Section 17
Intro to Transport - Drift, Mobility, Diffusion, Einstein Relationship

17.3 Carrier Concentration from Hall Effect

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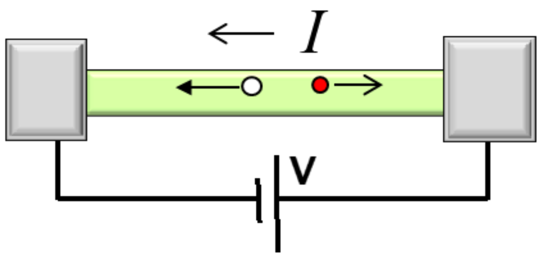


$I = G \times V$

$= q \times n \times v \times A$

charge density velocity area

Transport with scattering, non-equilibrium Stat. Mech.

- Drift-diffusion equation with recombination-generation

Vid

- 17.1 Drift Current $J_n = qn\mu_n\mathcal{E}$

Vid

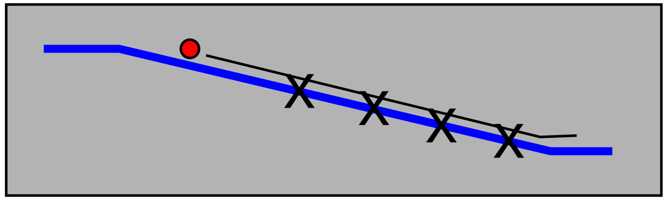
- 17.2 Mobility
  - »Matthiessen Rule $\frac{1}{\mu_n} = \frac{1}{\mu_{ph}} + \frac{1}{\mu_{II}}$
  - »High Field Effects
  - »Mobility Measurement

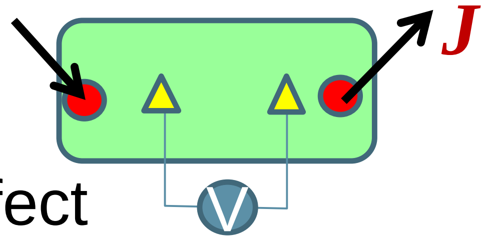
Vid

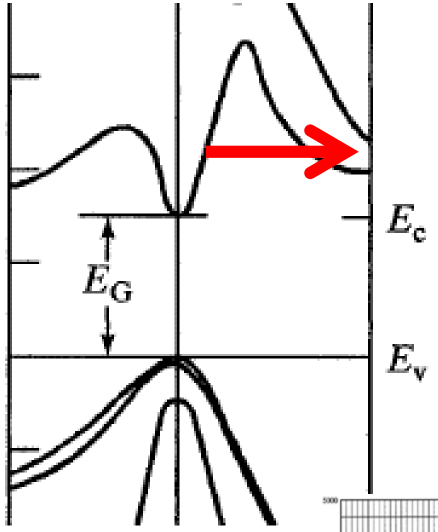
- 17.3 Carrier Concentration from Hall Effect

Vid

- 17.4

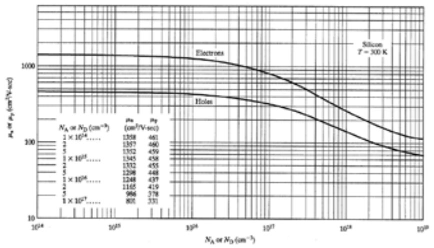


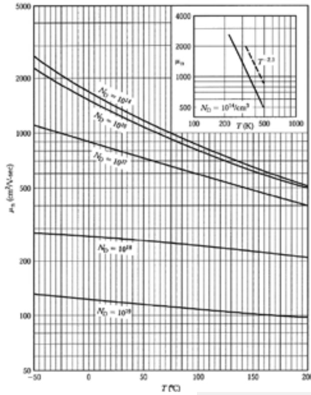




μ_n *and* N_D are coupled
need the electron density
=> get the doping

$$\rho_{n-type} = \frac{1}{q\mu_n N_D}$$

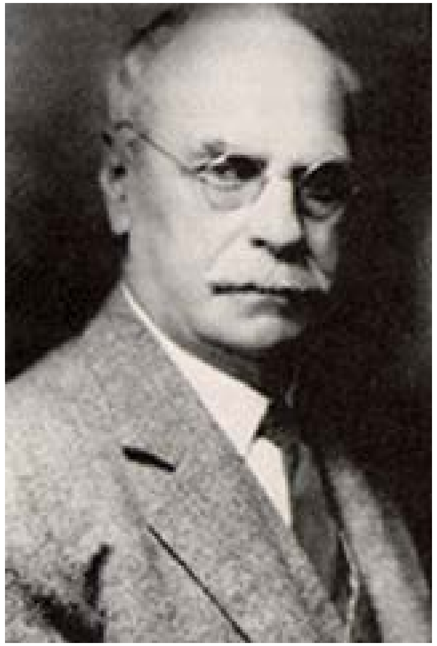




Consider system to be in local equilibrium

Measuring Electron Density
Hall Effect Trivia - FYI

- Discovered in 1879 by Edwin Herbert Hall while he was working on his doctoral degree at Johns Hopkins University in Baltimore, Maryland.
- Done 18 years before the electron was discovered.
- 4 Nobel Prizes associated directly with it.
- Read the original article:
  "*On a New Action of the Magnet on Electric Currents*"
  at http://www.stenomuseet.dk/skoletj/elmag/kilde9.html
  for a fascinating account of the discovery of the effect.



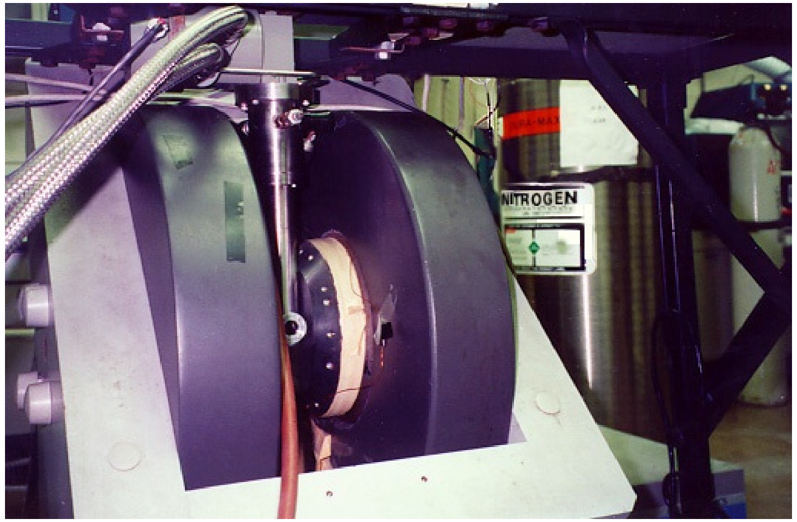
http://en.wikipedia.org/wiki/Hall_effect and http://en.wikipedia.org/wiki/Edwin_Hall

μ_n *and* N_D are coupled
need the electron density
=> get the doping

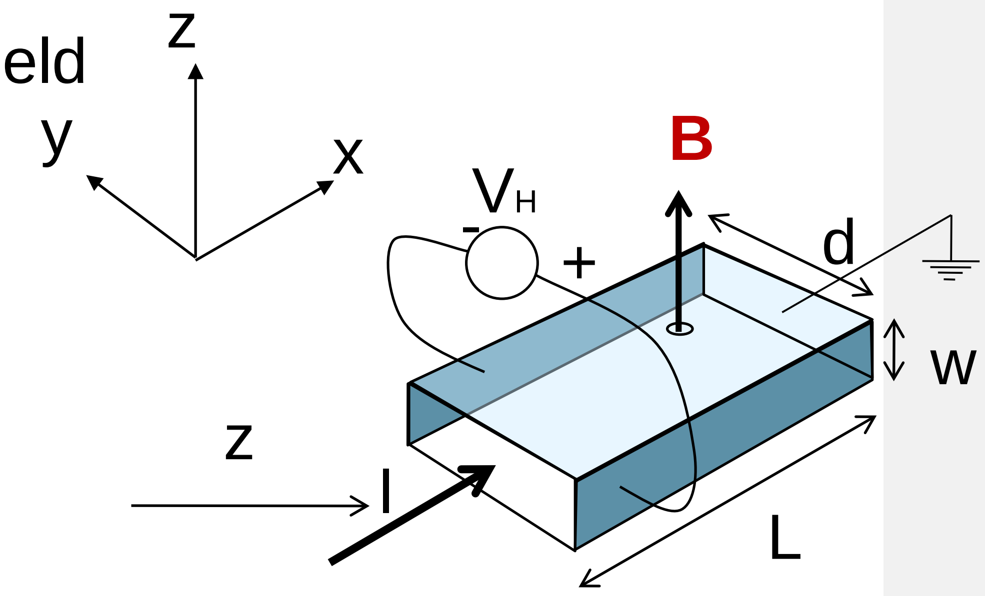
Set up for Hall Measurement

Force acting on a charged particle in a magnetic field

$$\vec{F} = q\vec{\mathcal{E}} + q\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}} + \frac{m^*\vec{\boldsymbol{v}}}{\tau}$$



UIC system: 4-300K, 0-1.5 T



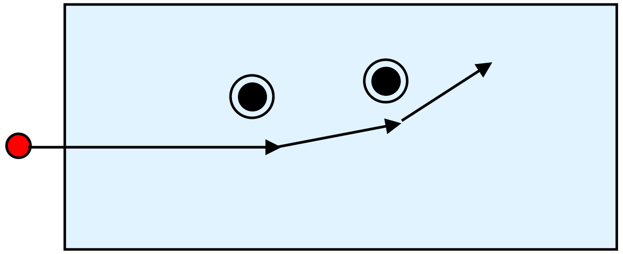
- Applying a magnetic field one 'pushes' carriers towards one face
- This induces a voltage across the two faces perpendicular to current flow.
- We will relate this voltage to the current and magnetic field and deduce the density of carriers.

μ_n *and* N_D are coupled
need the electron density
=> get the doping

Drude Model for electrons *

* Same model works for holes, but with +q instead of -q

Simple classical Newton's law expression
m – effective mass
v – drift velocity w/ scattering



$$\vec{F} = q\vec{\mathcal{E}} + q\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}} + \frac{m^*\vec{\boldsymbol{v}}}{\tau} = 0$$

$$m^*\boldsymbol{\upsilon} = -q\tau\mathcal{E} - q\tau\boldsymbol{\upsilon}\times\mathbf{B}$$

$$\approx -q\tau\mathcal{E} - q\tau\left(-\frac{q\tau\mathcal{E}}{m*}\right)\times\mathbf{B}$$

$$= -q\tau\mathcal{E} + \frac{q^2\tau^2}{m^*}\mathcal{E}\times\mathbf{B}$$

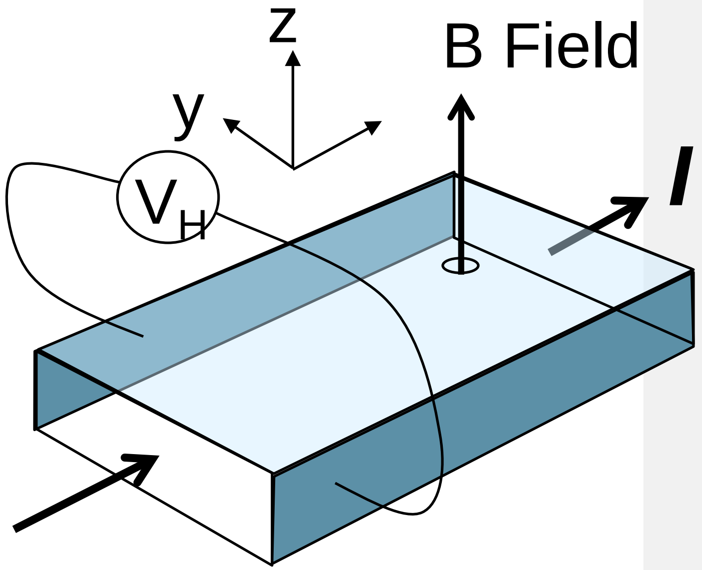
$$\upsilon = -\frac{q\tau\mathcal{E}}{m^*} + \frac{q^2\tau^2}{m^{*2}}\mathcal{E}\times\mathbf{B}$$

Must be "small"

Approximation for Weak **B=0** field …

$$-q\mathcal{E} - \frac{m^*\boldsymbol{\upsilon}}{\tau} \approx 0$$

$$\upsilon = \frac{-q\tau\mathcal{E}}{m^*}$$



Approximation works iff ….

$$\frac{q^2\tau^2 B}{m^* q\tau} = \frac{q\tau B}{m^*} \equiv \tau\omega_c << 1$$

Limit of strong scattering

Time for one magnetic orbit $\frac{1}{\omega_c} \gg \tau$ Relaxation time

$$\upsilon = -\frac{q\tau\mathcal{E}}{m^*} + \frac{q^2\tau^2}{m^{*2}}\mathcal{E}\times\mathbf{B}$$

$$\mathbf{J}_n = -qn\,\boldsymbol{\upsilon}$$

$$= \frac{q^2 n\tau}{m^*}\mathcal{E} - \frac{q^2 n\tau}{m^*}\frac{q\tau}{m^*}\mathcal{E}\times\mathbf{B}$$

$$= \sigma_0\mathcal{E} - \sigma_0\mu\,\mathcal{E}\times\mathbf{B}$$

$$\rho_{n-type} = \frac{1}{q\mu_n N_D}$$

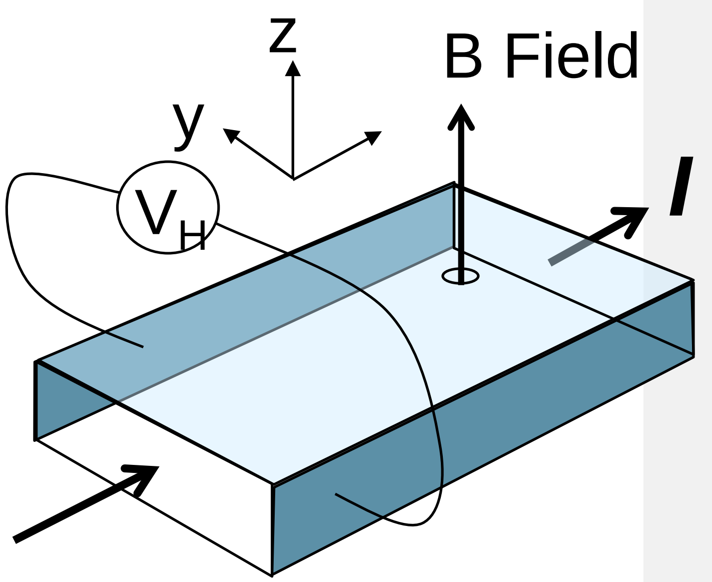
$$\sigma_{n-type} = q\mu_n N_D$$

$$\mu_n = \frac{q\tau_n}{m_n^*}$$

$$N_D \approx n$$

$$\sigma_{n-type} = q\frac{q\tau_n}{m_n^*}n$$

$$\sigma_{n-type} = \frac{q^2 n\tau}{m_n^*}$$



Drude Model & Hall Effect ...

$$\upsilon = -\frac{q\tau\mathcal{E}}{m^*} + \frac{q^2\tau^2}{m^{*2}}\mathcal{E}\times\mathbf{B}$$

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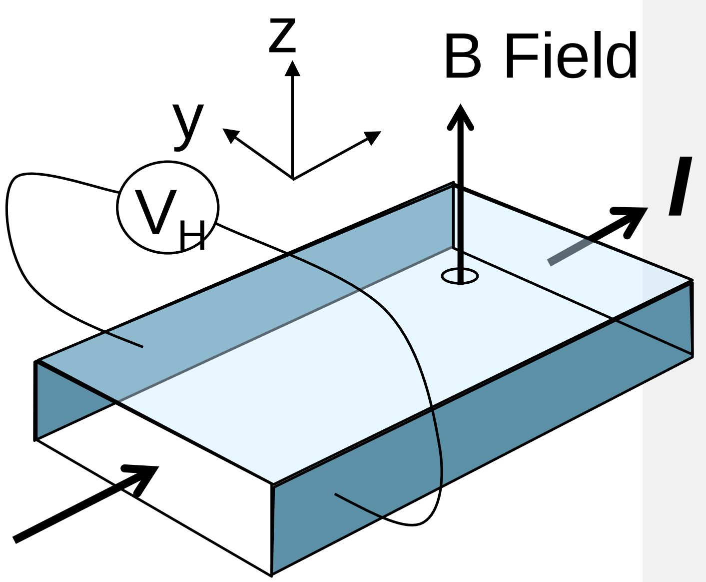
$$= \frac{q^2 n\tau}{m^*}\mathcal{E} - \frac{q^2 n\tau}{m^*}\frac{q\tau}{m^*}\mathcal{E}\times\mathbf{B}$$

$$= \sigma_0\mathcal{E} - \sigma_0\mu\,\mathcal{E}\times\mathbf{B}$$

$$\begin{vmatrix} \bar{x} & \bar{y} & \bar{z} \\ E_x & E_y & E_z \\ B_x & B_y & B_z \end{vmatrix}$$



$$N_D \approx n$$

$$\begin{bmatrix} J_x \\ J_y \end{bmatrix} = \begin{bmatrix} \sigma_0 & -\sigma_0\mu B_z \\ \sigma_0\mu B_z & \sigma_0 \end{bmatrix}\begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

$$\mu_n = \frac{q\tau_n}{m_n^*}$$

$$\sigma_{n-type} = \frac{q^2 n\tau}{m_n^*}$$

$$\rho_{n-type} = \frac{1}{q\mu_n N_D}$$

Hall Resistance

~0, since voltmeters have a very large internal resistance so very little current flows through

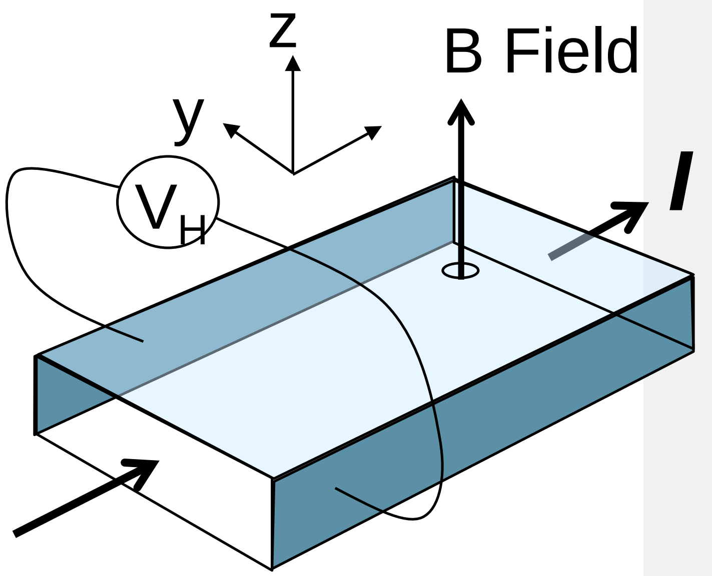
In the limit of small B

$$\begin{bmatrix} J_x \\ J_y \end{bmatrix} = \begin{bmatrix} \sigma_0 & -\sigma_0 \mu B_z \\ \sigma_0 \mu B_z & \sigma_0 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

$$\begin{bmatrix} J_x \\ J_y \end{bmatrix} \approx \begin{bmatrix} \sigma_0 & 0 \\ \sigma_0 \mu B_z & \sigma_0 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

$$\begin{bmatrix} J_x \\ 0 \end{bmatrix} = \begin{bmatrix} \sigma_0 & 0 \\ \sigma_0 \mu B_z & \sigma_0 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

$$R_H = \frac{E_y / B_z}{J_x} = -\frac{1}{qn}$$



B_z is known,
J_x is measured,
V_h is measured
=> E_y is known

$$N_D \approx n$$

$$\mu_n = \frac{q\tau_n}{m_n^*}$$

$$\sigma_{n-type} = \frac{q^2 n\tau}{m_n^*}$$

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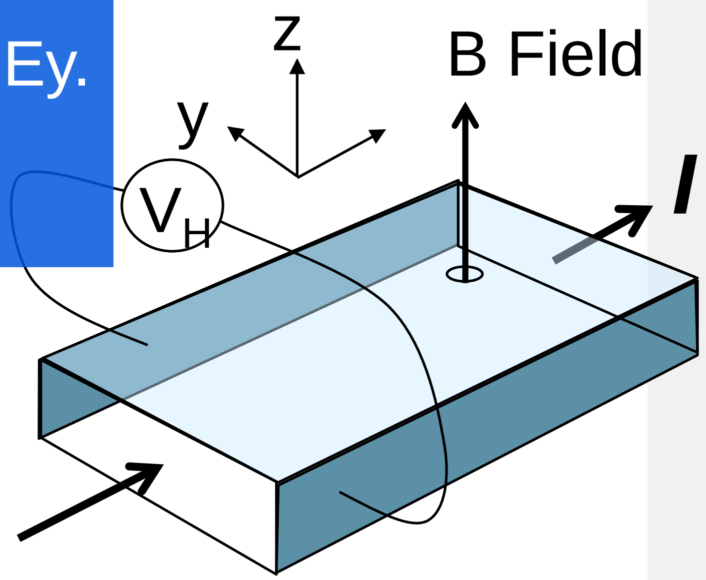
Hall Resistance

Why Bz set to zero in one eq and not the other?
=> analytical simplicity.
The system is also explicitly solvable in terms of Bz^2 and Ey.
Once Bz^z/Ey is small compared to 1/qn then one can neglect higher orders and come to the same result

In the limit of small B



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Hall Resistance

$$R_H = \frac{E_y / B_z}{J_x} = -\frac{1}{qn}$$

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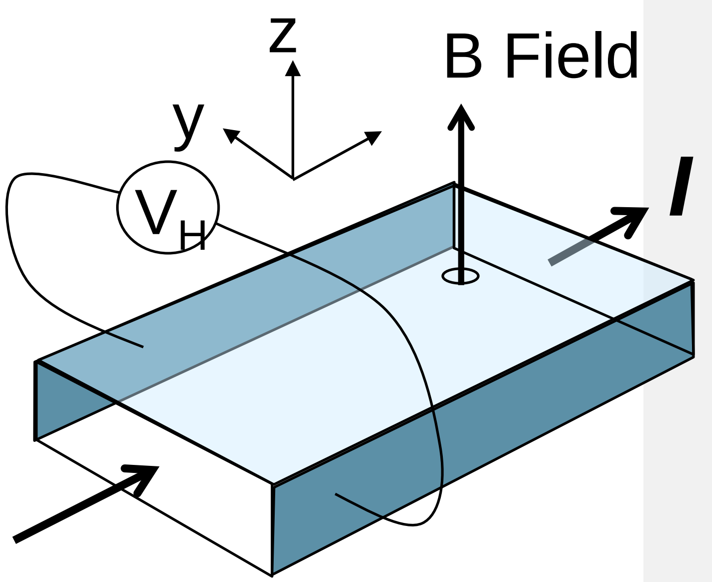

By a simple electrical measurement, we now know the concentration of electrons in the sample.
From the intrinsic concentration of carriers in the semiconductor and temperature we can deduce the doping concentration in the sample

$$N_D \approx n$$

$$\mu_n = \frac{q\tau_n}{m_n^*}$$

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Hall Resistance

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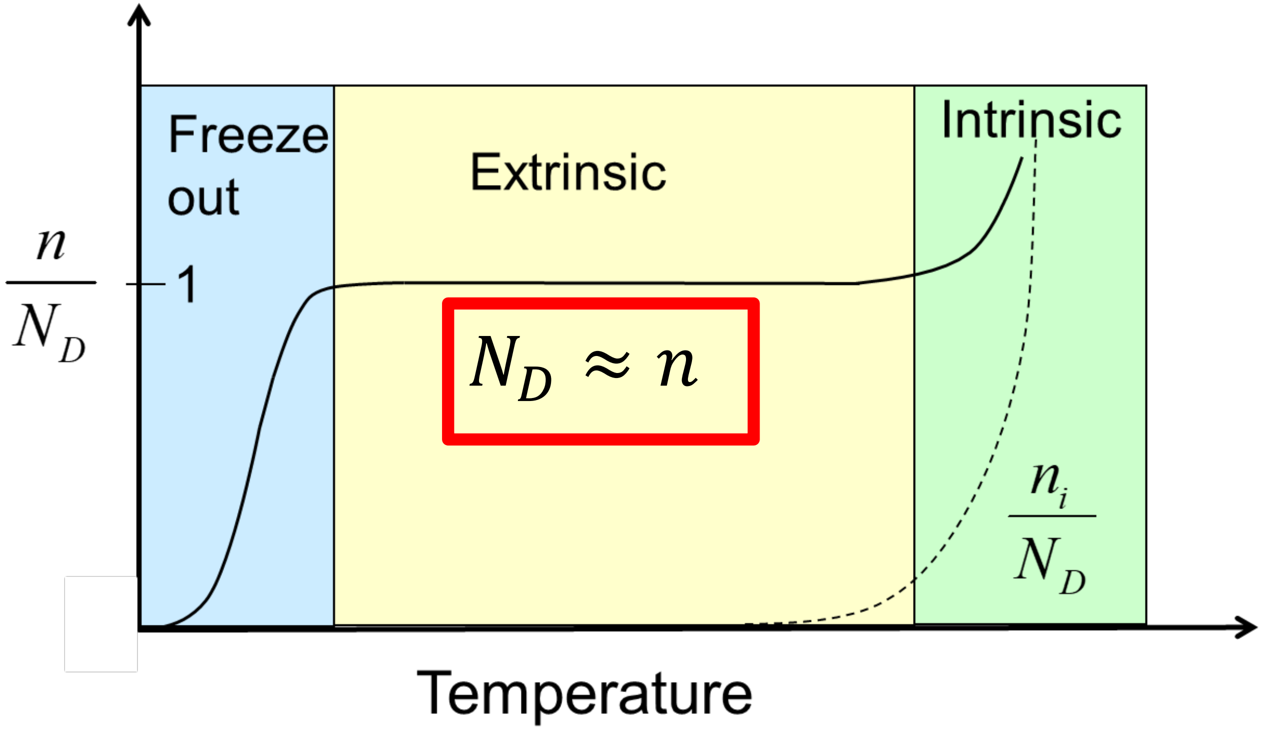
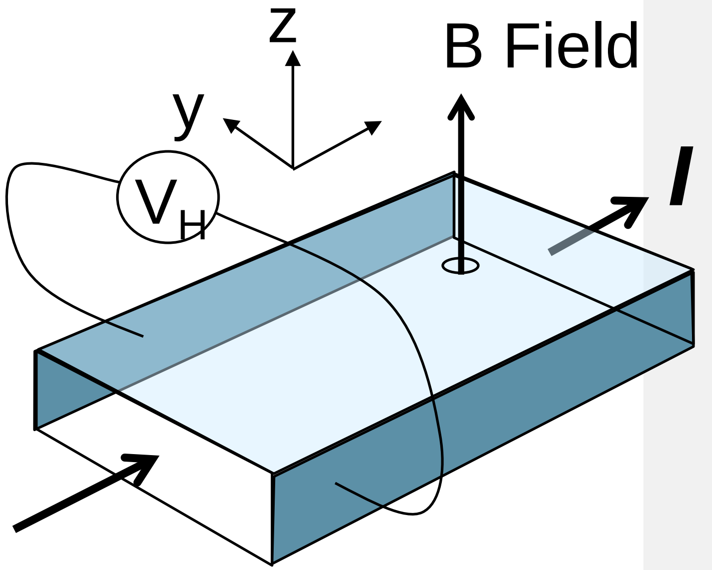
In the limit of small B





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Need multiple temperature dependent measurements to determine doping

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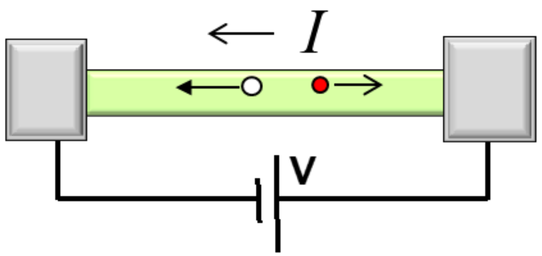

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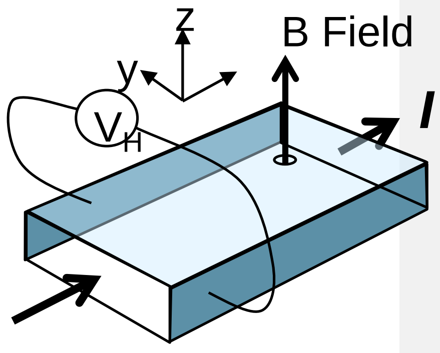
charge density velocity area

Transport with scattering, non-equilibrium Stat. Mech.
- Drift-diffusion equation with recombination-generation



Vid

- 17.1 Drift Current $J_n = qn\mu_n\mathcal{E}$

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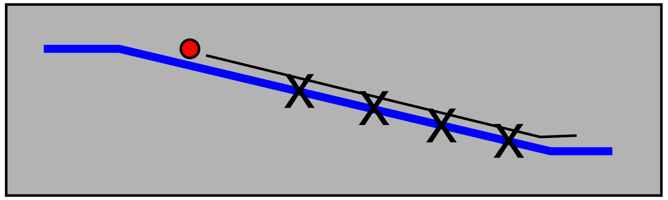
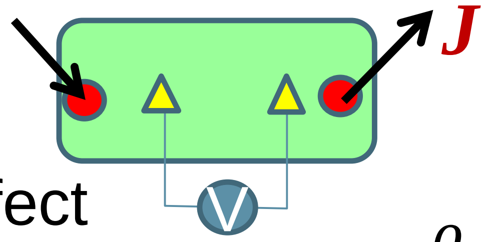
  - »High Field Effects
  - »Mobility Measurement





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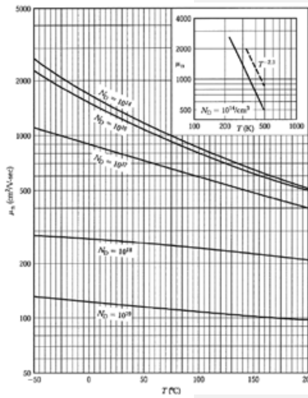

Vid

- 17.4

Consider system to be in local equilibrium

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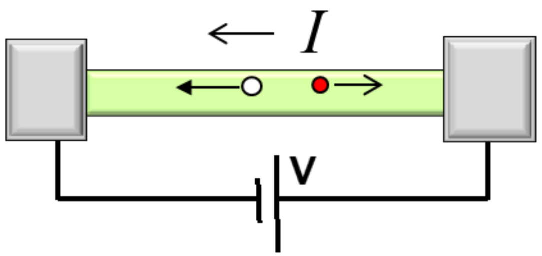

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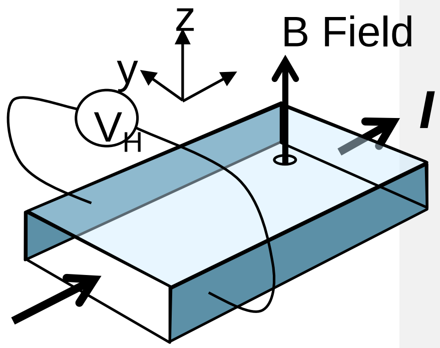

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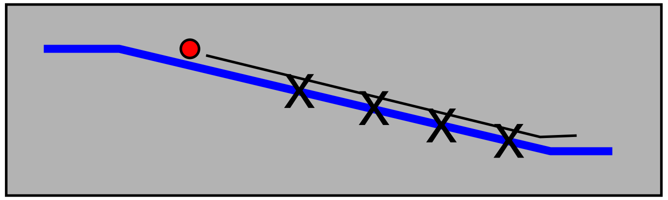

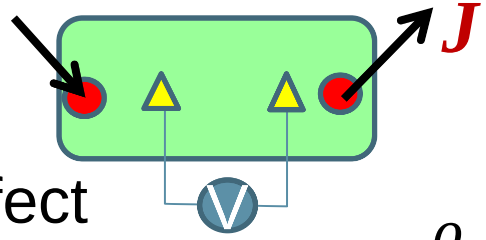

$$R_H = \frac{E_y / B_z}{J_x} = -\frac{1}{qn}$$

Vid

- 17.3 Carrier Concentration from Hall Effect

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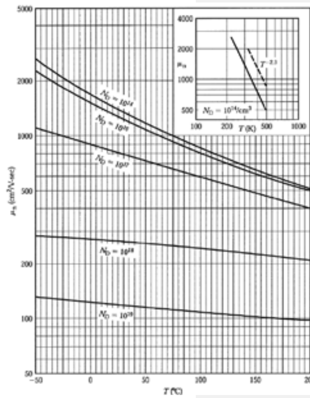
- 17.4 Physics of diffusion – Einstein Relationship

$$\rho_{n-type} = \frac{1}{q\mu_n N_D}$$



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