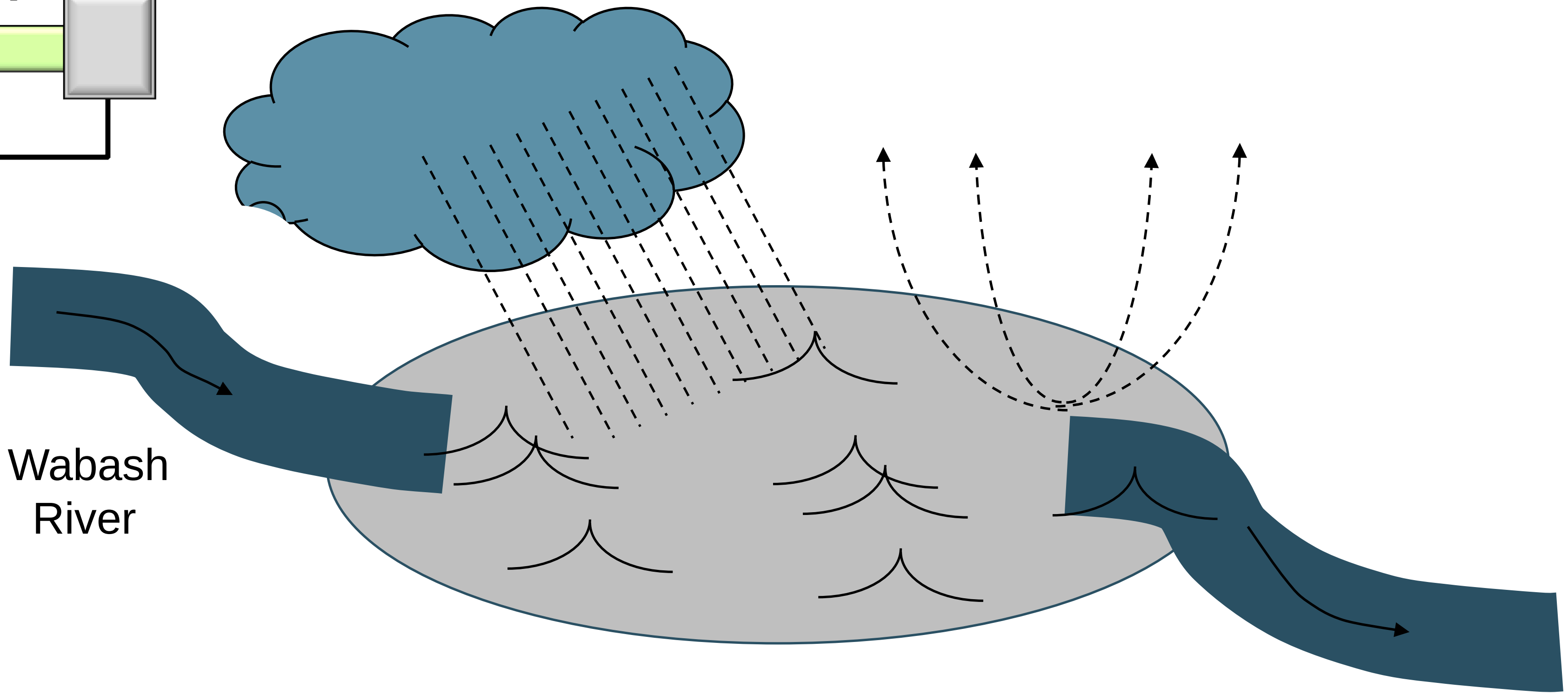
$$\nabla \bullet D = q\left(p - n + N_D^+ - N_A^-\right)$$

- Drift and Diffusion equations get modified when devices are so small that essentially no scattering takes place within the device.

$$\mathbf{J}_N = qn\mu_N E + qD_N \nabla n \qquad \mathbf{J}_P = qp\mu_P E - qD_P \nabla p$$

Continuity Equation prequel: A Good Analogy





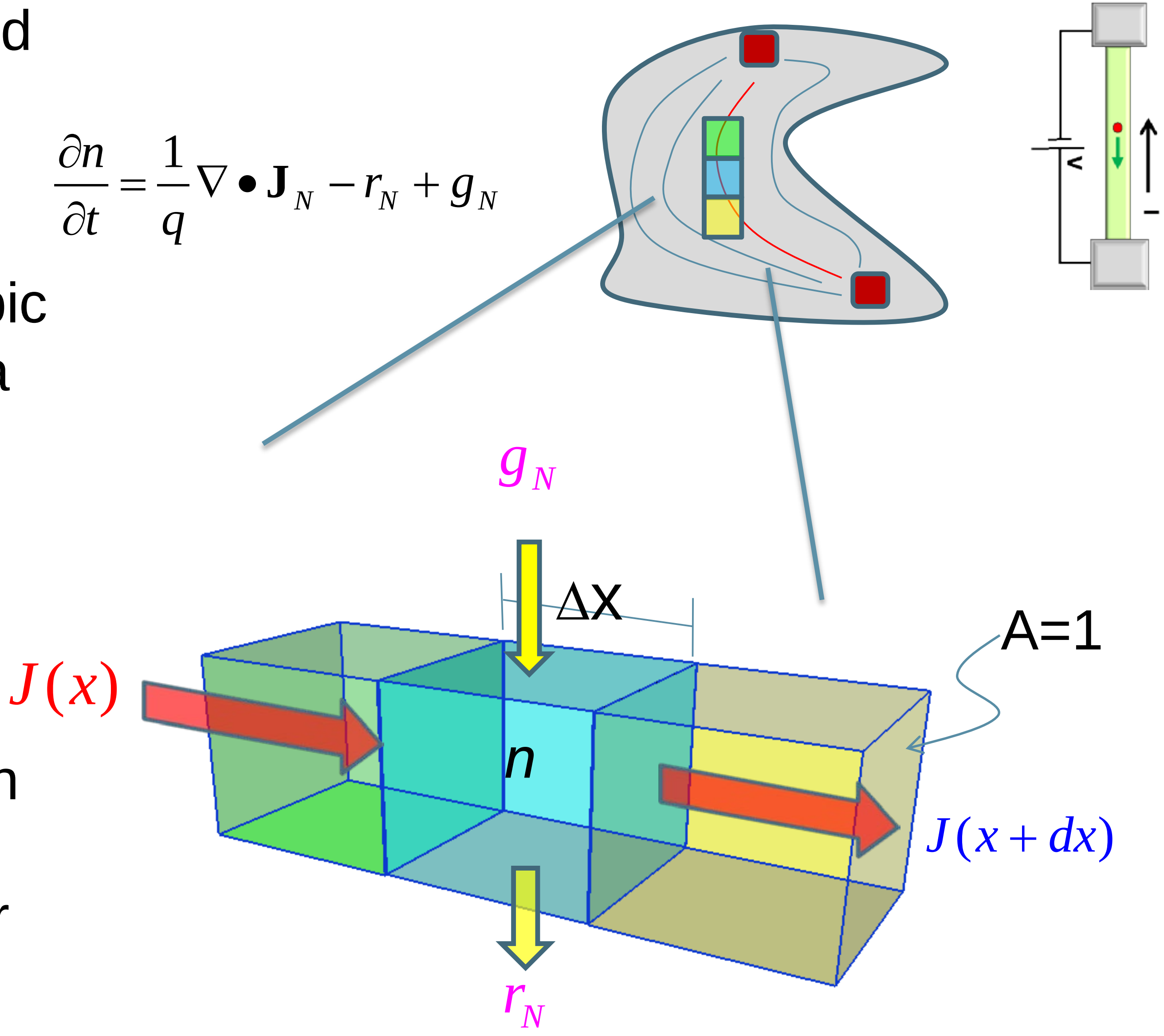
Rate of increase of water level in lake = (in flow - outflow) + rain - evaporation

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$

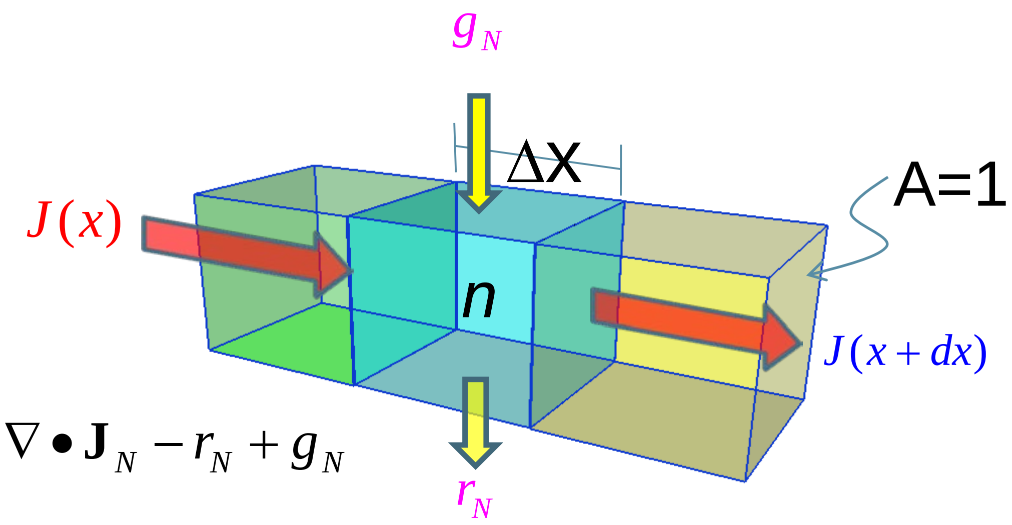
Now the Continuity Equations for electrons…

- Consider an arbitrarily shaped semiconductor with contacts that pump in current.
- Divide this arbitrary semiconductor into small cubic boxes of cross sectional area ‘A’ and length Δx.
- Boxes are large enough that concepts like effective mass, mobility etc. are valid inside these boxes.
- Electrons coming in are given by J(x), going out➔ J(x+Δx)
- Total number of electrons per cm^3 ➔n

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$



Now the Continuity Equations …

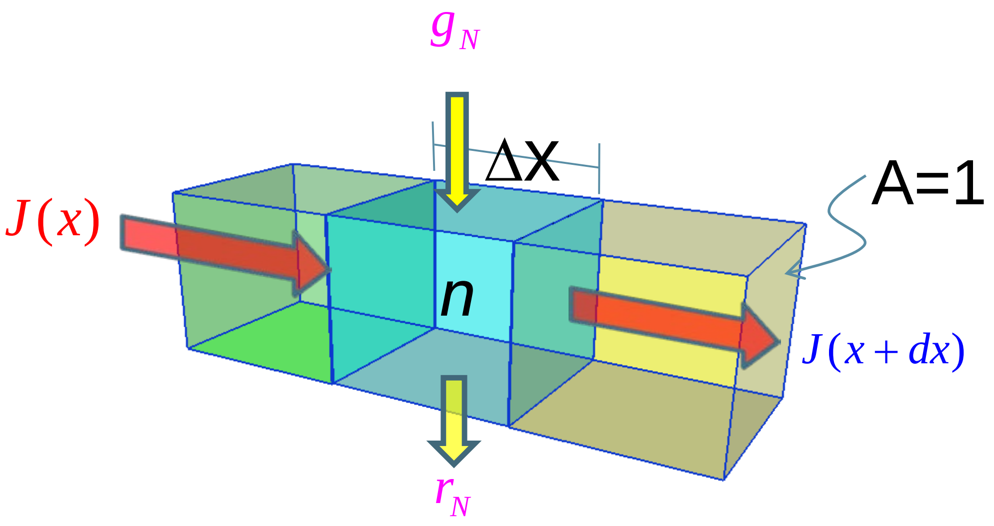


$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$

- g_N-> generation rate in (per cm^3 per sec) from external processes such as light.
- r_N-> recombination rate in (per cm^3 per sec) in the central box.

- We wish to relate all of these factors that affect the concentration of carriers in the central box.
- Our strategy (remember the analogy)– The *rate* of change of number of electrons INSIDE the central box should be equal to
  - No. of electrons coming in MINUS No. of electrons going out per sec (governed by current density J(x) and J(x+Δx)) PLUS
  - No. of electrons getting generated from external processes per sec (governed by generation rate g_N) MINUS
  - No. of electrons lost by recombination per sec (governed by recombination rate r_N)

Let's just write the equations down…

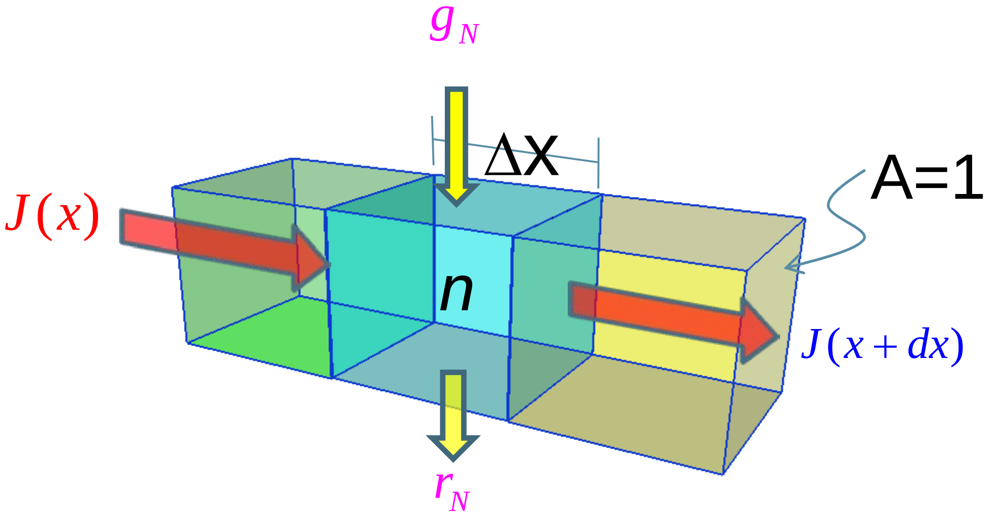


$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$

$$\frac{\partial(A\times\Delta x\times n)}{\partial t} = \frac{A\times J_n(x) - A\times J_n(x+dx)}{-q} + A\times g_N\Delta x - A\times r_N\Delta x$$

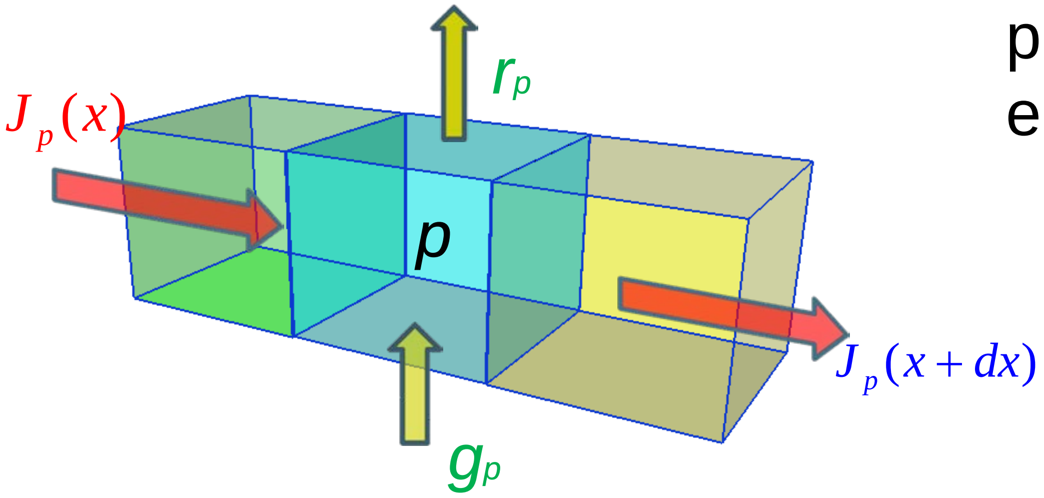
$$\frac{\partial n}{\partial t} = \frac{J_n(x) - J_n(x+dx)}{-q\Delta x} + g_N - r_N = \frac{1}{q}\nabla \bullet J_n + g_N - r_N$$

Continuity Equations for Electron/Holes



$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$

Usually generation and recombination rates for electrons and holes are the same since the same processes create/destroy an electron/hole



$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla \bullet J_P + g_P - r_P$$

Equations in place, learning to solve them...

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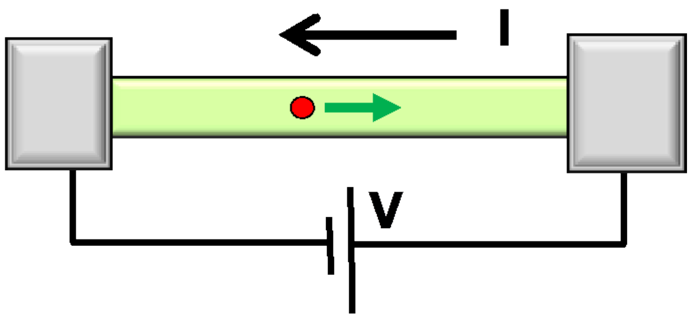
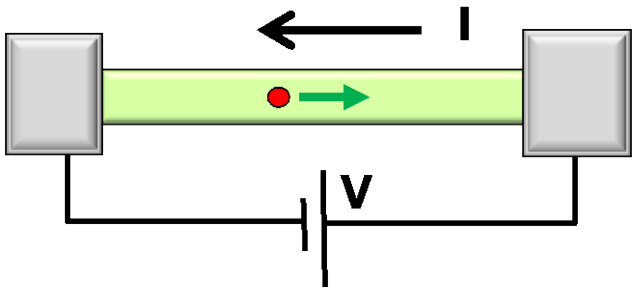


Two methods of solution:

Numerical and Analytical

Continuity Equations



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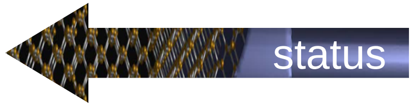

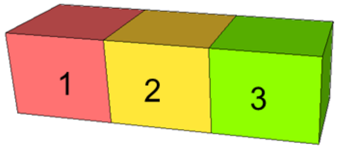

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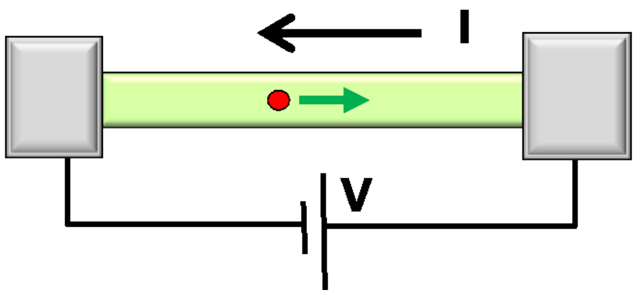

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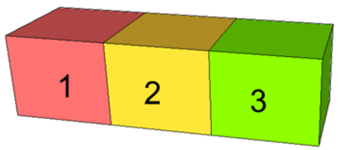

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