Section 17
Intro to Transport – Drift, Mobility, Diffusion, Einstein Relationship

17.2 Mobility

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PURDUE UNIVERSITY

School of Electrical and Computer Engineering

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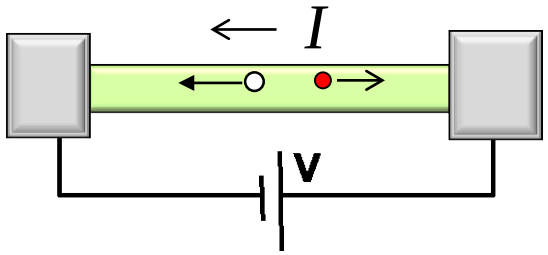

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charge density velocity area

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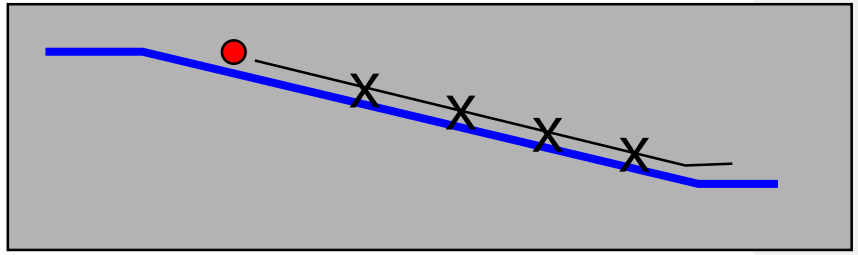

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Vid
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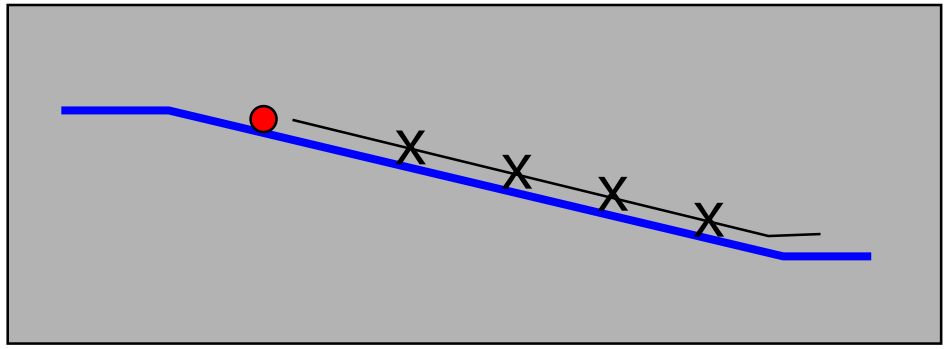
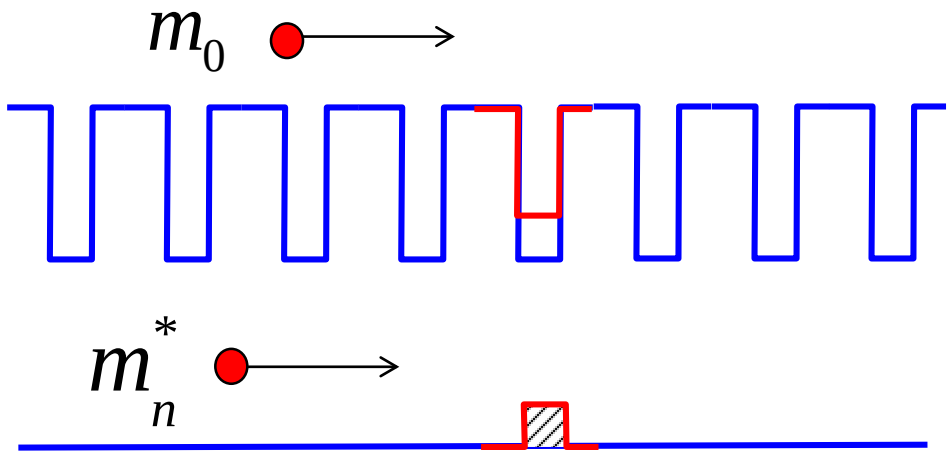
Consider system to be in local equilibrium

Mobility and Physics of Scattering Time



$$\mu_n = \frac{q\tau_n}{m_n^*}$$



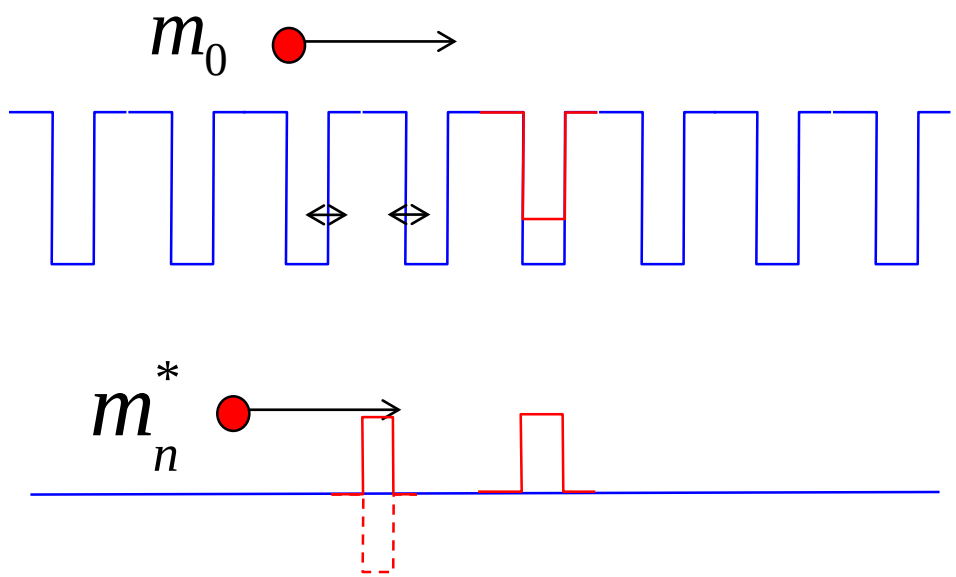
Fermi's Golden rule …

$$\tau_n^{-1} \sim \left| \frac{2\pi}{\hbar} \int_{-\infty}^{\infty} \psi^*(x) U(x) \psi(x) dx \right|^2$$

Phonon and Ionized Impurity Scattering

Ionized impurity

$$\tau_n \sim \frac{T^{3/2}}{N_D}$$

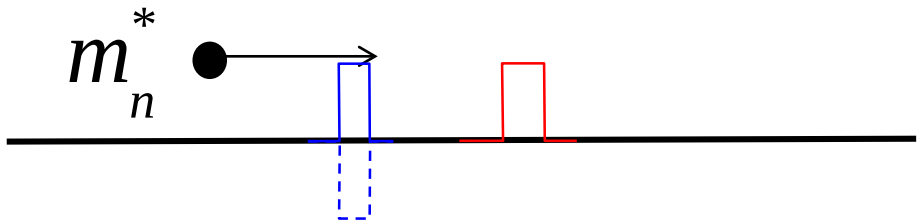


Higher temperature,
more phonon scattering

$$\tau_n \sim T^{-3/2}$$

Multiple Scattering Events

- Ionized impurity
- Phonon scattering
- others ….



$$\frac{1}{\mu_n} = \frac{1}{\mu_{ph}} + \frac{1}{\mu_{II}}$$

$$\Rightarrow \mu_n = \frac{\mu_{ph}\mu_{II}}{\mu_{ph} + \mu_{II}}$$

$$= \mu_{\min} + \left(\frac{\mu_{ph}\mu_{II}}{\mu_{ph} + \mu_{II}} - \mu_{\min} \right)$$

$$= \mu_{\min} + \left(\frac{\mu_0}{1 + \left(N_I / N_0 \right)^{\alpha}} \right)$$

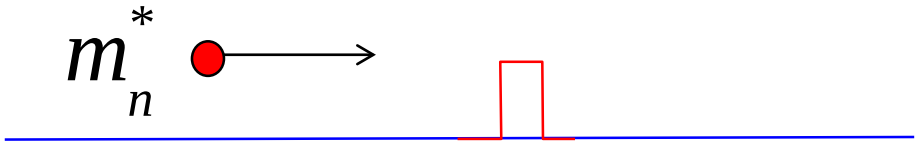
$$\frac{1}{\tau_n} = \frac{1}{\tau_{II}} + \frac{1}{\tau_{ph}} + \frac{1}{\tau_s} + \cdots$$

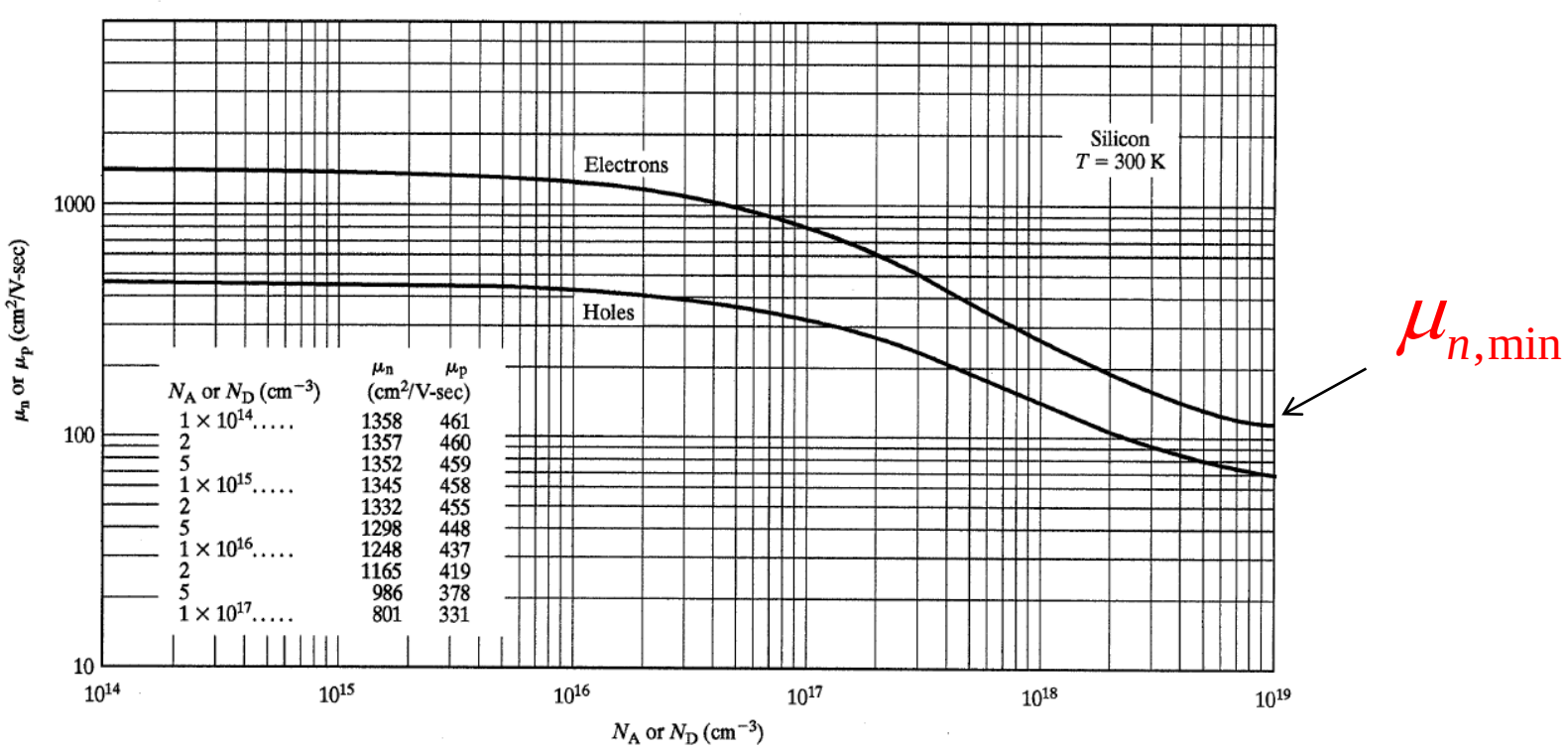
$$\frac{1}{\mu_n} = \frac{m_n^*}{q\tau_n}$$

Matthiessen Rule ….

Model for Ionized impurity Scattering

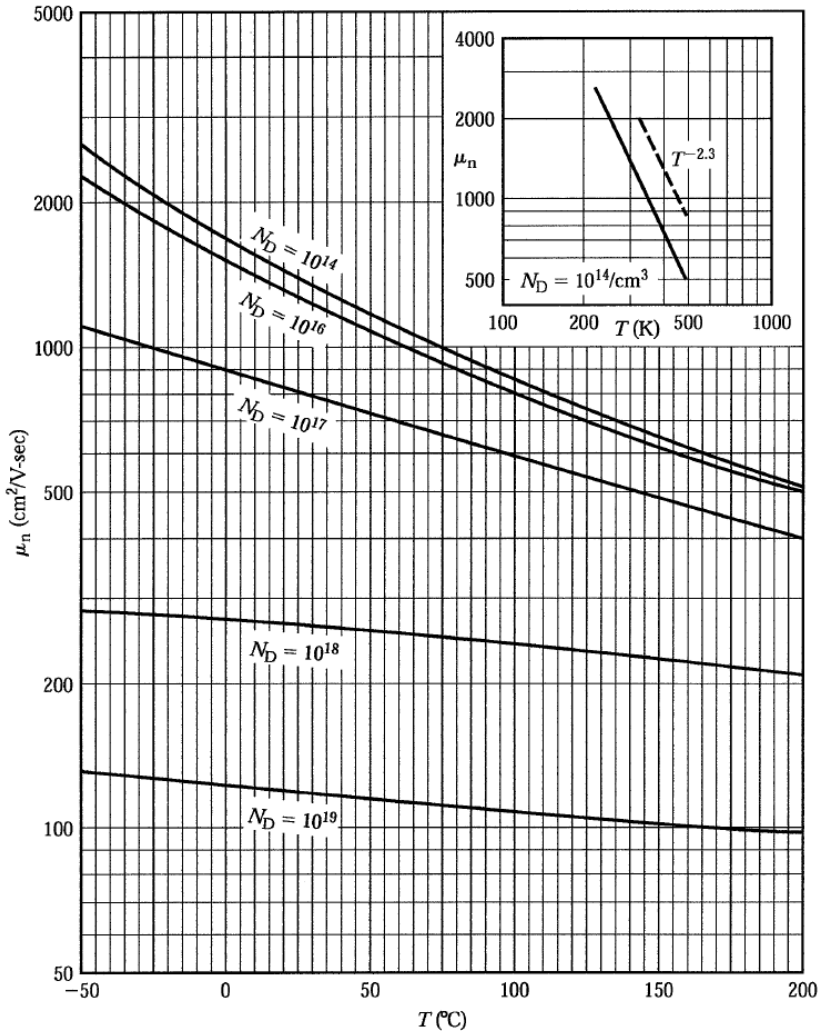
$$\mu_n = \mu_{n,\min} + \left(\frac{\mu_{0,n}}{1 + \left(N_I / N_{0,n} \right)^{\alpha_n}} \right)$$

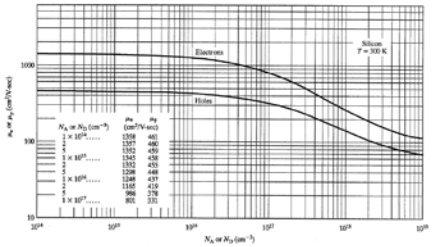




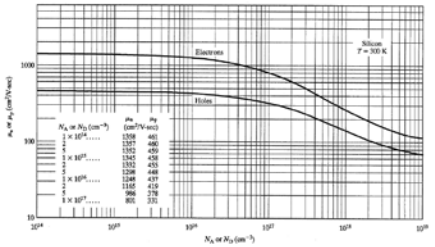
Temperature-dependent Mobility

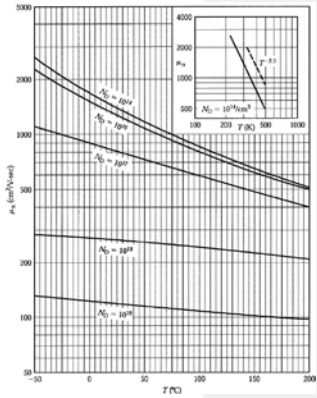
$$\mu_n \sim \tau_n \sim T^{\frac{-3}{2}}$$





Intermediate Summary





- Poisson and drift-diffusion equations
  - form a complete semi-classical transport model
  - explain wide variety of device phenomena.
- Drift current results from response of electrons/holes to electric field.
- The physics of mobility is complex and material dependent.
- Constancy of low-field mobility can be checked by experiments.

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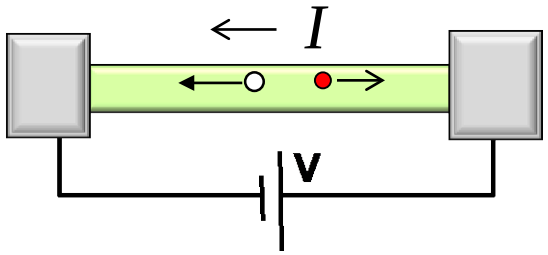

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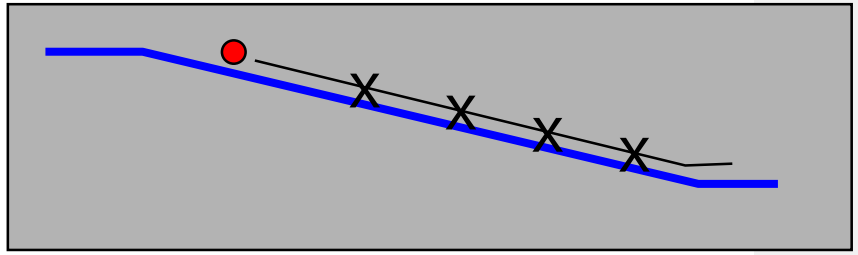

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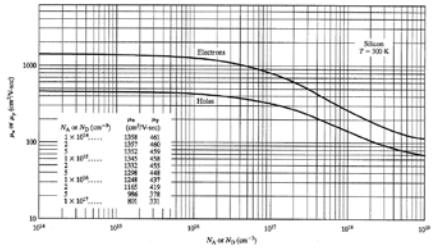

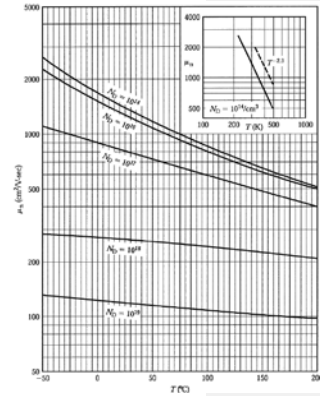

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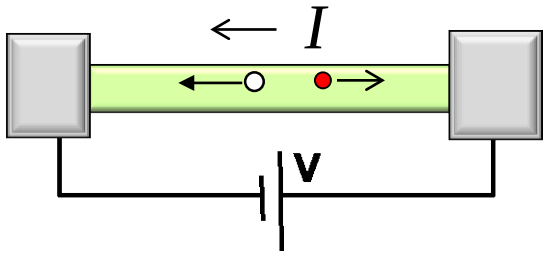

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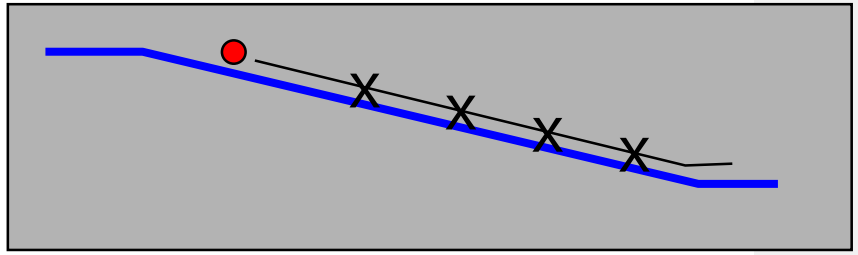

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status

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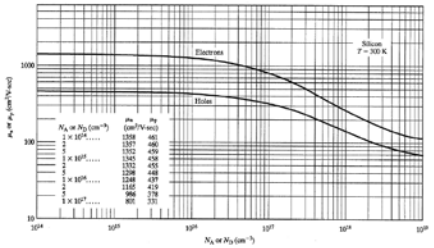

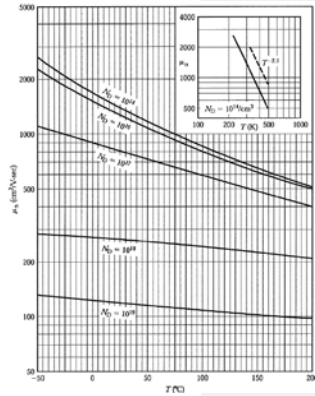

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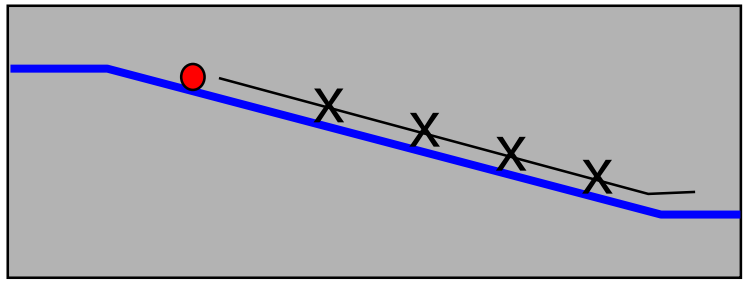
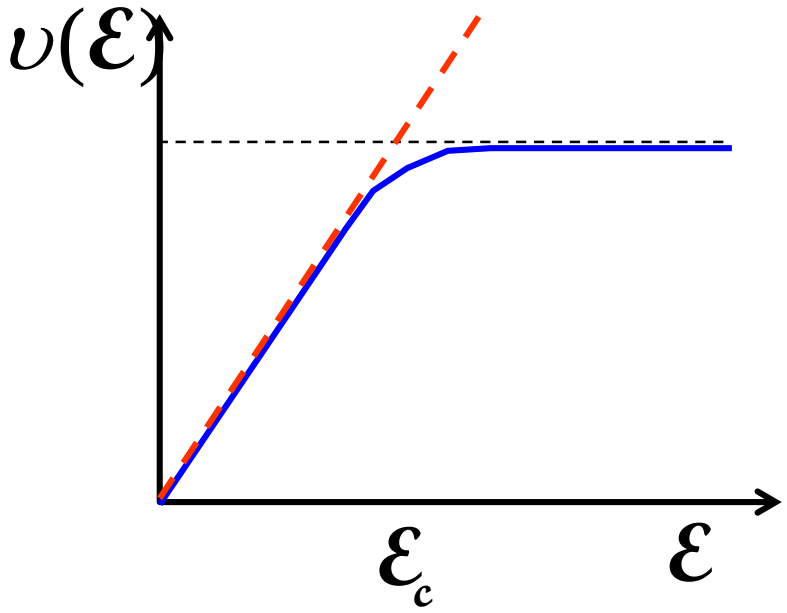
Mobility at High Fields?



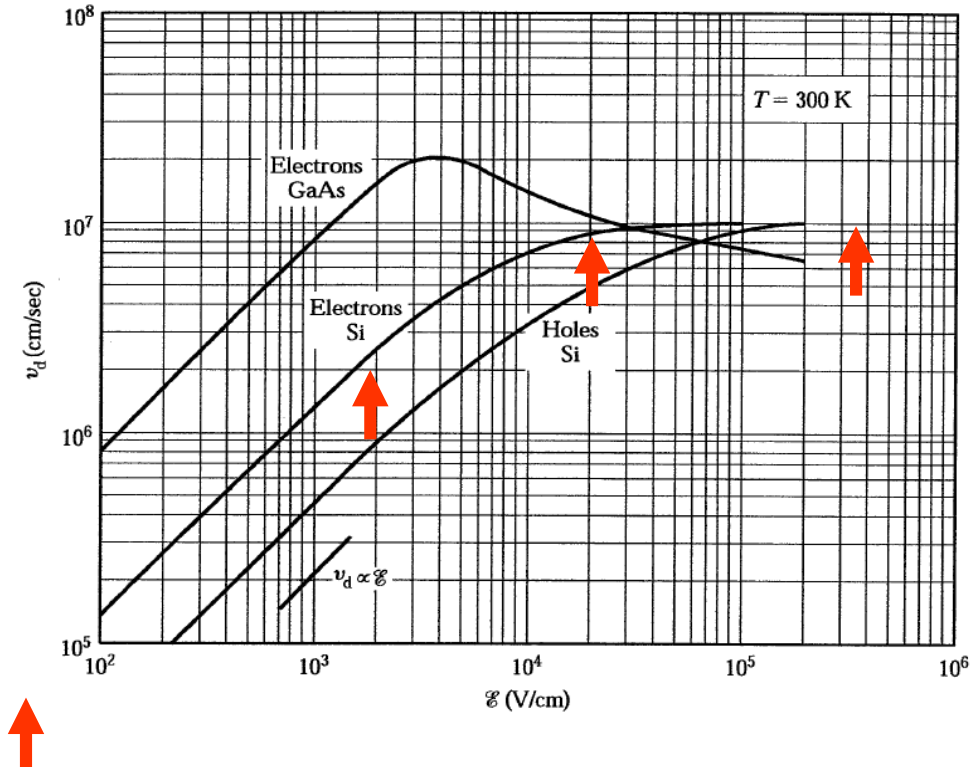
$$\upsilon = \frac{q\tau_N}{m_N^*}\mathcal{E}$$

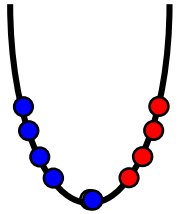


What causes velocity saturation at high fields?

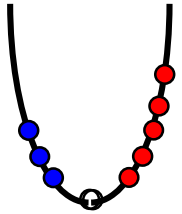
Where do the various mobility formula in device simulator come from?

Velocity Saturation in Si/Ge

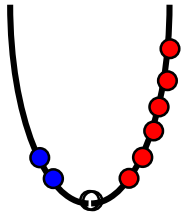




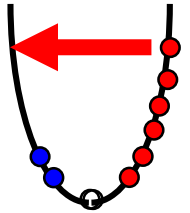
$\mathcal{E} = 0 \quad J_1 = J^+ - J^- = 0$



$\mathrm{E} \ll \mathrm{E}_c \quad J_2 = J^+ - J^- > J_1$

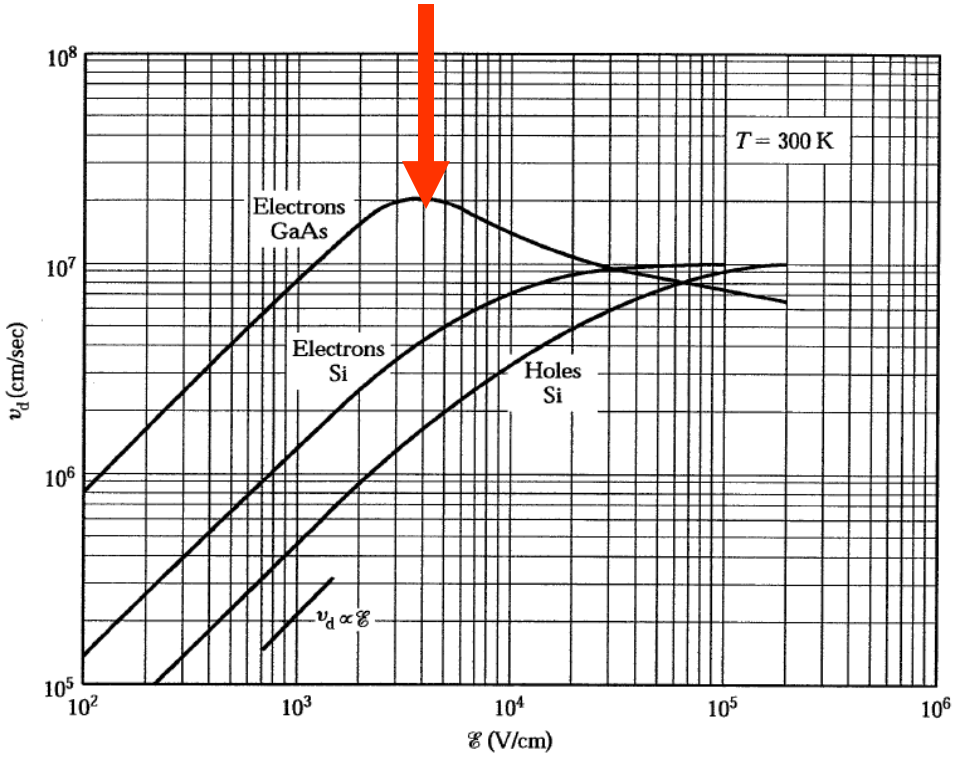


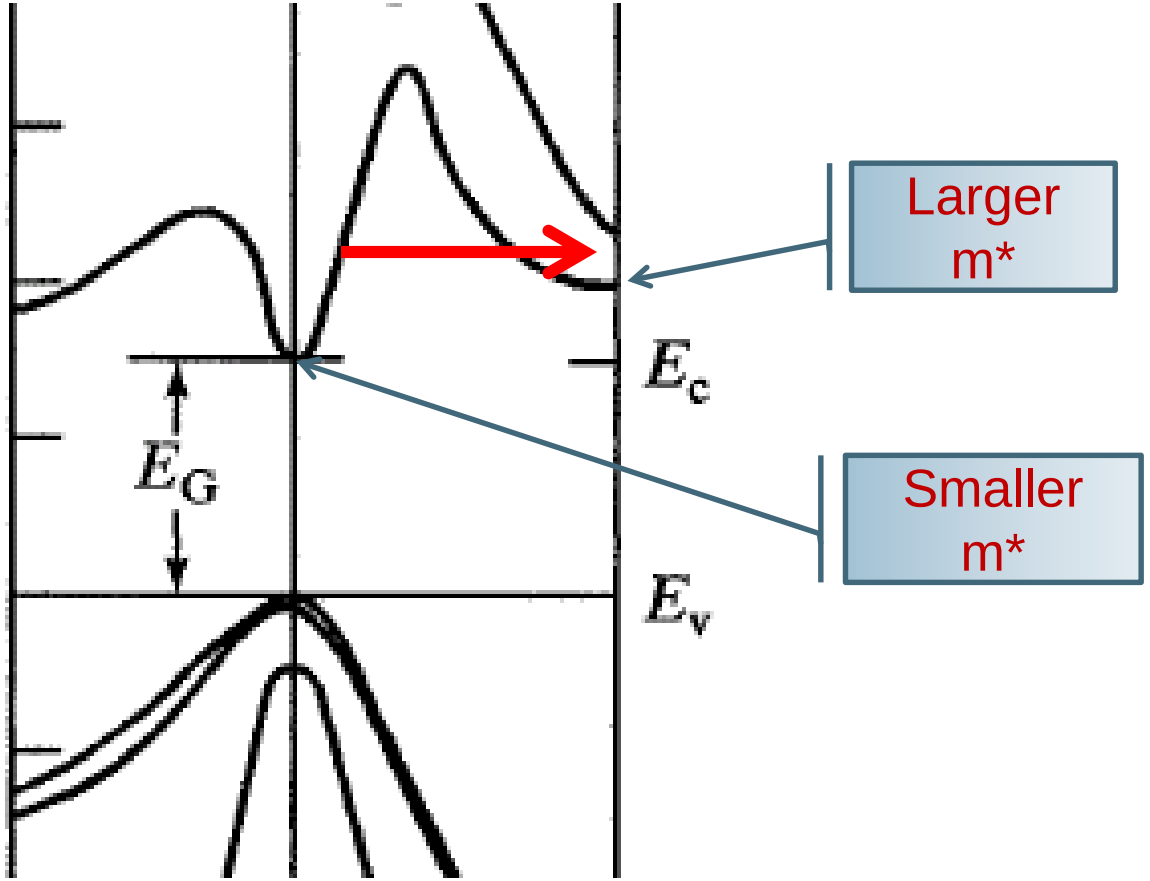
$\mathcal{E} \approx \mathcal{E}_c \quad J_3 = J^+ - J^- > J_2$



$\mathrm{E} \gg \mathrm{E}_c \quad J_4 = J^+ - J^- \approx J_3$

Velocity Overshoot & Inter-valley Transfer





What type of scattering would you need for inter-valley transfer?

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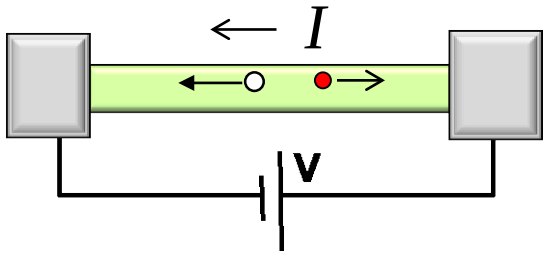

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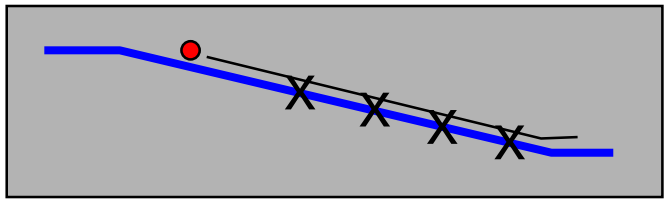

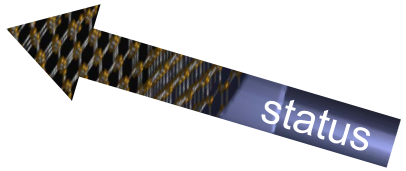

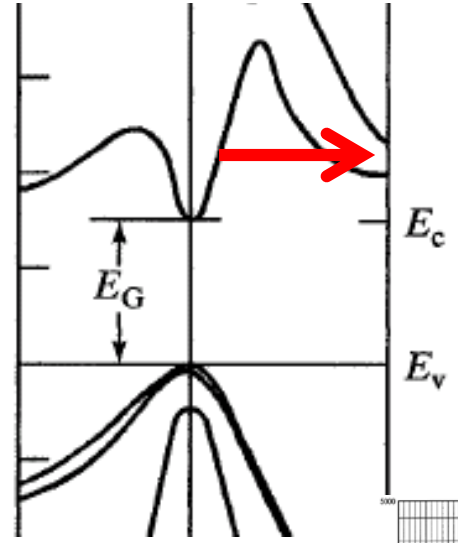

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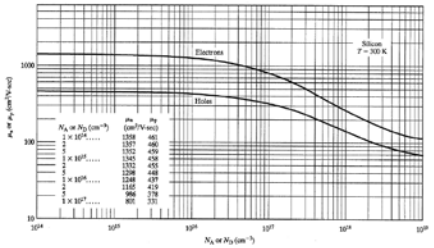

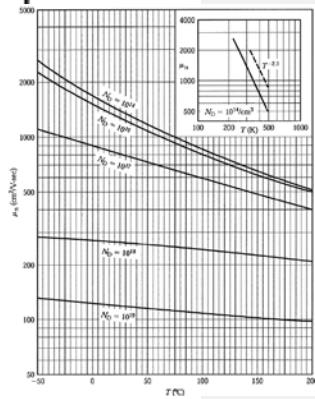

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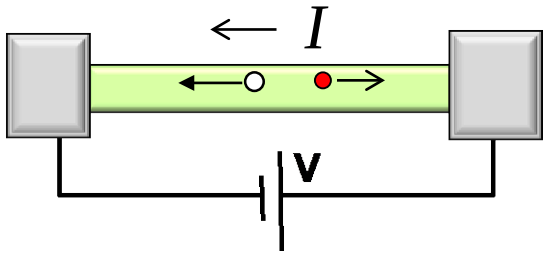

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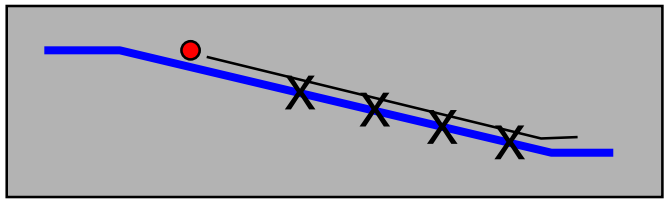

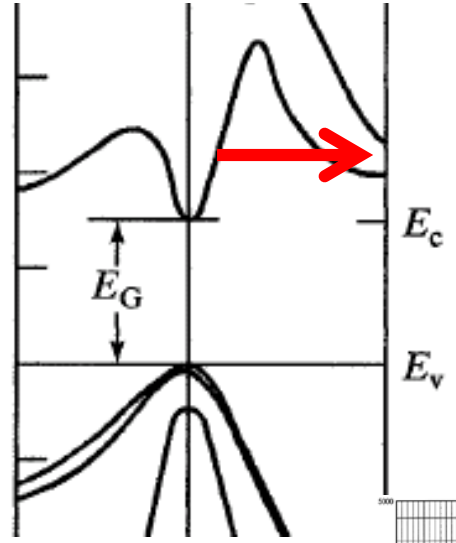

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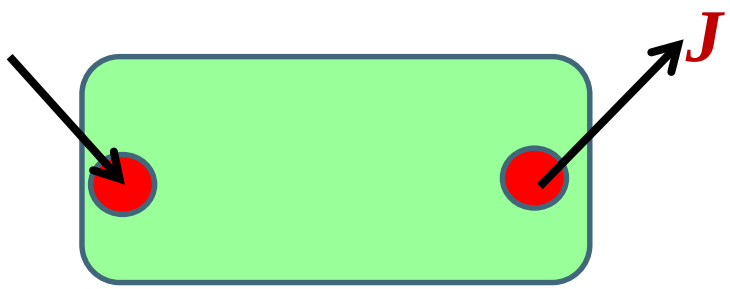
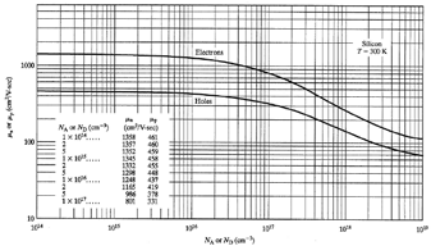
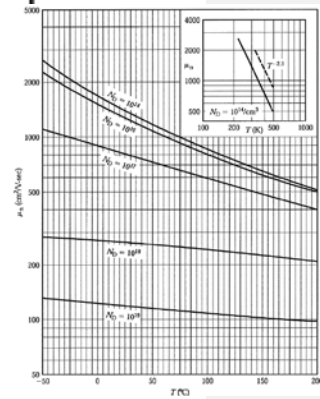
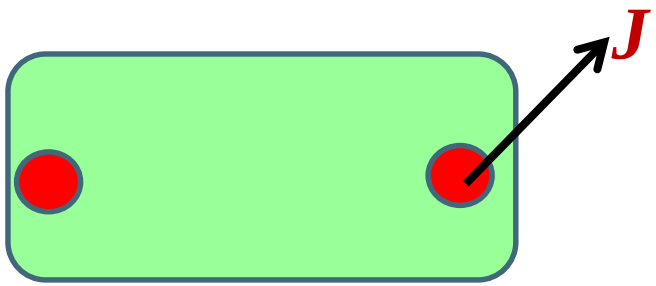


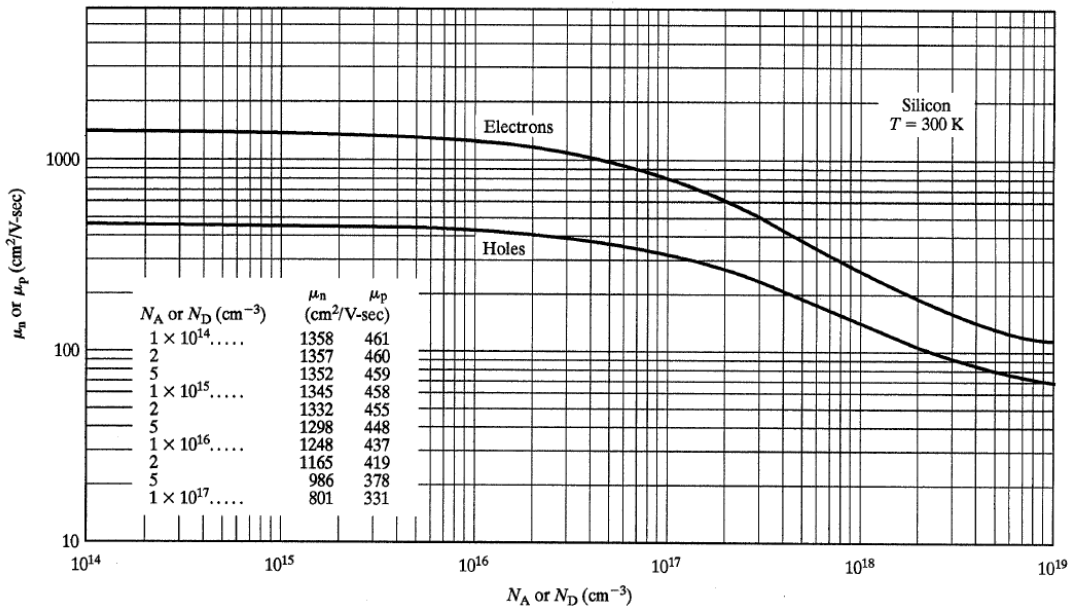




Consider system to be in local equilibrium

Problem of mobility measurement – unknown doping concentration





Can we find out the doping concentration and type by an *electrical measurement* without any knowledge of how the sample was prepared?

1 -http://en.wikipedia.org/wiki/Four-terminal_sensing

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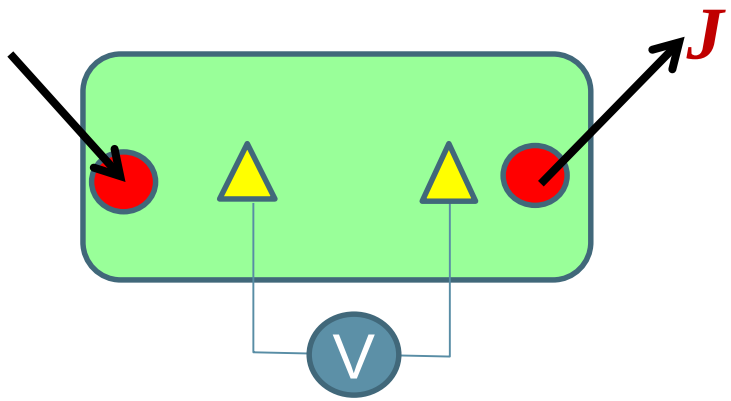
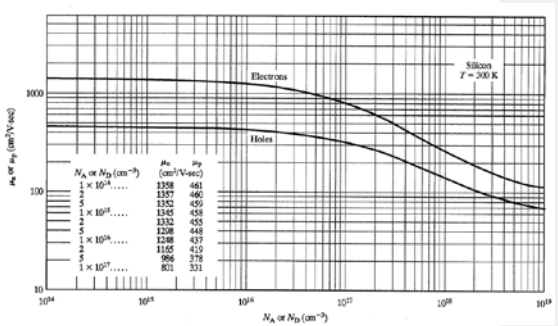


Four-Probe Measurement[1] measures voltage of device without measuring drop in current carrying wires



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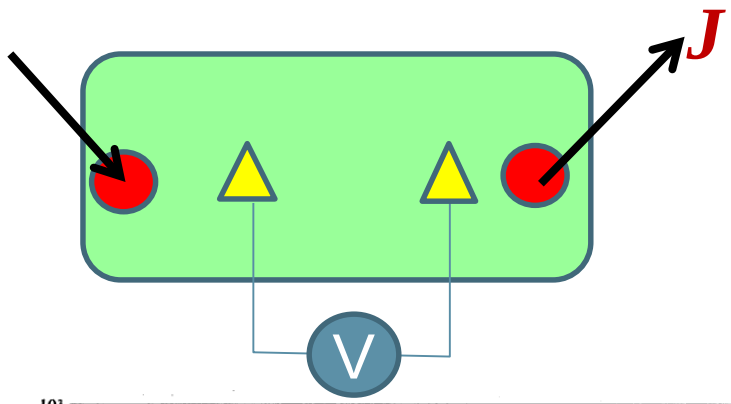
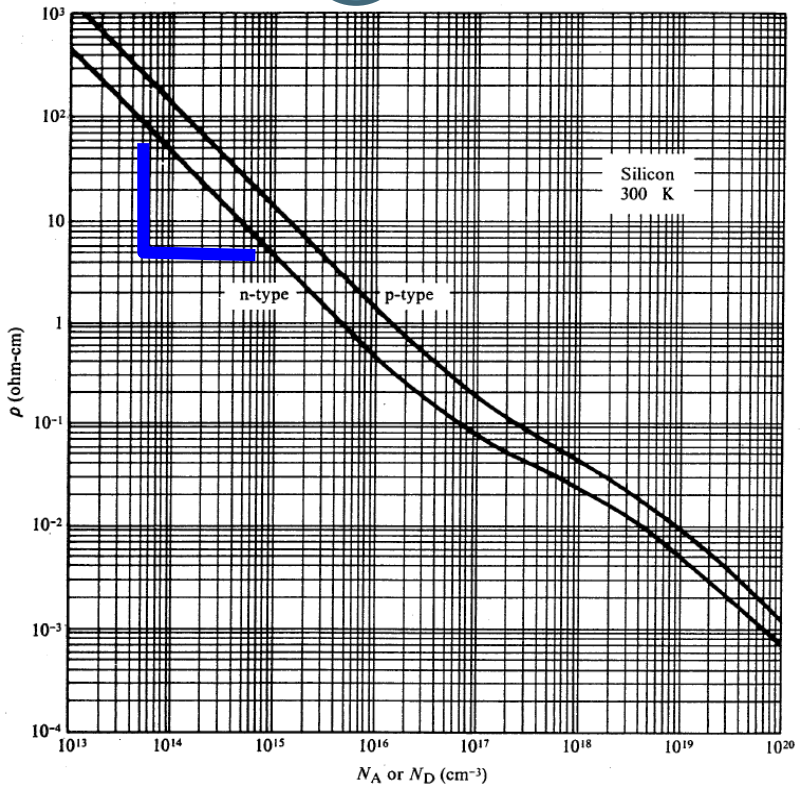
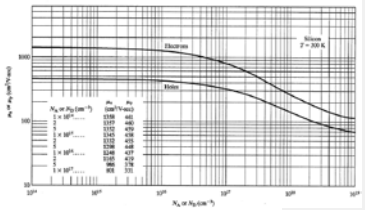




Four-Probe Measurement[1] measures voltage of device without measuring drop in current carrying wires



$$\mathcal{E} = \rho J$$

(in the low voltage limit where field-voltage curve is linear)

$$J = q(\mu_n n + \mu_p p)\mathcal{E}$$

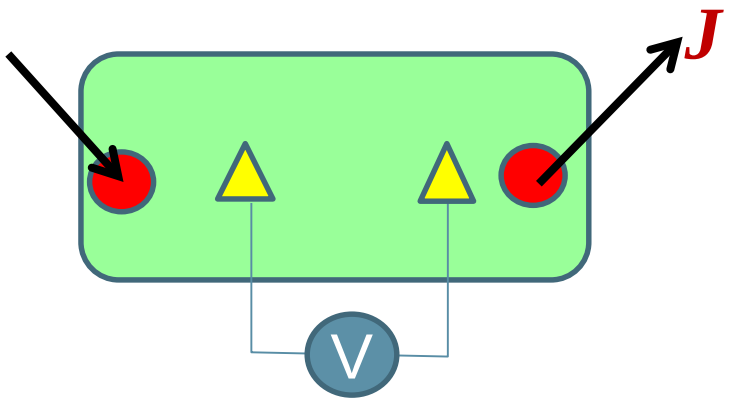
$$\rho = \frac{1}{q(\mu_n n + \mu_p p)}$$

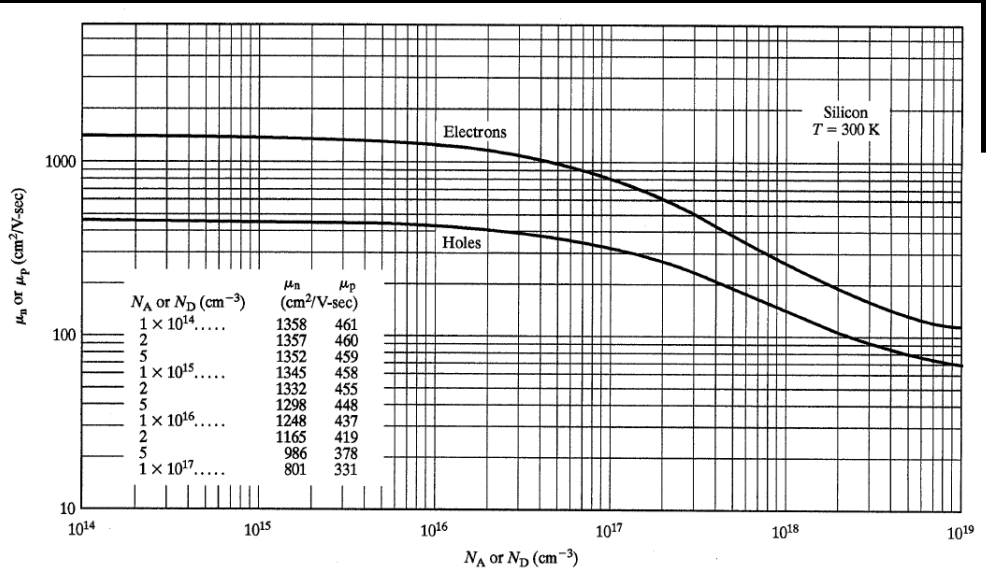
$$\rho_{n-type} = \frac{1}{q\mu_n N_D} \qquad N_D \approx n \gg p$$

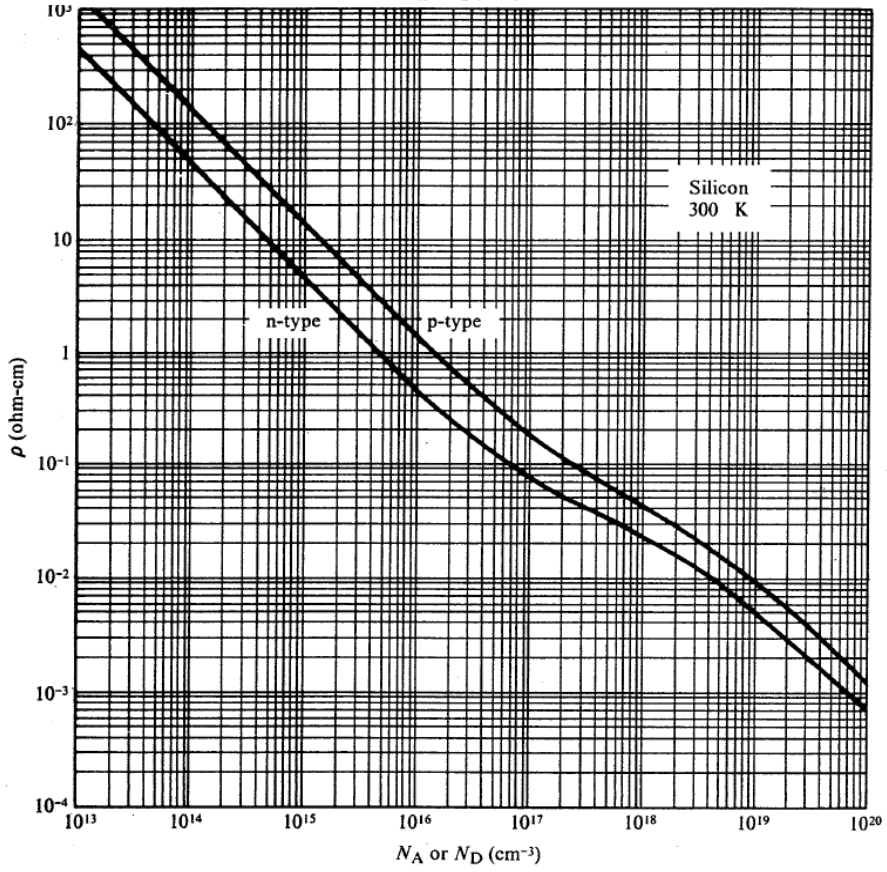
$$\rho_{p-type} = \frac{1}{q\mu_p N_A} \qquad N_A \approx p \gg n$$

1 -http://en.wikipedia.org/wiki/Four-terminal_sensing

Problem of mobility measurement

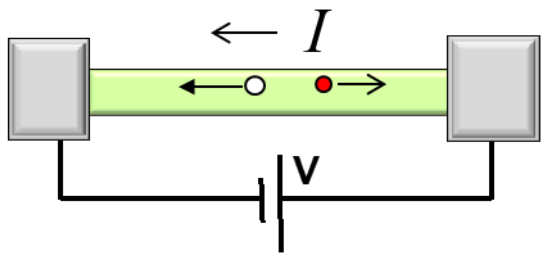






$$\rho_{n-type} = \frac{1}{q\mu_n N_D}$$

Section 17
Intro to Transport – Drift, Mobility, Diffusion, Einstein Relationship



$I = G \times V$

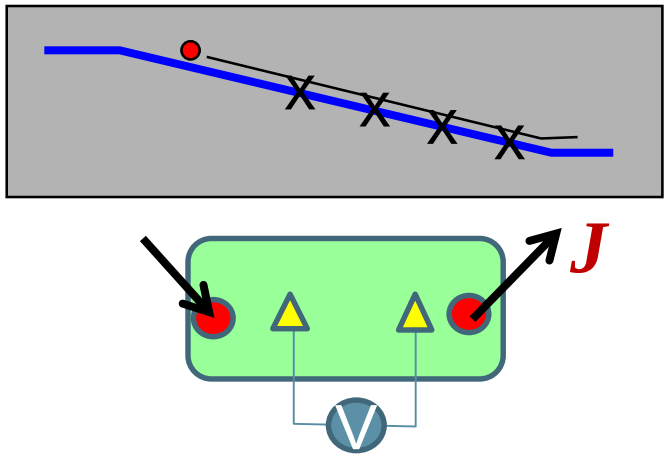
$= q \times n \times v \times A$

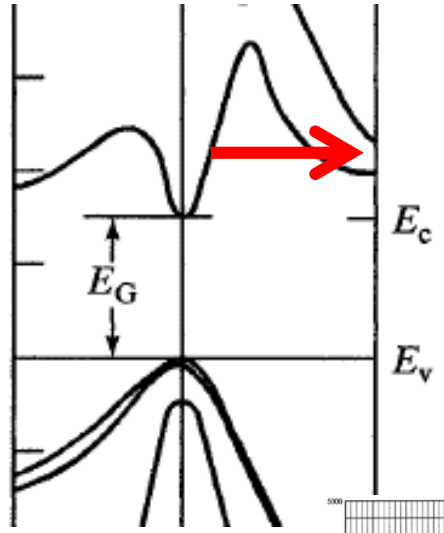
q: charge, n: density, v: velocity, A: area

Transport with scattering, non-equilibrium Stat. Mech.
- Drift-diffusion equation with recombination-generation

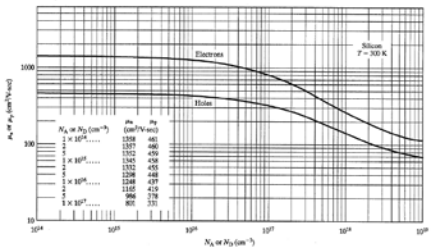
Vid
Vid
Vi
Vid

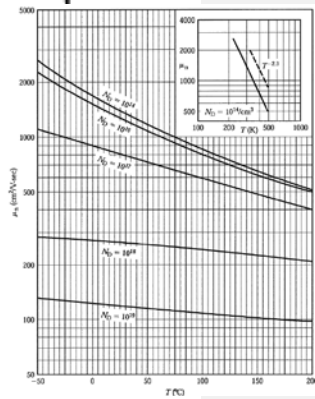
- 17.1 Drift Current $J_n = qn\mu_n\mathcal{E}$
- 17.2 Mobility
  - »Matthiessen Rule $\frac{1}{\mu_n} = \frac{1}{\mu_{ph}} + \frac{1}{\mu_{II}}$
  - »High Field Effects
  - »Mobility Measurement
- 17.3
- 17.4





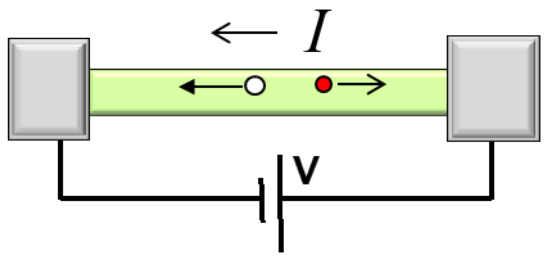
$$\rho_{n-type} = \frac{1}{q\mu_n N_D}$$





Consider system to be in local equilibrium

Section 17
Intro to Transport – Drift, Mobility, Diffusion, Einstein Relationship



$$I = G \times V$$
$$= q \times n \times v \times A$$

charge density velocity area

Transport with scattering, non-equilibrium Stat. Mech.
- Drift-diffusion equation with recombination-generation

Vid
Vid
Vi
Vid

- 17.1 Drift Current $J_n = qn\mu_n\mathcal{E}$
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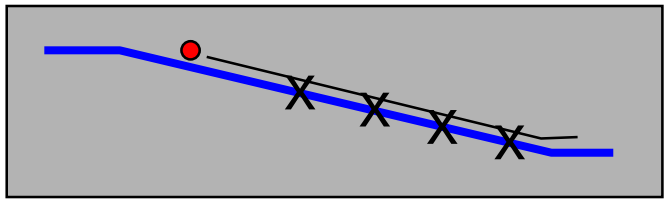
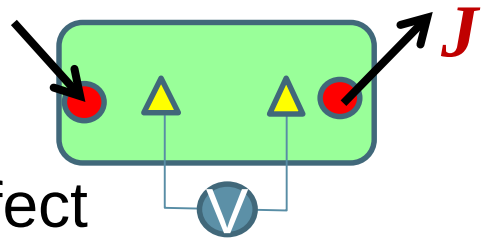
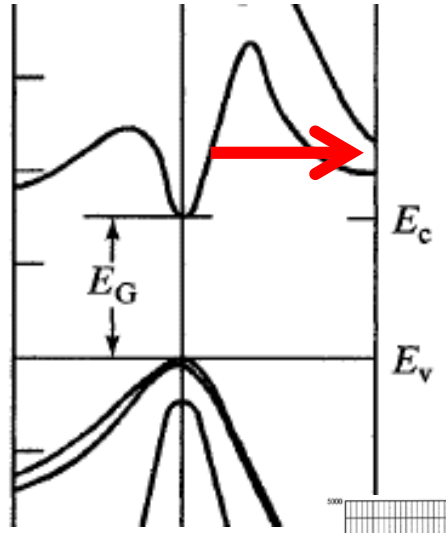
  - »Matthiessen Rule $\frac{1}{\mu_n} = \frac{1}{\mu_{ph}} + \frac{1}{\mu_{II}}$
  - »High Field Effects
  - »Mobility Measurement
- 17.3 Carrier Concentration from Hall Effect
- 17.4

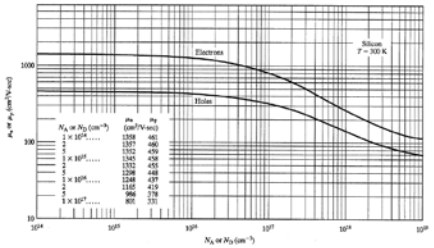


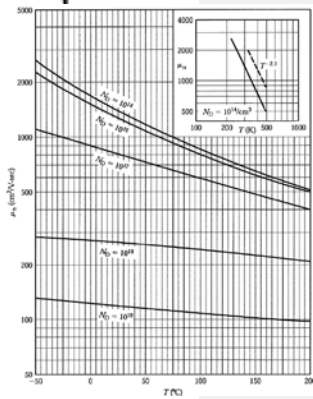




μ_n *and* N_D are coupled
need the electron density
=> get the doping

$$\rho_{n-type} = \frac{1}{q\mu_n N_D}$$





Consider system to be in local equilibrium