Section 17
Intro to Transport - Drift, Mobility, Diffusion, Einstein Relationship

17.4 Physics of diffusion – Einstein Relationship

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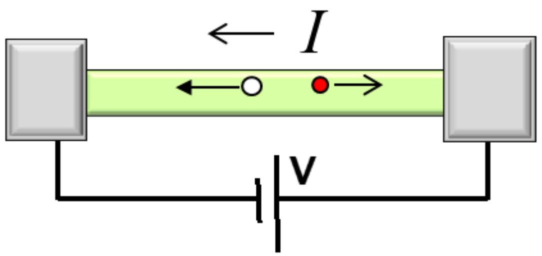
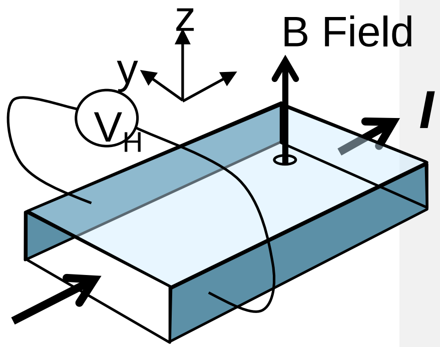


I = G x V

= q x n x v x A

charge density velocity area

Transport with scattering, non-equilibrium Stat. Mech.
- Drift-diffusion equation with recombination-generation



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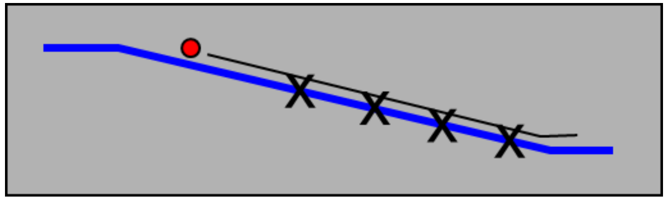
- 17.1 Drift Current $J_n = qn\mu_n\mathcal{E}$



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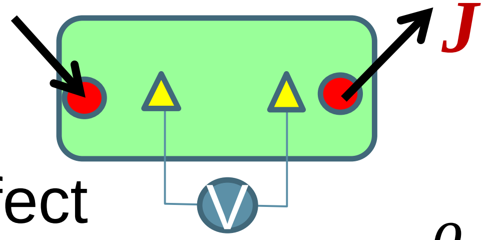
- 17.2 Mobility
  - »Matthiessen Rule $\frac{1}{\mu_n} = \frac{1}{\mu_{ph}} + \frac{1}{\mu_{II}}$
  - »High Field Effects
  - »Mobility Measurement

$$R_H = \frac{E_y/B_z}{J_x} = -\frac{1}{qn}$$



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- 17.3 Carrier Concentration from Hall Effect

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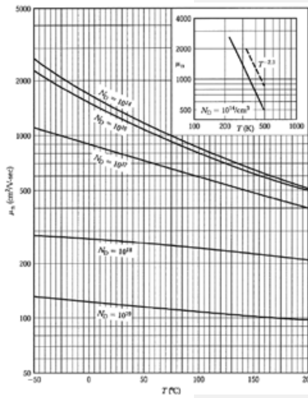
- 17.4 Physics of diffusion – Einstein Relationship

$$\rho_{n-type} = \frac{1}{q\mu_n N_D}$$



Consider system to be in local equilibrium

So far Considered: Drift Term

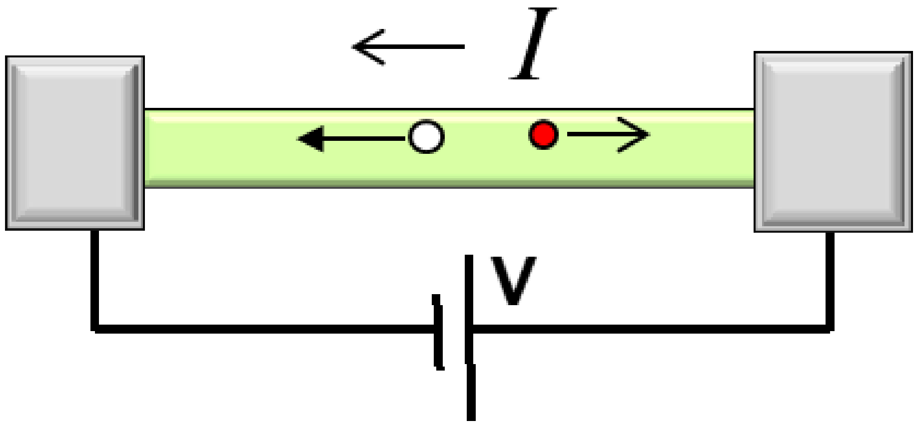
$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$

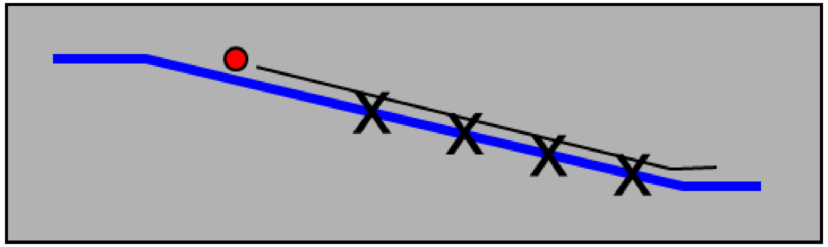
$$\mathbf{J}_N = qn\mu_N E + qD_N \nabla n$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla \bullet \mathbf{J}_P - r_P + g_P$$

$$\mathbf{J}_P = qp\mu_P E - qD_P \nabla p$$

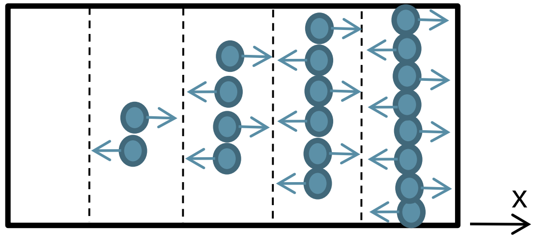
$$\nabla \bullet D = q\left(p - n + N_D^+ - N_A^-\right)$$





Consider system to be in local equilibrium

Now the Diffusion term…



$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$

$$\mathbf{J}_N = qn\mu_N E + qD_N \nabla n$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla \bullet \mathbf{J}_P - r_P + g_P$$

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$$\nabla \bullet D = q\left(p - n + N_D^+ - N_A^-\right)$$

~~So far Considered: Drift Term~~

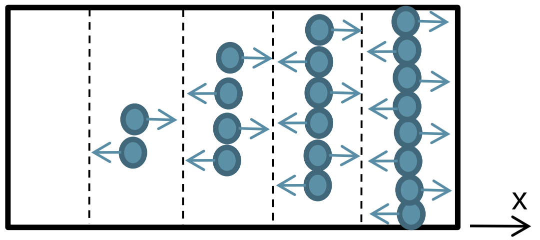
~~Consider system to be in local equilibrium~~

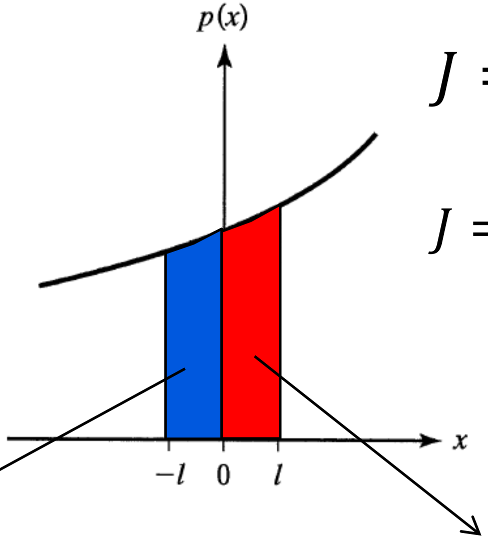
Diffusion Flux ...

- Assuming independent random motion
- Electrons move on average (at each x)
  - half to left,
  - half to right

➔NET movement of electrons to left

- i.e. against the gradient.
- Opposite sign for holes.





$$J = qp\upsilon_{th}$$

$$J = q[-p(blue\ area) + p(red\ area)]\upsilon_{th}$$

$$J = q\left[-\left(\frac{p(0) + p(0) - \frac{dp}{dx}l}{2}\right) \times l + \left(\frac{p(0) + p(0) + \frac{dp}{dx}l}{2} \times l\right)\right] \Bigg/ \frac{l}{\upsilon_{th}}$$

$$J = ql\upsilon\frac{dp}{dx} \qquad D = l\upsilon \qquad J = qD\frac{dp}{dx}$$

Diffusion Flux …

- Assuming independent random motion
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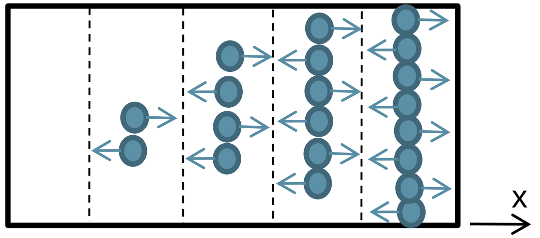
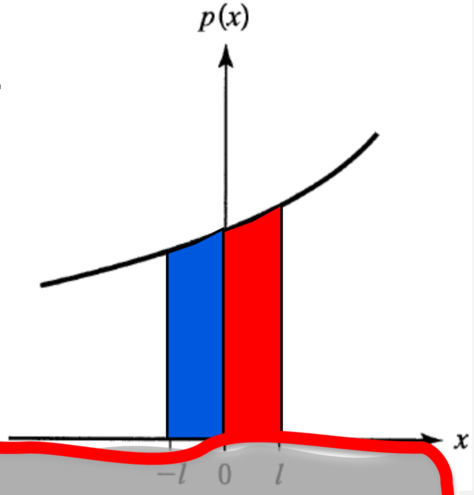


$$J = qD\frac{dp}{dx}$$

$$D = lv$$



This looks like a completely classical derivation

Where is the Quantum Mechanics?

Diffusion and Mobility must be related!

Quantum Mechanics is in the Diffusion coefficient and the Drift Velocity.

⇒Determines the available states

⇒Determines the capability to carry current

Scattering is built in here! Interactions of many, many, many electrons and he surrounding.

Without scattering one could not explain the equal partitioning into 2 directions!

$$J = qD\frac{dp}{dx} \qquad D = lv \qquad \mu = \frac{q\tau}{m^*}$$

Diffusion and Mobility must be related!

… because scattering dominates both phenomena

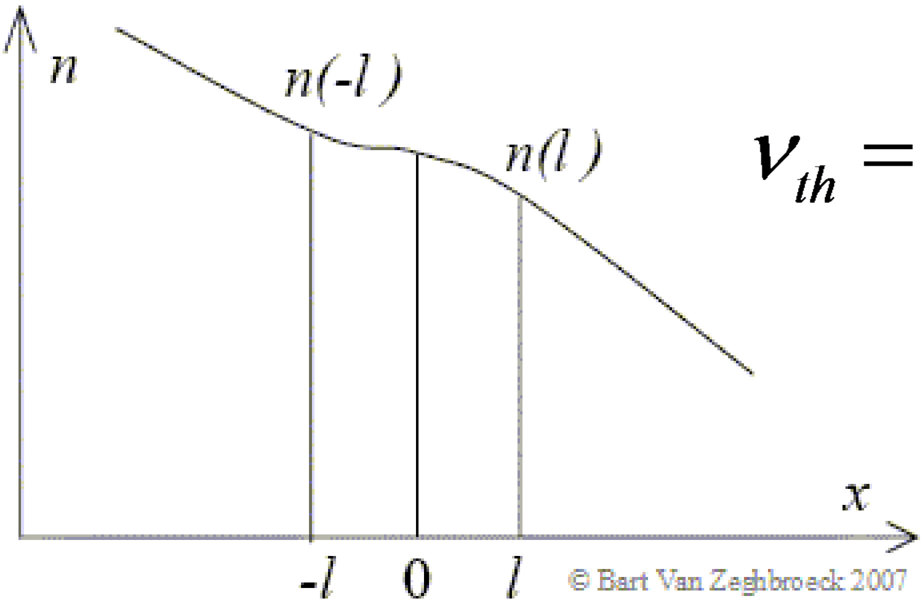
$$l = v\tau$$

$$\frac{D}{\mu} = \frac{lv}{\frac{q\tau}{m_0^*}} = \frac{(v\tau)\times v}{\frac{q\tau}{m_0^*}} = \frac{2(\frac{1}{2}m_0^* v^2)}{q} \qquad \frac{D}{\mu} = \frac{k_B T}{q}$$

$$E_{thermal}^{1D} = \frac{k_B T}{2}$$

An alternate derivation

Diffusion and Mobility must be related!



$$v_{th} = \frac{l}{\tau}$$

RMS velocity of carriers is decided by the average mean free path and average scattering time

$$\frac{D}{\mu} = \frac{k_B T}{q}$$

$$E_{thermal}^{1D} = \frac{k_B T}{2}$$

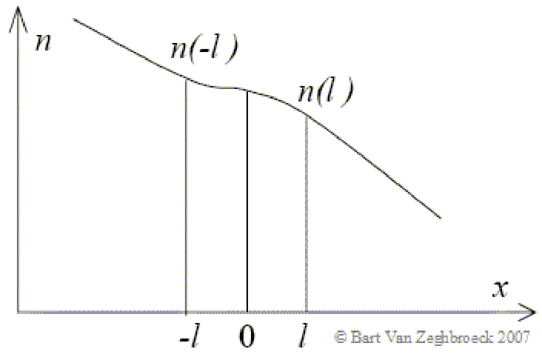
$$l = v\tau$$

Carriers that arrive at x=0 do so by travelling one mean free path from right to left or vice versa.

Net current from left to right (at x=0) = charge on carrier*net flux of carriers from left to right

Figures and derivation from http://ecee.colorado.edu/~bart/book/book/chapter2/ch2_7.htm#fig2_7_8

Derivation for electron diffusion current



Electron flux at x=0 from left to right

$$\phi_{n,left\rightarrow right} = \frac{1}{2} v_{th} n(x=-l)$$

The factor half appears because the other half at *x=-l* travels towards the left

Electron flux at x=0 from right to left

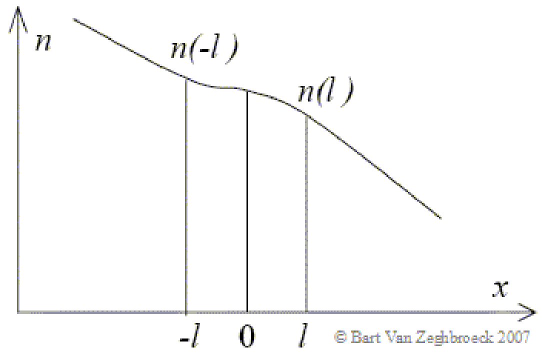
$$\phi_{n,right\rightarrow left} = \frac{1}{2} v_{th} n(x=l)$$

Net Flux

$$\phi_n = \phi_{n,left\rightarrow right} - \phi_{n,right\rightarrow left} = \frac{1}{2} \upsilon_{th} [n(x=-l) - n(x=l)]$$

Derivation continued

$$\phi_n = \phi_{n,left\rightarrow right} - \phi_{n,right\rightarrow left} = \frac{1}{2}\upsilon_{th}[n(x=-l)-n(x=l)]$$



$$\phi_n = l\upsilon_{th}\frac{[n(x=-l)-n(x=l)]}{2l}$$

If the mean free path is small enough, then we can write this as

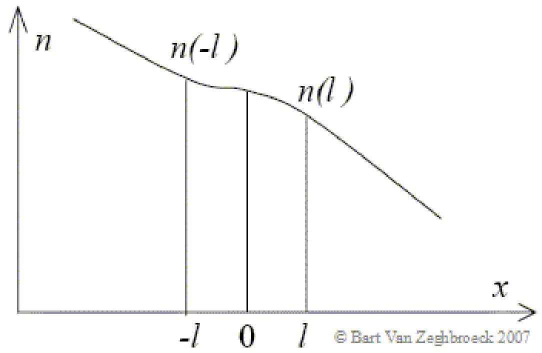
$$\phi_n = -l\upsilon_{th}\frac{dn}{dx}$$

Consider system to be in local equilibrium

The negative sign arises because we take the gradient is usually measured for increasing values of x. The current density is then given by.

$$J_n = -q\phi_n = ql\upsilon_{th}\frac{dn}{dx}$$

For holes, we repeat the derivation and find…



$$J_n = -q\phi_n = q l \upsilon_{th} \frac{dn}{dx}$$

$$J_p = q\phi_p = -q l_{holes} \upsilon_{th,holes} \frac{dp}{dx}$$

Lump together the second and third terms to form a ‘diffusion constant’

$$D_p = l_{holes} \upsilon_{th,holes} \qquad D_n = l_n \upsilon_{th,n}$$

$$J_p = -qD_p \frac{dp}{dx} \qquad J_n = qD_n \frac{dn}{dx}$$

For thermally distributed carriers...

$$E_{thermal}^{1D} = \frac{k_B T}{2}$$

$$E_{thermal}^{1D} = \frac{1}{2} m_0^* v^2$$

$$m_0^* v^2 = k_B T$$

Equipartition theorem states that in equilibrium each carrier has thermal energy of kT/2 per degree of freedom.

$$D_n = l_n v_{th,n}$$

valid for electrons and holes
AT EQULIBRIUM

Our derivation is in one dimension➔ one degree of freedom

http://en.wikipedia.org/wiki/Equipartition_theorem#Derivations

Alternate way to obtain the diffusion constants

$$D = l\upsilon_{th} = \tau\upsilon_{th}^2 = \frac{q\tau}{m^*}\frac{m^*\upsilon_{th}^2}{q} = \mu\frac{k_B T}{q}$$

$$\mu = \frac{q\tau}{m^*}$$

$$D_n = l_n \upsilon_{th,n}$$

$$m_0^* \upsilon^2 = k_B T$$

$$\frac{D}{\mu} = \frac{k_B T}{q}$$

valid for electrons and holes
AT EQULIBRIUM

$$l = \upsilon\tau$$

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{k_B T}{q}$$

The equation shows how thermal quantities governing diffusion can be related to those governing drift (mobility depends on the effective mass which describes motion of carriers in the presence of a field and scatterers)

An alternate derivation of Einstein's relationship

No current in equilibrium

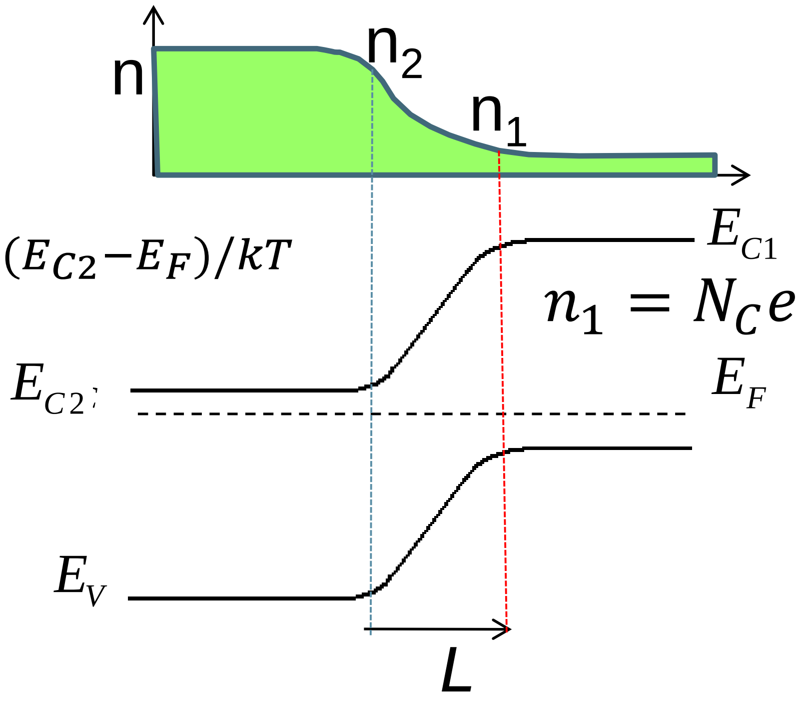
$$J_n = qn\mu_n \mathrm{E} + qD_n \frac{dn}{dx} = 0$$

$$\Rightarrow \frac{1}{n}\frac{dn}{dx} = -\frac{\mu_n \mathrm{E}}{D_n}$$

$$n_2 = n_1 e^{-\int_0^L \frac{\mu_n \mathcal{E}}{D_n}} = n_1 e^{\frac{\mu_n V}{D_n}}$$

$$\frac{n_2}{n_1} = e^{\mu_n V/D_n}$$

$$\ln\left(\frac{n_2}{n_1}\right) = \frac{\mu_n V}{D_n} = \frac{qV}{kT}$$



$$n_2 = N_C e^{-(E_{C2}-E_F)/kT}$$

$$n_1 = N_C e^{-(E_{C1}-E_F)/kT}$$

$$\frac{n_2}{n_1} = \frac{N_C e^{-(E_{C2}-E_F)/kT}}{N_C e^{-(E_{C1}-E_F)/kT}} = e^{-(E_{C2}-E_{C1})/kT} = e^{qV/kT}$$

$$\frac{D}{\mu} = \frac{k_B T}{q}$$

Similar to relationship between c_n and e_n discussed in Section 16

Section 17
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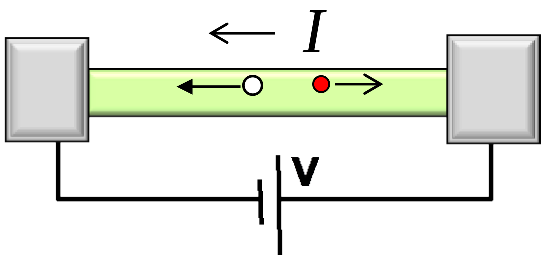
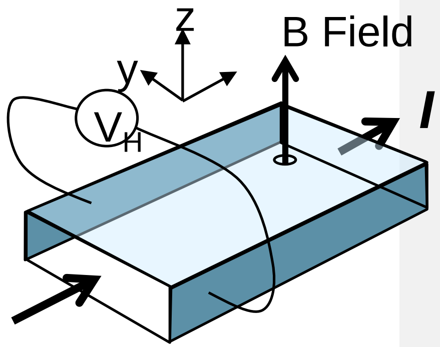


I = G × V

= q × n × v × A

charge density velocity area

Transport with scattering, non-equilibrium Stat. Mech.
- Drift-diffusion equation with recombination-generation



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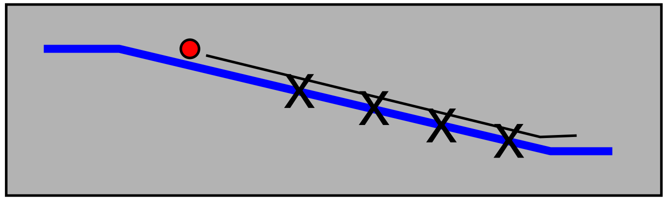
- 17.1 Drift Current

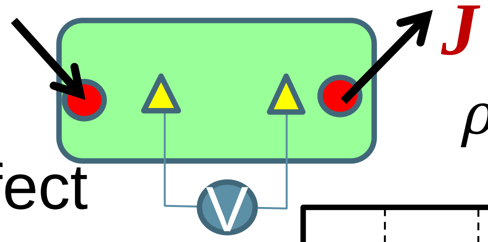
$$J_n = qn\mu_n\mathcal{E}$$

- 17.2 Mobility
  - »Matthiessen Rule
  - »High Field Effects
  - »Mobility Measurement

$$\frac{1}{\mu_n} = \frac{1}{\mu_{ph}} + \frac{1}{\mu_{II}}$$



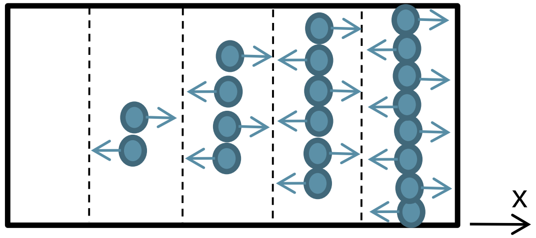
$$R_H = \frac{E_y/B_z}{J_x} = -\frac{1}{qn}$$

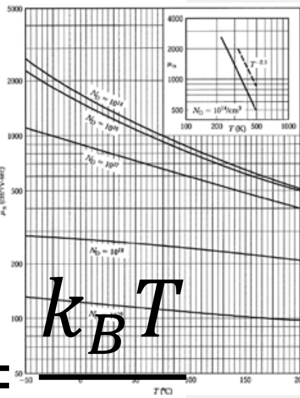


$$\rho_{n-type} = \frac{1}{q\mu_n N_D}$$

- 17.3 Carrier Concentration from Hall Effect
- 17.4 Physics of diffusion – Einstein Relationship

REF: ADF, Chapter 5, pp. 190-202





$$\frac{D}{\mu} = \frac{k_B T}{q}$$

- Drift, diffusion, and recombination-generation => elemental processes in semiconductor device physics.
- Measurement of mobility and carrier concentration => characterize semiconductor devices.

Consider system to be in local equilibrium