Section 16
Recombination & Generation
16.2 Derivation of SRH formula (Shockley, Reed, Hall)
16.2.2 Capture and emission relationship (n_1 and p_1)

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n_T electron-filled traps

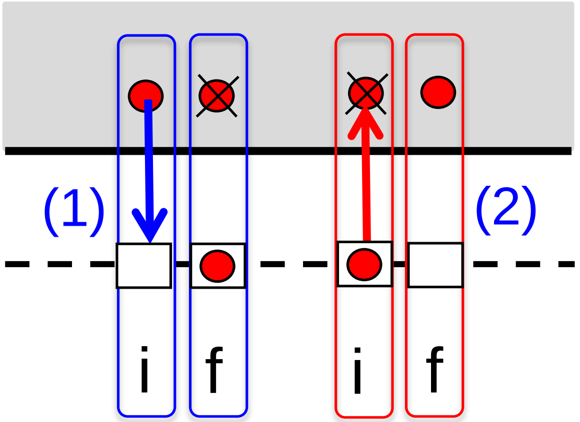
p_T hole-filled (empty) traps

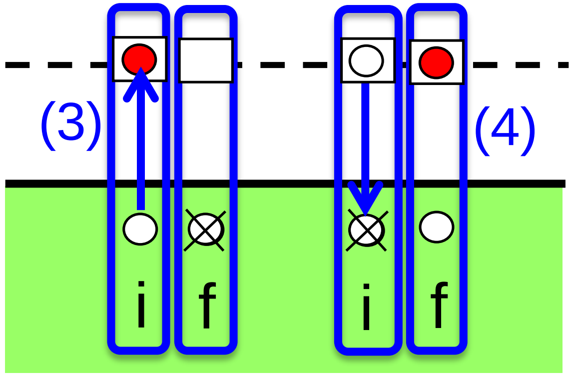
c_n electron capture coef.

c_p hole capture coef.

e_n electron emission coef.

e_p hole emission coef.





$$\left.\frac{\partial n}{\partial t}\right|_{1,2} = -c_n n p_T + e_n n_T (1 - f_c)$$

$$\left.\frac{\partial p}{\partial t}\right|_{3,4} = -c_p p n_T + e_p p_T f_v$$

Non-Degeneracy Assumption

n_T electron-filled traps

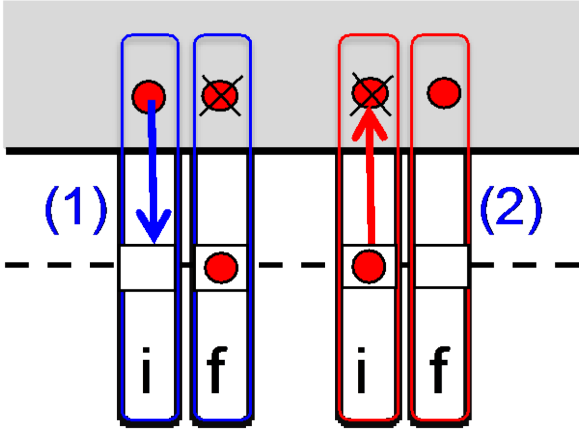
p_T hole-filled (empty) traps

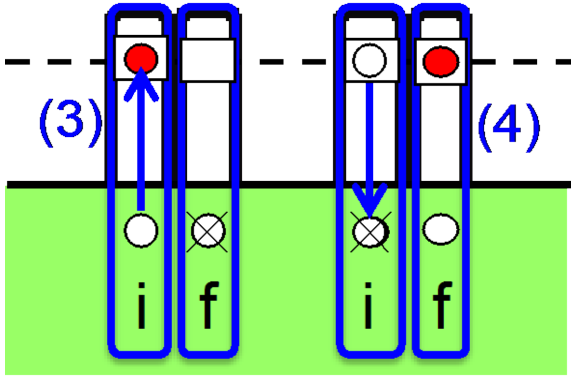
c_n electron capture coef.

c_p hole capture coef.

e_n electron emission coef.

e_p hole emission coef.





$$\left.\frac{\partial n}{\partial t}\right|_{1,2} = -c_n n p_T + e_n n_T (1 - f_c)$$

$$\left.\frac{\partial p}{\partial t}\right|_{3,4} = -c_p p n_T + e_p p_T f_v$$

Assume non-degenerate

Empty conduction band

$$(1 - f_c) \approx 1$$

Empty valence band

$$f_v \approx 1$$

$$\left.\frac{\partial n}{\partial t}\right|_{1,2} = -c_n n p_T + e_n n_T$$

$$\left.\frac{\partial p}{\partial t}\right|_{3,4} = -c_p p n_T + e_p p_T$$

Equilibrium - Detailed Balance

n_T electron-filled traps

p_T hole-filled (empty) traps

c_n electron capture coef.

c_p hole capture coef.

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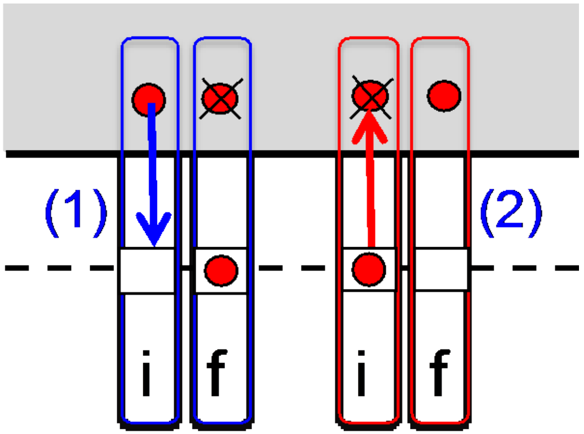
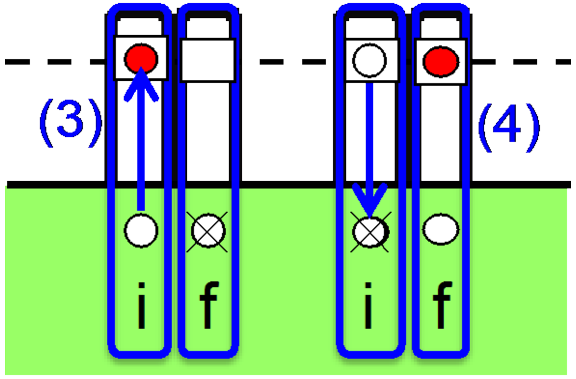
e_p hole emission coef.





$$\left.\frac{\partial n}{\partial t}\right|_{1,2} = -c_n n p_T + e_n n_T$$

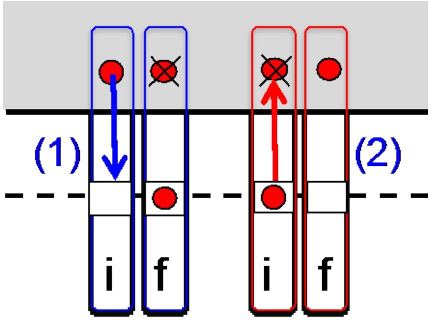
$$\left.\frac{\partial p}{\partial t}\right|_{3,4} = -c_p p n_T + e_p p_T$$

Subscripts 0
indicate equilibrium

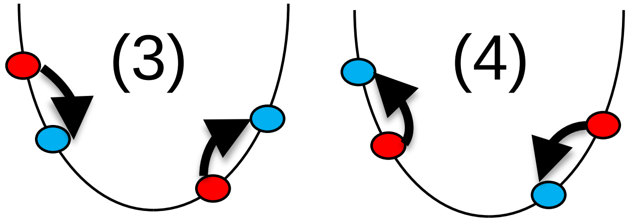
$$\left.\frac{\partial n_0}{\partial t}\right|_{1,2} = -c_{n0} n_0 p_{T0} + e_{n0} n_{T0}$$

$$\left.\frac{\partial p_0}{\partial t}\right|_{3,4} = -c_{p0} p_0 n_{T0} + e_{p0} p_{T0}$$

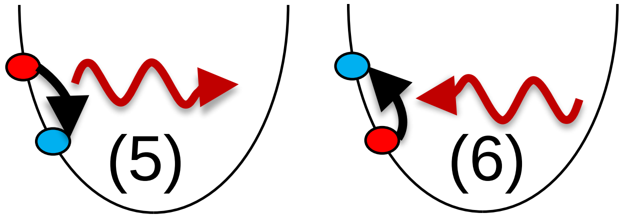
Equilibrium – vs. Steady State



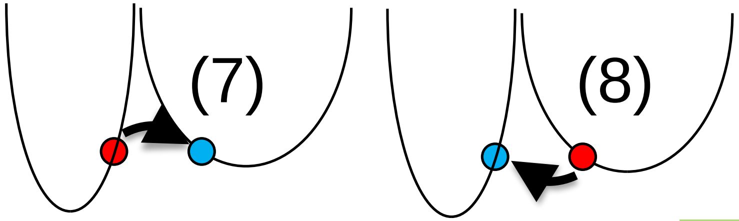
R-G $\left.\frac{\partial n_0}{\partial t}\right|_{1,2} = -c_{n0} n_0 p_{T0} + e_{n0} n_{T0}$



e-e $\left.\frac{\partial n_0}{\partial t}\right|_{3,4} = R_3 + R_4$



e-ph $\left.\frac{\partial n_0}{\partial t}\right|_{5,6} = R_5 + R_6$

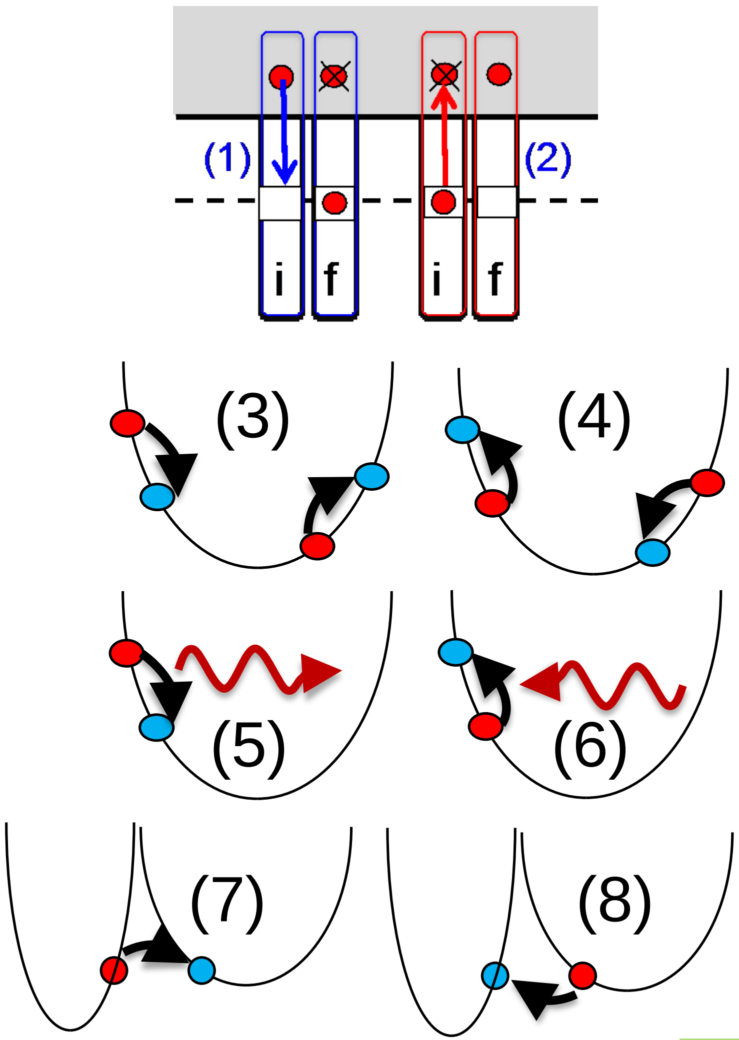


Γ-X $\left.\frac{\partial n_0}{\partial t}\right|_{7,8} = R_7 + R_8$

Steady State $\frac{d}{dt}=0$

$$\frac{dn_0}{dt} = \left.\frac{\partial n_0}{\partial t}\right|_{1,2} + \left.\frac{\partial n_0}{\partial t}\right|_{3,4} + \left.\frac{\partial n_0}{\partial t}\right|_{5,6} + \left.\frac{\partial n_0}{\partial t}\right|_{7,8} + \ldots = 0$$

Equilibrium – vs. Steady State



Equilibrium

R-G $\left.\frac{\partial n_0}{\partial t}\right|_{1,2} = -c_{n0} n_0 p_{T0} + e_{n0} n_{T0}$ =0

e-e $\left.\frac{\partial n_0}{\partial t}\right|_{3,4} = R_3 + R_4$ =0 $\quad R_3 = -R_4$

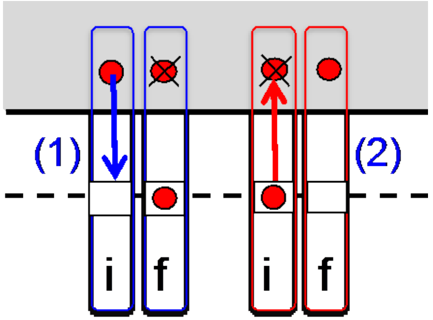
e-ph $\left.\frac{\partial n_0}{\partial t}\right|_{5,6} = R_5 + R_6$ =0 $\quad R_5 = -R_6$

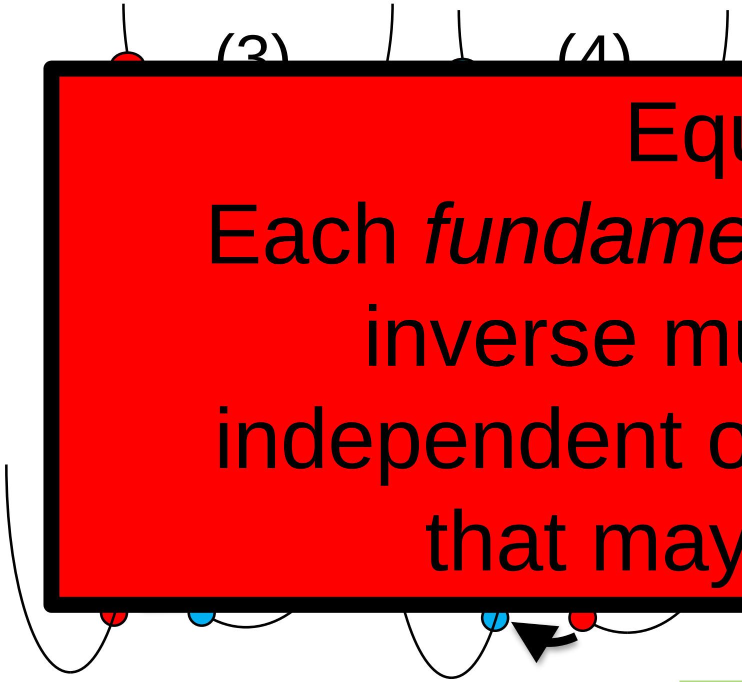
Γ-X $\left.\frac{\partial n_0}{\partial t}\right|_{7,8} = R_7 + R_8$ =0 $\quad R_7 = -R_7$

Steady State $\frac{d}{dt}$=0

$$\frac{dn_0}{dt} = \left.\frac{\partial n_0}{\partial t}\right|_{1,2} + \left.\frac{\partial n_0}{\partial t}\right|_{3,4} + \left.\frac{\partial n_0}{\partial t}\right|_{5,6} + \left.\frac{\partial n_0}{\partial t}\right|_{7,8} + \ldots \quad =0$$

Equilibrium – vs. Steady State





Equilibrium

R-G $\left.\frac{\partial n_0}{\partial t}\right|_{1,2} = -c_{n0} n_0 p_{T0} + e_{n0} n_{T0}$ =0

$R_3 = -R_4$

$R_5 = -R_6$

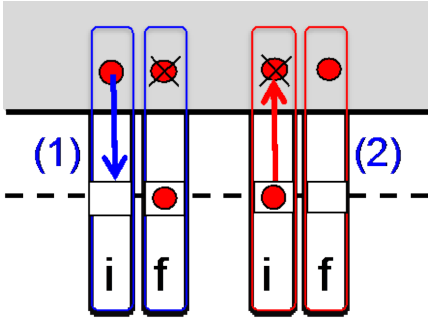
$R_7 = -R_7$

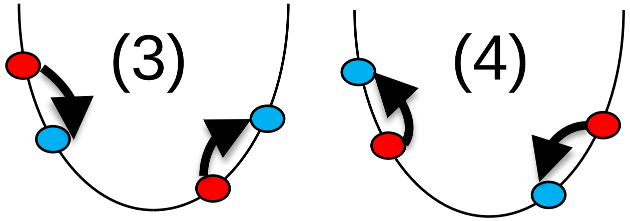
Equilibrium:
Each *fundamental* process and its inverse must *self-balance* independent of any other process that may be occurring.

Steady State $\frac{d}{dt}$=0

$\frac{dn_0}{dt} = \left.\frac{\partial n_0}{\partial t}\right|_{1,2} + \left.\frac{\partial n_0}{\partial t}\right|_{3,4} + \left.\frac{\partial n_0}{\partial t}\right|_{5,6} + \left.\frac{\partial n_0}{\partial t}\right|_{7,8} + \ldots$ =0

Equilibrium - Detailed Balance for Fermi Function





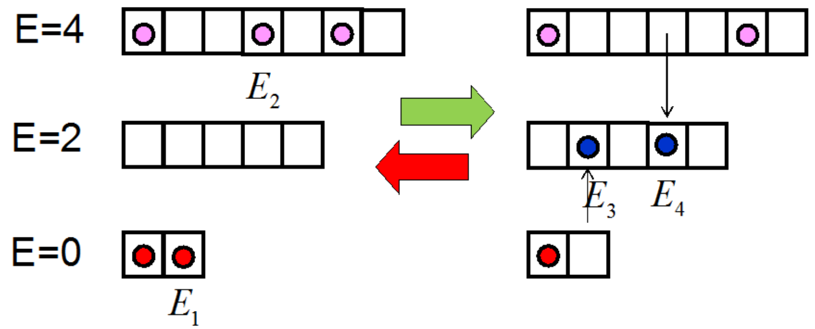
Equilibrium

R-G $\left.\frac{\partial n_0}{\partial t}\right|_{1,2} = -c_{n0} n_0 p_{T0} + e_{n0} n_{T0}$ =0

e-e $\left.\frac{\partial n_0}{\partial t}\right|_{3,4} = R_3 + R_4$ =0

$R_3 = -R_4$

Derivation by Detailed Balance



Detailed Balance in Equilibrium

$f_0(E_1) f_0(E_2)[1-f_0(E_3)][1-f_0(E_4)]$ Pauli Exclusion

$= f_0(E_3) f_0(E_4)[1-f_0(E_1)][1-f_0(E_2)]$

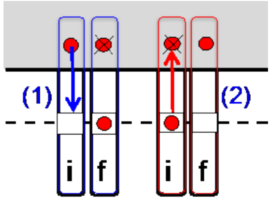
Energy conversation

$E_1 + E_2 = E_3 + E_4$ Only solution is …. $f_0(E) = \frac{1}{1+e^{\beta(E-E_F)}}$

- Pauli Principle, energy, and number conservation all satisfied

A Note on the Use of the Principle of Detailed Balance in Obtaining the Fermi-Dirac Distribution Function, John E. Walsh: American Journal of Physics38, 392 (1970); doi: 10.1119/1.1976344

Electron Emission in Equilibrium



n_T electron-filled traps

p_T hole-filled (empty) traps

c_n electron capture coef.

c_p hole capture coef.

e_n electron emission coef.

e_p hole emission coef.

R-G $$\left.\frac{\partial n_0}{\partial t}\right|_{1,2} = -c_{n0} n_0 p_{T0} + e_{n0} n_{T0} = 0$$

$$e_{n0} n_{T0} = c_{n0} n_0 p_{T0}$$

$$e_{n0} = c_{n0} \frac{n_0 p_{T0}}{n_{T0}}$$

$$el.\,emission = (el.\,capture) \times (el.\,density)$$

$$e_{n0} = c_{n0} \times n_1$$

$$e_{n0} = c_{n0}\, n_1 \qquad n_1 \equiv \frac{n_0 p_{T0}}{n_{T0}}$$

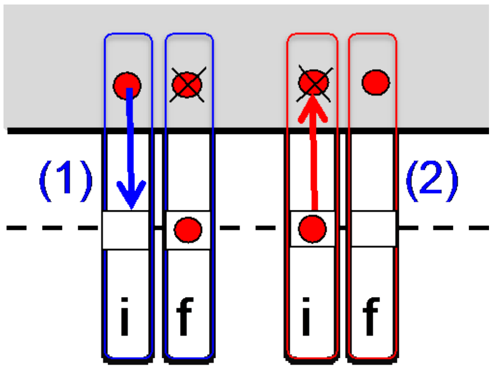
$$n_0 = n_i e^{\beta(E_F - E_i)} \qquad \beta = \frac{1}{k_T T}$$

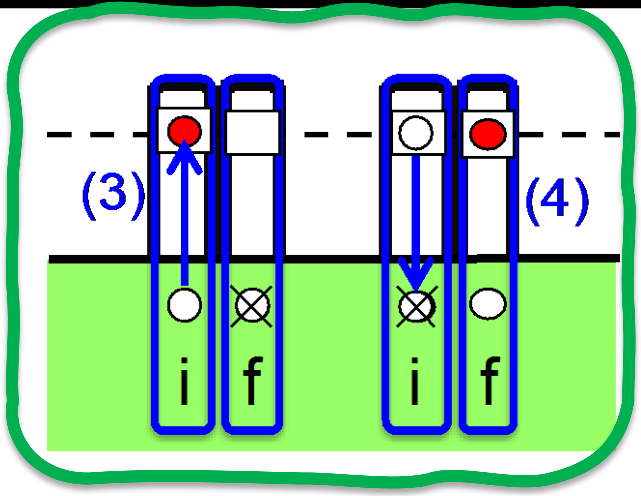
$$n_{T0} = N_T \frac{1}{1 + e^{\beta(E_T' - E_F)}}$$

$$p_{T0} = N_T - n_{T0}$$

computable constant

Hole Emission in Equilibrium





n_T electron-filled traps

p_T hole-filled (empty) traps

c_n electron capture coef.

c_p hole capture coef.

e_n electron emission coef.

e_p hole emission coef.

$$\left.\frac{\partial n_0}{\partial t}\right|_{1,2} = -c_{n0} n_0 p_{T0} + e_{n0} n_{T0} = 0$$

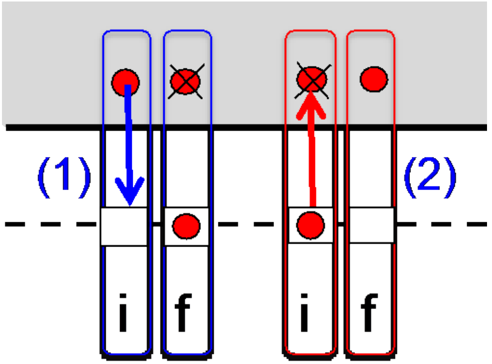
$$e_{n0} = c_{n0}\ n_1 \qquad n_1 \equiv \frac{n_0 p_{T0}}{n_{T0}}$$

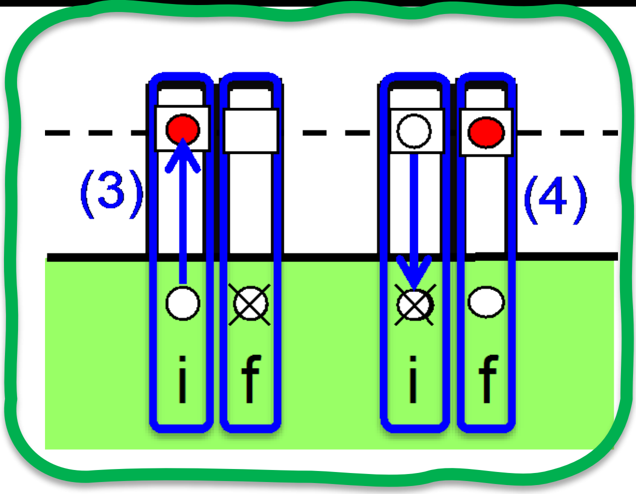
$$n_{T0} = N_T \frac{1}{1 + e^{\beta(E_T' - E_F)}}$$

$$n_0 = n_i e^{\beta(E_F - E_i)} \qquad \beta = \frac{1}{k_T T}$$

$$p_{T0} = N_T - n_{T0}$$

Hole Emission in Equilibrium





n_T electron-filled traps

p_T hole-filled (empty) traps

c_n electron capture coef.

c_p hole capture coef.

e_n electron emission coef.

e_p hole emission coef.

$$\left.\frac{\partial n_0}{\partial t}\right|_{1,2} = -c_{n0} n_0 p_{T0} + e_{n0} n_{T0} = 0$$

$$\left.\frac{\partial p_0}{\partial t}\right|_{3,4} = -c_{p0} p_0 n_{T0} + e_{p0} p_{T0} = 0$$

$$e_{n0} = c_{n0}\ n_1 \qquad n_1 \equiv \frac{n_0 p_{T0}}{n_{T0}}$$

$$e_{p0} = c_{p0}\ p_1 \qquad p_1 \equiv \frac{n_0 n_{T0}}{p_{T0}}$$

$$n_{T0} = N_T \frac{1}{1 + e^{\beta(E_T' - E_F)}}$$

$$n_0 = n_i e^{\beta(E_F - E_i)} \qquad \beta = \frac{1}{k_T T}$$

$$p_{T0} = N_T - n_{T0}$$

Relationship between n_1 and p_1

$$n_1 \equiv \frac{n_0 p_{T0}}{n_{T0}} \qquad p_1 \equiv \frac{n_0 n_{T0}}{p_{T0}}$$

$$n_1 p_1 = \frac{n_0 p_{T0}}{n_{T0}} \times \frac{p_0 n_{T0}}{p_{T0}} = n_0 p_0 = n_i^2$$

$$p_1 n_1 = n_i^2$$

Law of Mass Action (again)

n_T electron-filled traps

p_T hole-filled (empty) traps

c_n electron capture coef.

c_p hole capture coef.

e_n electron emission coef.

e_p hole emission coef. $\beta = \frac{1}{k_T T}$

$$n_0 = n_i e^{\beta(E_F - E_i)}$$

$$n_{T0} = N_T \frac{1}{1 + e^{\beta(E'_T - E_F)}}$$

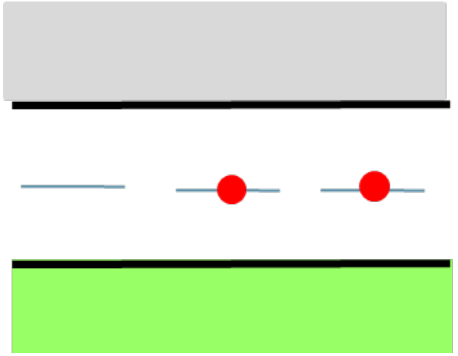
$$p_{T0} = N_T - n_{T0}$$

Expressions for (n_1) and (p_1)

$$n_1 \equiv \frac{n_0 p_{T0}}{n_{T0}} \qquad p_1 \equiv \frac{n_0 n_{T0}}{p_{T0}}$$

Trap is like a donor!

f_{00} empty trap prob.



E_T

n_T electron-filled traps

p_T hole-filled (empty) traps

c_n electron capture coef.

c_p hole capture coef.

e_n electron emission coef.

e_p hole emission coef. $\beta = \dfrac{1}{k_T T}$

$$n_{T0} = N_T \frac{1}{1 + e^{\beta(E_T' - E_F)}}$$

$$n_0 = n_i e^{\beta(E_F - E_i)}$$

$$n_{T0} = N_T(1 - f_{00}) = \frac{N_T}{1 + g_D e^{\beta(E_T - E_F)}} = N_T \frac{1}{1 + g\ exp}$$

$$p_{T0} = N_T - n_{T0}$$

$$g\ exp \equiv g_D e^{\beta(E_T - E_F)}$$

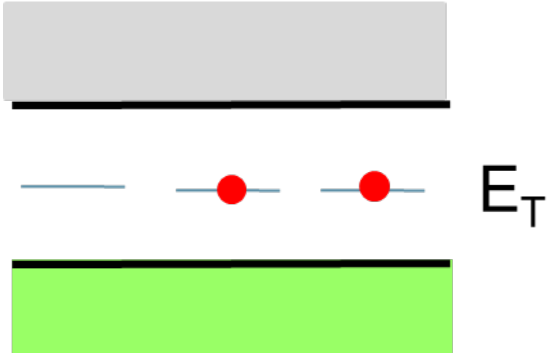
$$p_1 n_1 = n_i^2$$

Expressions for (n_1) and (p_1)

$$n_1 \equiv \frac{n_0 p_{T0}}{n_{T0}} \qquad p_1 \equiv \frac{n_0 n_{T0}}{p_{T0}}$$

Trap is like a donor!

f_{00} empty trap prob.



$$n_{T0} = N_T \frac{1}{1 + e^{(E_T' - E_F)/k_T T}}$$

$$n_{T0} = N_T(1 - f_{00}) = \frac{N_T}{1 + g_D e^{\beta(E_T - E_F)}} = N_T \frac{1}{1 + g\ exp}$$

$$p_{T0} = N_T - n_{T0}$$

$$p_{T0} = N_T\ f_{00}$$

n_T electron-filled traps

p_T hole-filled (empty) traps

c_n electron capture coef.

c_p hole capture coef.

e_n electron emission coef.

e_p hole emission coef. $\quad \beta = \frac{1}{k_T T}$

$$n_0 = n_i e^{\beta(E_F - E_i)}$$

$$g\ exp \equiv g_D e^{\beta(E_T - E_F)}$$

$$p_1 n_1 = n_i^2$$

Some Algebra to get Expressions for (n_1) and (p_1)

$$n_{T0} = N_T(1 - f_{00}) = N_T \frac{1}{1 + g\ exp}$$

$$(1 - f_{00}) = \frac{1}{1 + g\ exp}$$

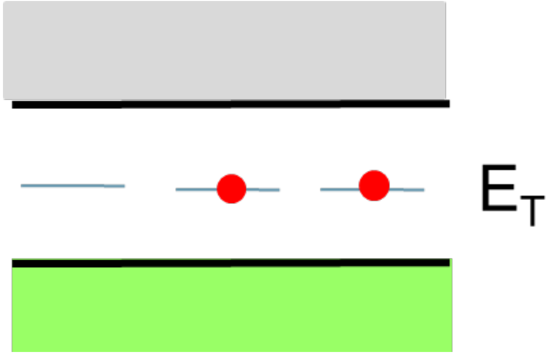
$$f_{00} = 1 - \frac{1}{1 + g\ exp}$$

$$f_{00} = \frac{g\ exp}{1 + g\ exp}$$

$$\frac{f_{00}}{1 - f_{00}} = \frac{g\ exp}{1 + g\ exp} \Big/ \frac{1}{1 + g\ exp}$$

$$\frac{f_{00}}{1 - f_{00}} = g\ exp$$

$$n_1 \equiv \frac{n_0 p_{T0}}{n_{T0}} \qquad p_1 \equiv \frac{n_0 n_{T0}}{p_{T0}}$$



$$\beta = \frac{1}{k_T T}$$

f_{00} empty trap prob.

$$n_0 = n_i e^{\beta(E_F - E_i)}$$

$$g\ exp \equiv g_D e^{\beta(E_T - E_F)}$$

$$n_1 \equiv \frac{n_0 p_{T0}}{n_{T0}}$$

$$p_1 \equiv \frac{n_0 n_{T0}}{p_{T0}}$$

$$p_1 n_1 = n_i^2$$

$$n_{T0} = N_T(1 - f_{00})$$

$$p_{T0} = N_T f_{00}$$

Expressions for (n_1) and (p_1)

$$n_1 \equiv \frac{n_0 p_{T0}}{n_{T0}}$$

$$p_1 \equiv \frac{n_0 n_{T0}}{p_{T0}}$$

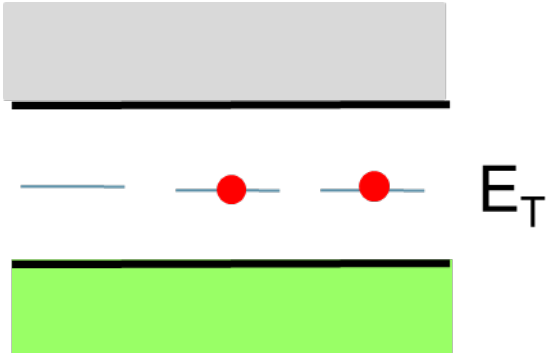
$$\beta = \frac{1}{k_T T}$$

f_{00} empty trap prob.

$$n_0 = n_i e^{\beta(E_F - E_i)}$$

$$n_{T0} = N_T(1 - f_{00})$$

$$p_{T0} = N_T\, f_{00}$$



$$= n_0 \frac{(N_T f_{00})}{N_T(1 - f_{00})}$$

$$\frac{f_{00}}{1 - f_{00}} = g\ exp$$

$$g\ exp \equiv g_D e^{\beta(E_T - E_F)}$$

$$p_1 n_1 = n_i^2$$

$$= n_0\ g\ exp$$

Expressions for (n_1) and (p_1)

$$n_1 \equiv \frac{n_0 p_{T0}}{n_{T0}}$$

$$p_1 \equiv \frac{n_0 n_{T0}}{p_{T0}}$$

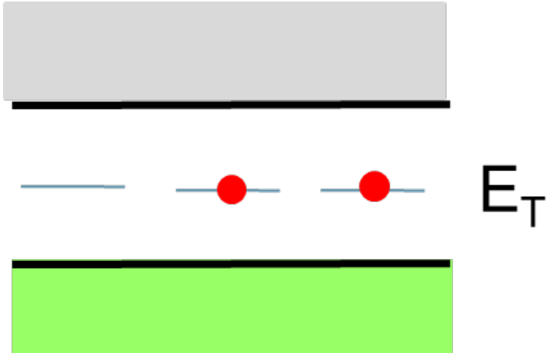
$$\beta = \frac{1}{k_T T}$$

f_{00} empty trap prob.

$$n_0 = n_i e^{\beta(E_F - E_i)}$$

$$n_{T0} = N_T(1 - f_{00})$$

$$p_{T0} = N_T\, f_{00}$$



$$= n_0 \frac{(N_T f_{00})}{N_T(1 - f_{00})}$$

$$\frac{f_{00}}{1 - f_{00}} = g\ exp$$

$$= n_0\ g\ exp$$

$$g\ exp \equiv g_D e^{\beta(E_T - E_F)}$$

$$= n_0 g_D e^{\beta(E_T - E_F)}$$

$$p_1 n_1 = n_i^2$$

Expressions for (n_1) and (p_1)

$$n_1 \equiv \frac{n_0 p_{T0}}{n_{T0}}$$

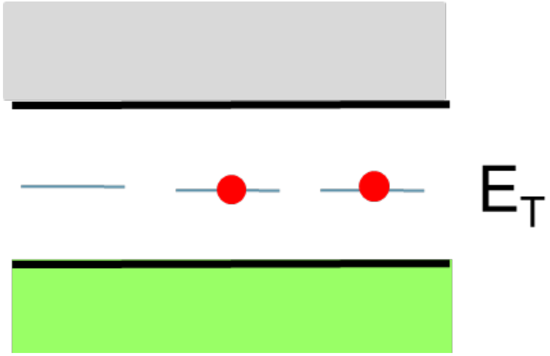
$$p_1 \equiv \frac{n_0 n_{T0}}{p_{T0}}$$

$$\beta = \frac{1}{k_T T}$$

f_{00} empty trap prob.

$$n_{T0} = N_T(1 - f_{00})$$

$$p_{T0} = N_T\, f_{00}$$



$$= n_0 \frac{(N_T f_{00})}{N_T(1 - f_{00})}$$

$$\frac{f_{00}}{1 - f_{00}} = g\ exp$$

$$p_1 n_1 = n_i^2$$

$$= n_0\, g\ exp$$

$$g\ exp \equiv g_D e^{\beta(E_T - E_F)}$$

$$= n_0 g_D e^{\beta(E_T - E_F)}$$

$$n_0 = n_i e^{\beta(E_F - E_i)}$$

$$= n_i e^{\beta(E_F - E_i)} g_D e^{\beta(E_T - E_F)}$$

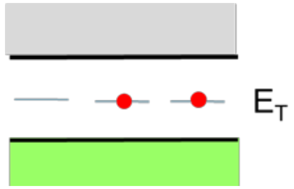
$$n_1 = n_i g_D e^{\beta(E_T - E_i)}$$

Expressions for (n_1) and (p_1)

$$\beta = \frac{1}{k_T T}$$

$$n_1 \equiv \frac{n_0 p_{T0}}{n_{T0}} \qquad p_1 \equiv \frac{n_0 n_{T0}}{p_{T0}}$$

f_{00} empty trap prob.



$$p_1 n_1 = n_i^2$$

$$n_1 = n_i g_D e^{\beta(E_T - E_i)} \quad \rightarrow \quad p_1 = n_i^2 / n_1$$

$$p_1 = n_i g_D{}^{-1} e^{-\beta(E_T - E_i)}$$

$$p_1 = n_i g_D{}^{-1} e^{\beta(E_i - E_T)}$$

Expressions for (n_1) and (p_1)

$$n_1 \equiv \frac{n_0 p_{T0}}{n_{T0}}$$

$$p_1 \equiv \frac{n_0 n_{T0}}{p_{T0}}$$

$$n_1 = n_i g_D e^{\beta(E_T - E_i)}$$

$$p_1 = n_i g_D{}^{-1} e^{\beta(E_i - E_T)}$$

$$p_1 n_1 = n_i^2$$

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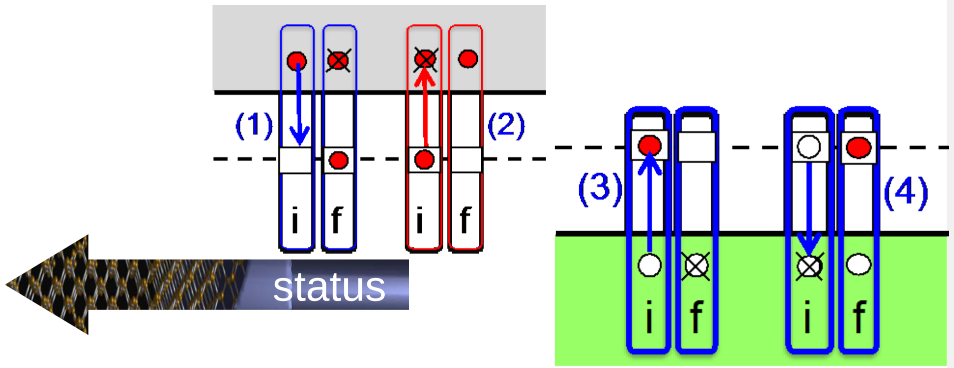


$$N_T = p_T + n_T \qquad p_T = N_T - n_T$$

$$\left.\frac{\partial n_0}{\partial t}\right|_{1,2} = -c_{n0} n_0 p_{T0} + e_{n0} n_{T0} = 0$$

$$\left.\frac{\partial p_0}{\partial t}\right|_{3,4} = -c_{p0} p_0 n_{T0} + e_{p0} p_{T0} = 0$$

$$e_{n0} = c_{n0}\ n_1$$

$$e_{p0} = c_{p0}\ p_1$$

$$n_1 \equiv \frac{n_0 p_{T0}}{n_{T0}}$$

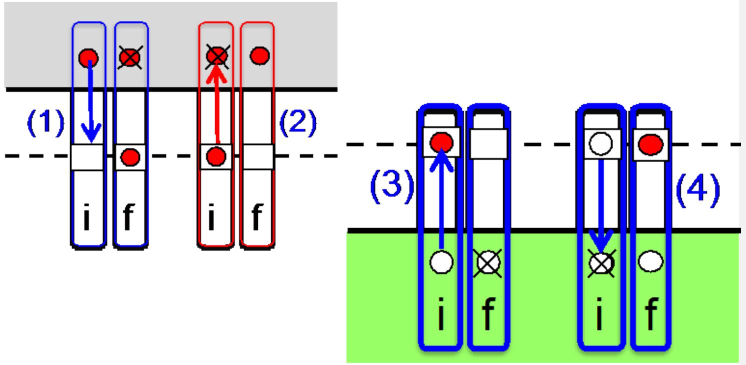
$$p_1 \equiv \frac{n_0 n_{T0}}{p_{T0}}$$

$$n_1 = n_i g_D e^{\beta(E_T - E_i)}$$

$$p_1 = n_i g_D{}^{-1} e^{\beta(E_i - E_T)}$$

$$p_1 n_1 = n_i^2$$

- 16.1 Capture coefficient & Capture Cross Section
- 16.2 Derivation of SRH formula (Shockley, Reed, Hall)
  - » 16.2.1 Trap Assisted Recombination Rates
  - » 16.2.2 Capture and emission relationship (n_1 and p_1)
  - » 16.2.3 Steady State Trap Population ← status
  - » 16.2.4 Recombination-Generation Rate



$$N_T = p_T + n_T \qquad p_T = N_T - n_T$$

$$\left.\frac{\partial n_0}{\partial t}\right|_{1,2} = -c_{n0} n_0 p_{T0} + e_{n0} n_{T0} = 0$$

$$\left.\frac{\partial p_0}{\partial t}\right|_{3,4} = -c_{p0} p_0 n_{T0} + e_{p0} p_{T0} = 0$$

$$e_{n0} = c_{n0}\; n_1$$

$$e_{p0} = c_{p0}\; p_1$$

$$n_1 \equiv \frac{n_0 p_{T0}}{n_{T0}}$$

$$p_1 \equiv \frac{n_0 n_{T0}}{p_{T0}}$$

$$n_1 = n_i g_D e^{\beta(E_T - E_i)}$$

$$p_1 = n_i g_D{}^{-1} e^{\beta(E_i - E_T)}$$

$$p_1 n_1 = n_i^2$$