Section 14
Doping

14.2 Statistics of donor and acceptor levels

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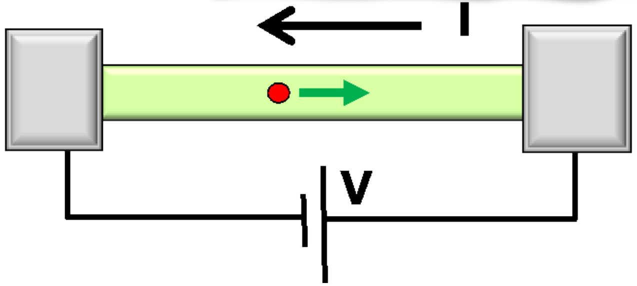
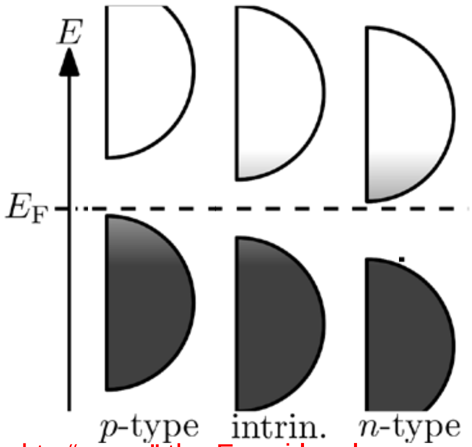
Section 14
Doping

Not Enough Electrons available



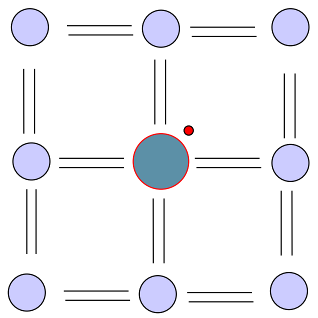
$$I = G \times V$$
$$= q \times n \times v \times A$$

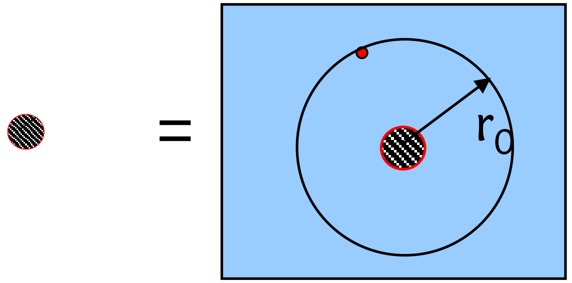
charge density velocity area



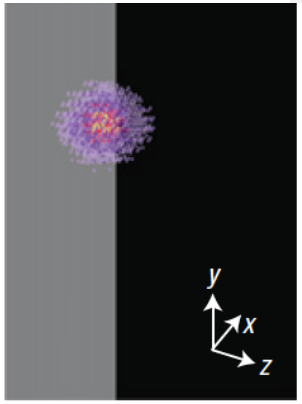
» Need to “move” the Fermi level
» ”add” electrons – n-type doping – E_F close to E_c
» “add” holes – p-type doping – E_F close to E_v

- 14.1 Basic concepts of donors and acceptors





A doping atom behaves like a H-atom, except that the dielectric constant and effective masses are given by by those of the host atom.



- 14.2 Statistics of donor and acceptor levels
- 14.3

- 14.4

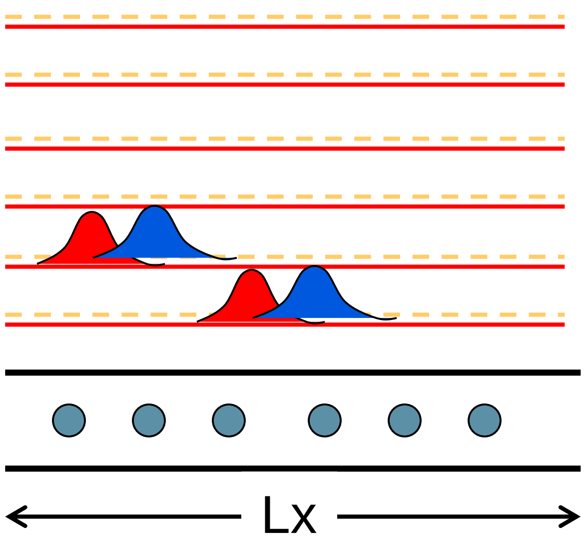
Video

Video

Video

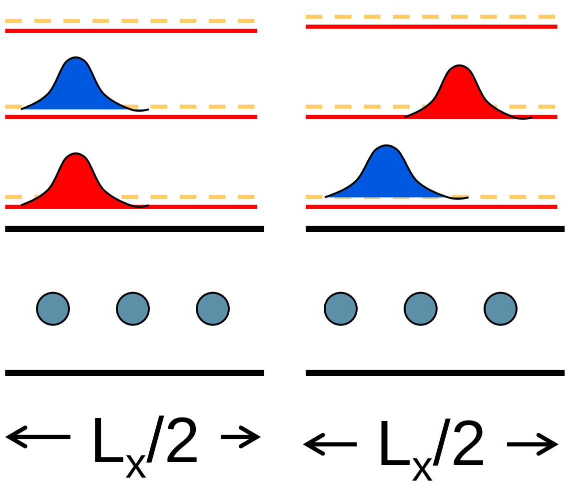
Video

Localized vs. Band Electrons



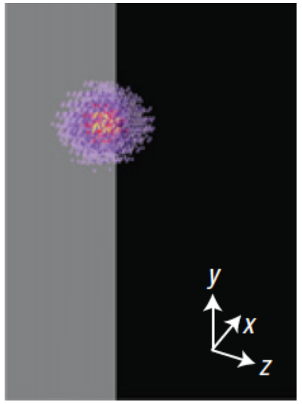
$E6 \leftarrow 12\pi/L_x$
$E5 \leftarrow 10\pi/L_x$
$E4 \leftarrow 8\pi/L_x$
$E3 \leftarrow 6\pi/L_x$
$E2 \leftarrow 4\pi/L_x$
$E1 \leftarrow 2\pi/L_x$

Two electrons (even with opposite spin) can not be at the same position and same energy because of electrostatic repulsion



$E3' \leftarrow 6\pi/(L_x/2)$
$E2' \leftarrow 4\pi/(L_x/2)$
$E1' \leftarrow 2\pi/(L_x/2)$

Band electrons (with opposite spin) need not be at the same position, so they can share occupy same energy level.

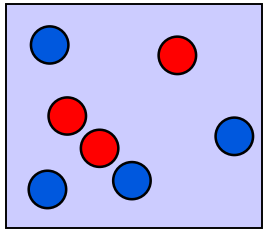


When we divide space by a factor of 2, the number of states (e.g. 6 here) does not change.

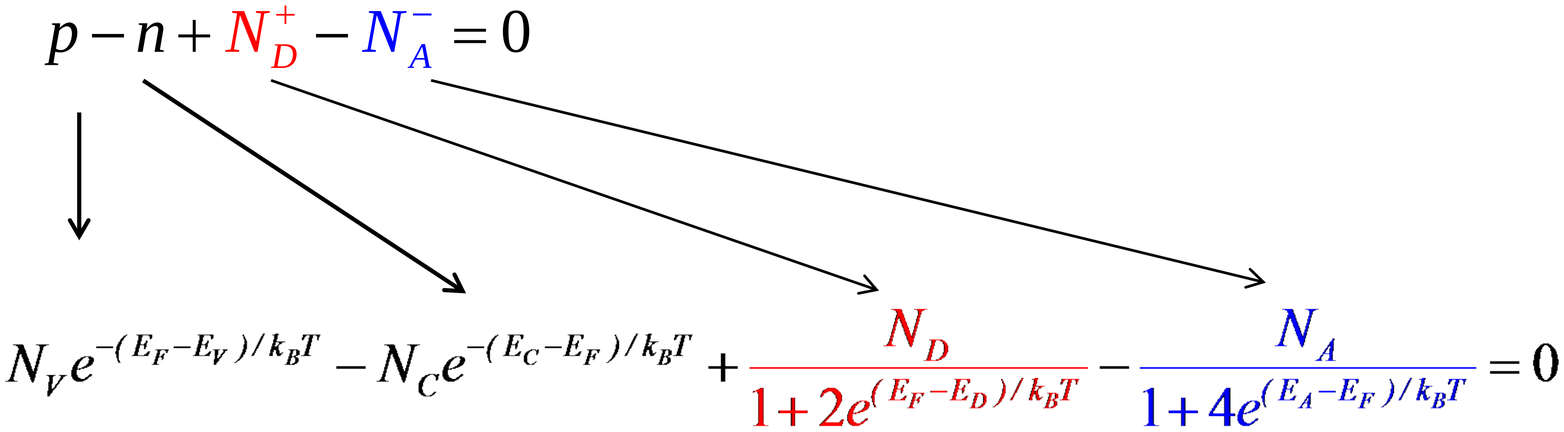
Looking ahead: Carrier-Density w/Doping

A bulk material must be charge neutral over all …

$$\int \left[p - n + N_D^+ - N_A^- \right] dV = 0$$



Further if the material is *spatially homogenous*

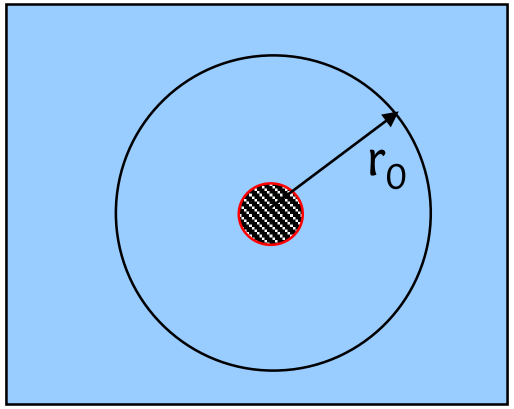


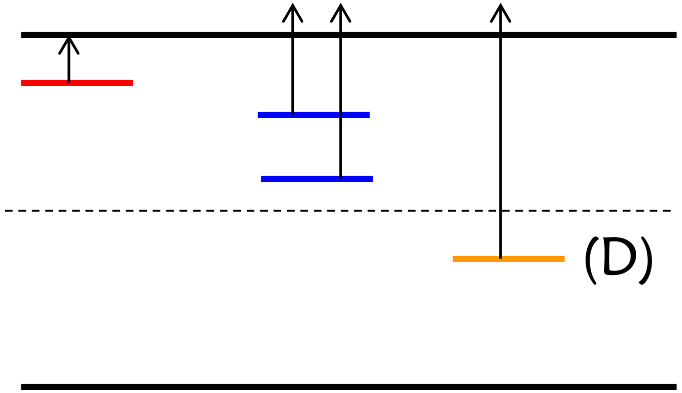
$$p - n + N_D^+ - N_A^- = 0$$

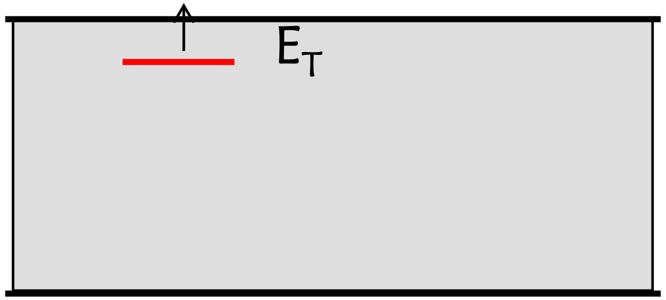
$$N_V e^{-(E_F - E_V)/k_B T} - N_C e^{-(E_C - E_F)/k_B T} + \frac{N_D}{1 + 2e^{(E_F - E_D)/k_B T}} - \frac{N_A}{1 + 4e^{(E_A - E_F)/k_B T}} = 0$$

Let us see how the formula comes about …

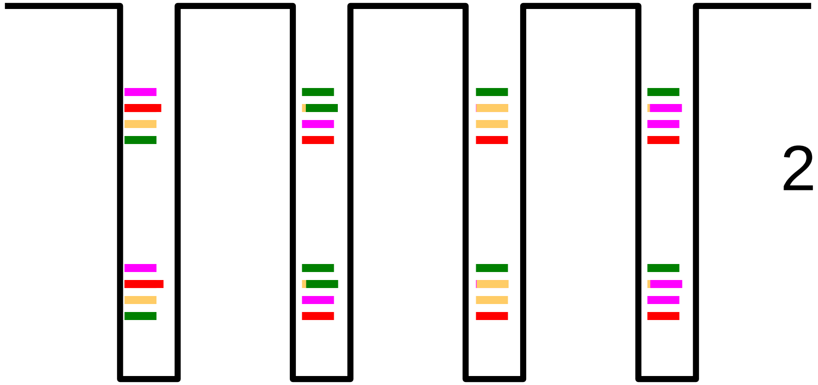
Characteristics of Donor Atoms



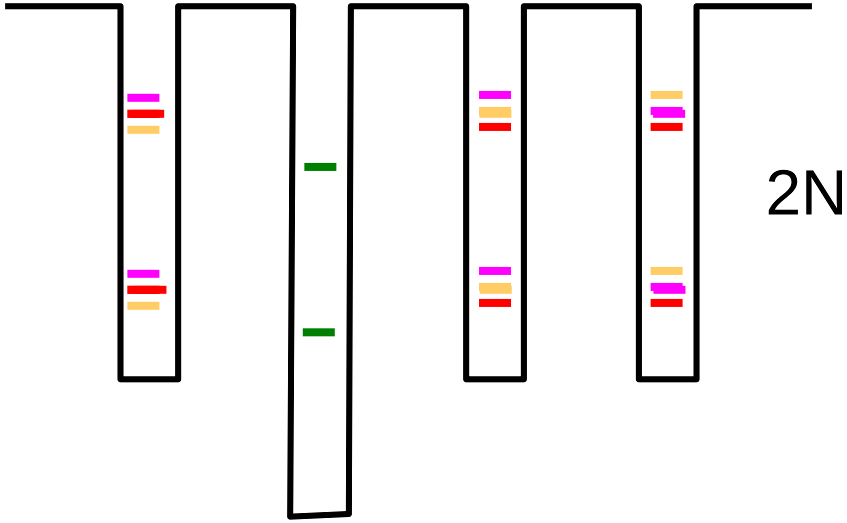




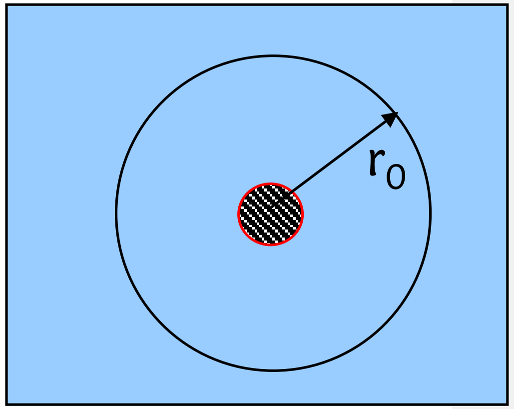
Localized vs. Delocalized States



2N states/per-band (with spin)

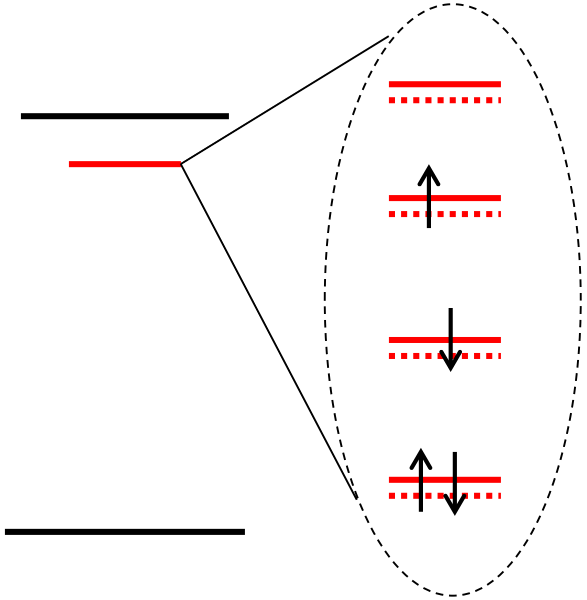


2N-2 states/per-band (with spin)



Statistics of Donor Levels

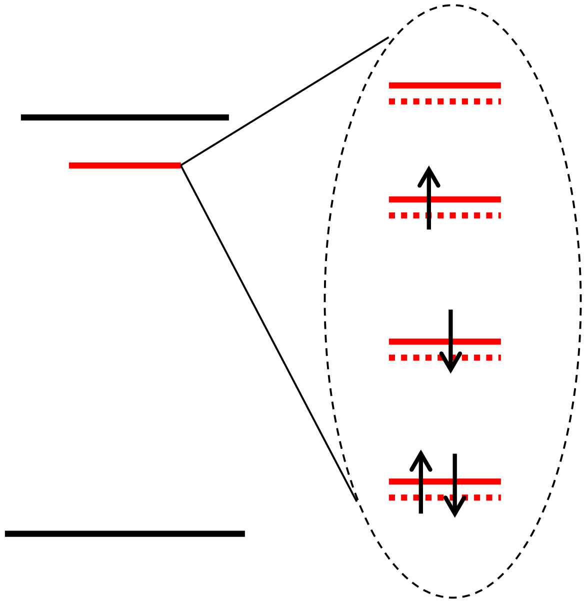
$$P_i = \frac{e^{-(E_i - N_i E_F)/k_B T}}{\sum_i e^{-(E_i - N_i E_F)/k_B T}} \equiv \frac{e^{-(E_i - N_i E_F)/k_B T}}{Z}$$



u/d	E_i	N_i	P_i
$0/0$	0	0	$1/Z$

Statistics of Donor Levels

$$P_i = \frac{e^{-(E_i - N_i E_F)/k_BT}}{\sum_i e^{-(E_i - N_i E_F)/k_BT}} \equiv \frac{e^{-(E_i - N_i E_F)/k_BT}}{Z}$$



u/d	E_i	N_i	P_i
$0/0$	0	0	$1/Z$
$1/0$	1	1	$e^{-\frac{(E_i-E_F)}{k_BT}}\Big/Z$
$0/1$	1	1	$e^{-\frac{(E_i-E_F)}{k_BT}}\Big/Z$
1/1	**x**	**x**	x

Coulomb interaction
forbids this configuration

Statistics of Donor Levels

u/d	E_i	N_i	P_i
$0/0$	0	0	$1/Z$
$0/1$	1	1	$e^{-\frac{(E_i-E_F)}{k_B T}}\Big/ Z$
$1/0$	1	1	$e^{-\frac{(E_i-E_F)}{k_B T}}\Big/ Z$

Probability that the donor is empty (charged)

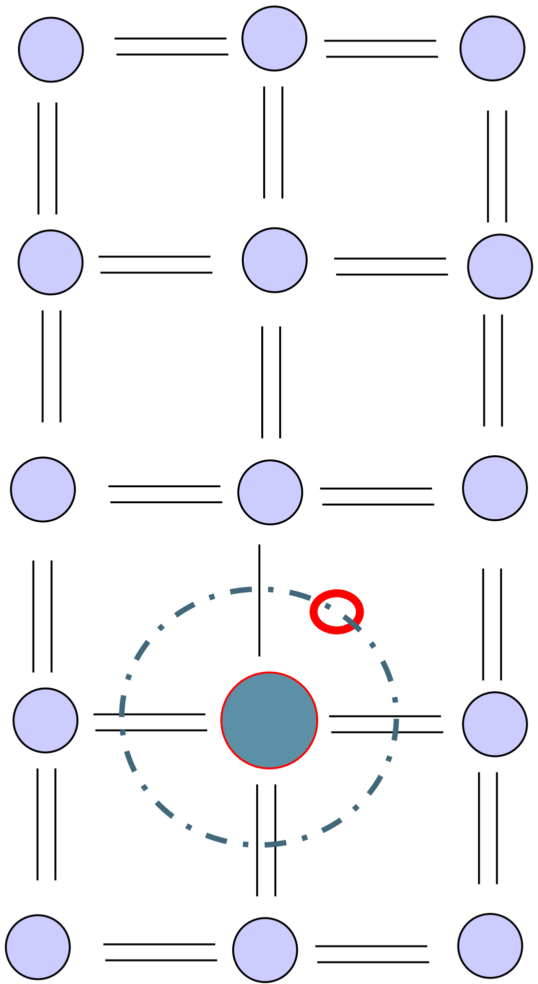
$$f_{00} = \frac{P_{00}}{P_{00}+P_{01}+P_{10}} = \frac{1/Z}{1/Z + 2e^{-(E_i-E_F)/k_B T}/Z} = \frac{1}{1+2e^{(E_F-E_i)/k_B T}}$$

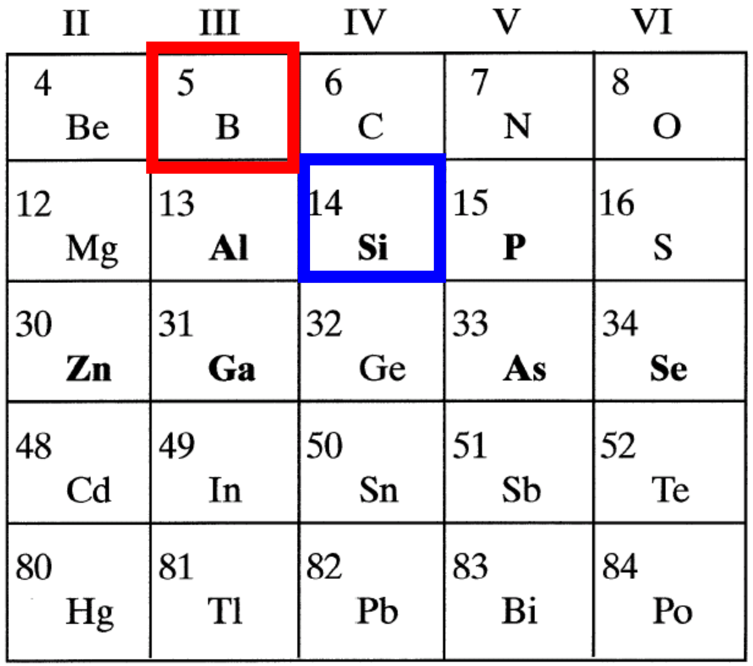
Probability that the donor is filled with at least one electron (neutral)

$$1 - f_{00} = 1 - \frac{1}{1+2e^{(E_F-E_i)/k_B T}} = \frac{1}{1+\frac{1}{2}e^{(E_i-E_F)/k_B T}}$$

Note the extra factor ....

Acceptor Atoms



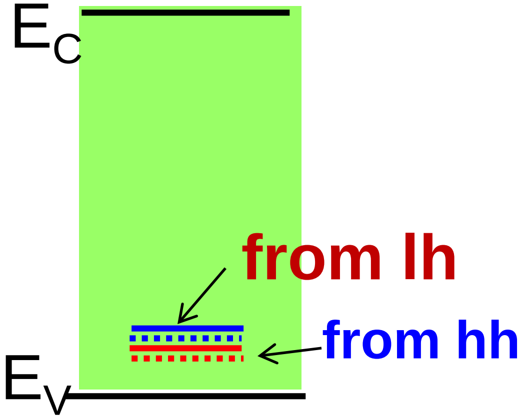


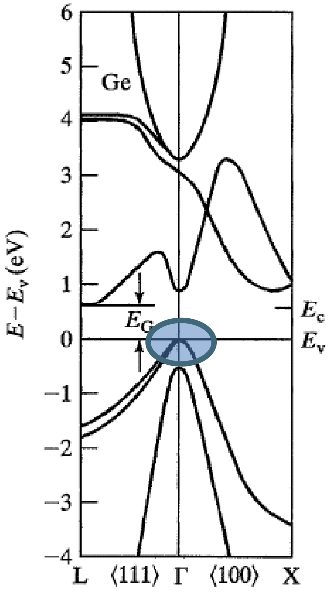
II	III	IV	V	VI
4 Be	5 B	6 C	7 N	8 O
12 Mg	13 **Al**	14 **Si**	15 **P**	16 S
30 **Zn**	31 **Ga**	32 Ge	33 **As**	34 **Se**
48 Cd	49 In	50 Sn	51 Sb	52 Te
80 Hg	81 Tl	82 Pb	83 Bi	84 Po

State [1] …. Hole present … N-1 charges

State [0] … Hole filled …. N charges

Statistics of Acceptor Levels in Si and Ge





1. Each atom contributes 2 states (up & down spin) to a band, therefore a band has 2N states.

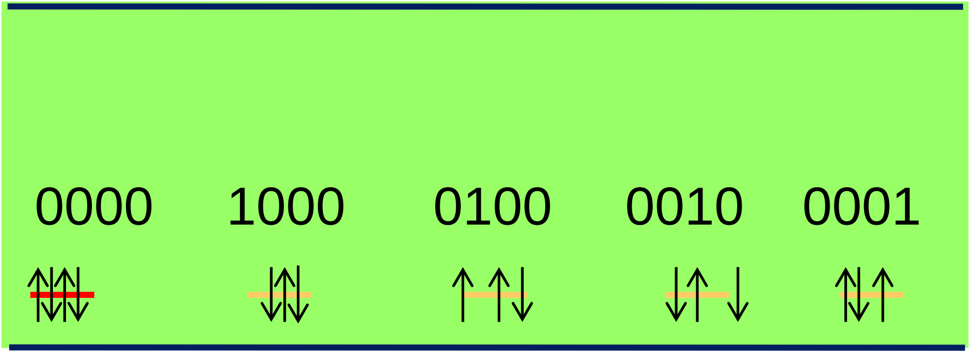
2. Every time a host atom is replaced by a impurity atom, 2 states are disappear per a band and appear as localized states (sort of).

3. Therefore an acceptor atom close to hh and lh bands removes four states from those bands.

4. Because of Coulomb interaction only 1 hole can seat in these 4 states: the states are 0000, 0001, 0010, 0100, 1000.

5. One now uses P_i to compute the occupation of acceptors.

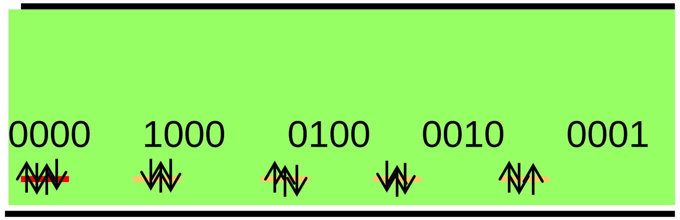
Number and Energy Considerations …



4N-4 States
In HH/LH bands

1) [0000] is the charged state as it has N electrons, but N-1 protons.
2) Single hole configuration [0001] is uncharged, as we have N-1 electrons, and N-1 protons … same is true for [0010], [0100], [1000] states.
3) Going from [0000] to [0001] states, the number of electrons goes down by 1 (N_i=-1).
4) Going from [0000] to [0001] states energy goes down by $-E_A$, because one electron is no longer occupying the high energy level at E_A.

Statistics of Acceptor Levels



$$P_{0000} = \frac{e^{-(0-0xE_F)/k_BT}}{\sum_i e^{-(E_i-N_iE_F)/k_BT}} \equiv \frac{1}{Z}$$

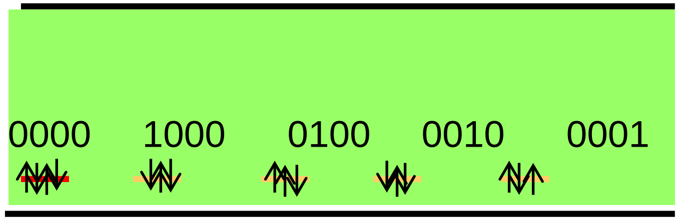
Steps 3 & 4

$$P_{0001} = P_{0010} = P_{0100} = P_{1000} = \frac{e^{-(-E_A-(-1)E_F)/k_BT}}{\sum_i e^{-(E_i-N_iE_F)/k_BT}} \equiv \frac{e^{(E_A-E_F)/k_BT}}{Z}$$

$$f_{0000} = \frac{P_{0000}}{P_{0000}+P_{1000}+P_{0100}+P_{0010}+P_{0001}} = \frac{1}{1+4e^{(E_A-E_F)/k_BT}}$$

Filled and empty Donor/Acceptor Levels

$$N_A^{filled} \equiv N_A^- = N_A\left[f_{0000} \right] = N_A \frac{1}{1+4e^{(E_A-E_F)/k_BT}}$$

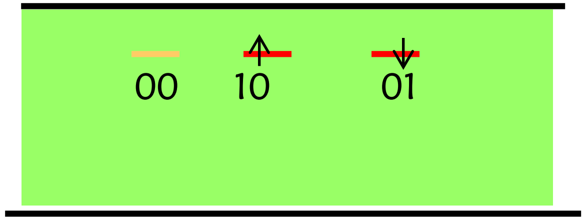


$$f_{0000} = \frac{1}{1+4e^{(E_A-E_F)/k_BT}}$$

4N-4 States
In HH/LH bands
(Two holes can not seat together)

$$f_D = \frac{N_D}{1+g_D e^{(E_F-E_D)/k_BT}}$$

2N-2 states



$$N_D^{empty} \equiv N_D^+ = N_D f_{00} = N_D \frac{1}{1+2e^{(E_F-E_i)/k_BT}}$$

Distributions are physical …. Define an effective donor level

$$f_D = \frac{N_D}{1+g_D e^{(E_F-E_D)/k_BT}}$$

Degeneracy factor …

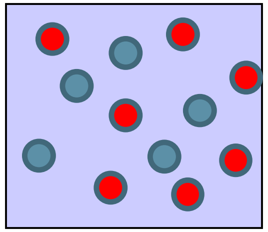
$$f_D = \frac{N_D}{1+e^{\varepsilon/k_BT}e^{(E_F-E_D)/k_BT}} = \frac{N_D}{1+e^{(E_F-E_D')/k_BT}}$$

Effective donor level

Carrier-density with Uniform Doping

A bulk material must be charge neutral over all …

$$\int \left[p - n + N_D^+ + N_A^- \right] dV = 0$$



Further if the doping is spatially homogenous

$$p - n + N_D^+ + N_A^- = 0$$

FD integral vs. FD function ?

$$N_V \frac{2}{\sqrt{\pi}} F_{1/2}\left[\beta\left(E_F - E_V\right)\right] - N_A \frac{2}{\sqrt{\pi}} F_{1/2}\left[\beta\left(E_C - E_F\right)\right] + \frac{N_D}{1+2e^{\beta(E_F - E_D)}} - \frac{N_A}{1+4e^{\beta(E_A - E_F)}} = 0$$

$$N_V e^{-(E_F - E_V)/k_B T} - N_C e^{-(E_C - E_F)/k_B T} + \frac{N_D}{1+2e^{(E_F - E_D)/k_B T}} - \frac{N_A}{1+4e^{(E_A - E_F)/k_B T}} = 0$$

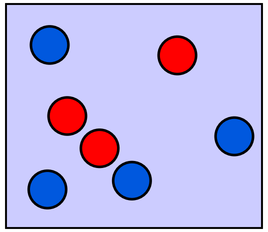
(approx.)

Once you know E_F, you can calculate n, p, N_D^+, N_A^-.

Summary ...

A bulk material must be charge neutral over all …

$$\int \left[p - n + N_D^+ + N_A^- \right] dV = 0$$

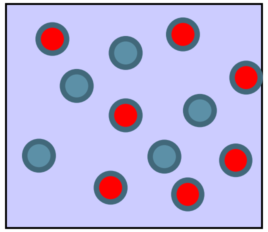


Further if the material is *spatially homogenous*

$$p - n + N_D^+ + N_A^- = 0$$

$$N_V e^{-(E_F - E_V)/k_B T} - N_C e^{-(E_C - E_F)/k_B T} + \frac{N_D}{1 + 2e^{(E_F - E_D)/k_B T}} - \frac{N_A}{1 + 4e^{(E_A - E_F)/k_B T}} = 0$$

Intrinsic Concentration



$$p - n + N_D^+ + N_A^- = 0$$

$$N_V e^{-(E_F-E_V)/k_BT} - N_C e^{-(E_C-E_F)/k_BT} + \cancel{\frac{N_D}{1+2e^{(E_F-E_D)/k_BT}}} - \cancel{\frac{N_A}{1+4e^{(E_A-E_F)/k_BT}}} = 0$$

$$n - p = 0 \Rightarrow N_C e^{-\beta(E_c-E_F)} = N_V e^{+\beta(E_v-E_F)}$$

$$E_F \equiv E_i = \frac{E_G}{2} + \frac{1}{2\beta} ln \frac{N_V}{N_C}$$

Section 14
Doping

Video

Video

Video

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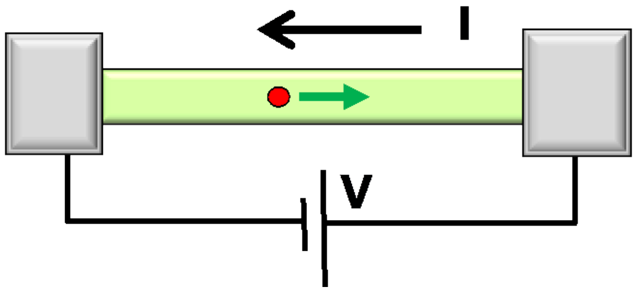

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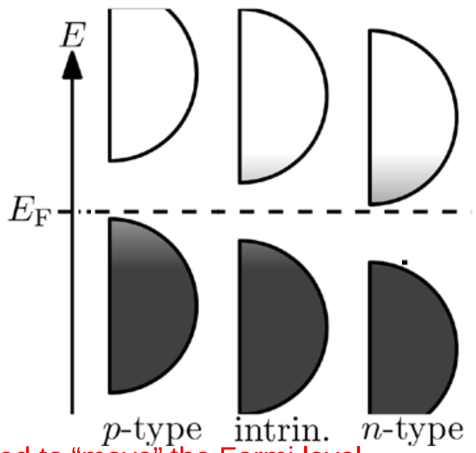
charge density velocity area



» Need to “move” the Fermi level

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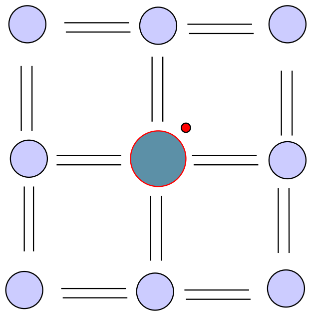
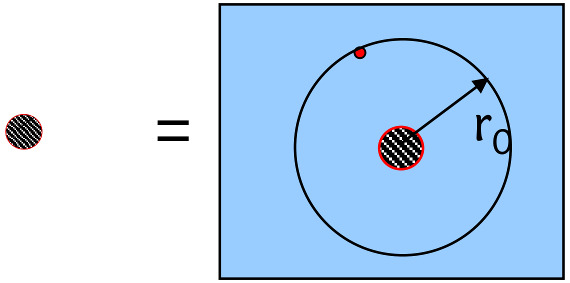
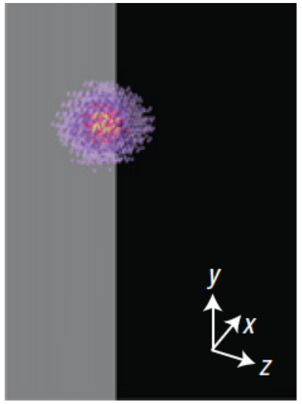
- 14.1 Basic concepts of donors and acceptors







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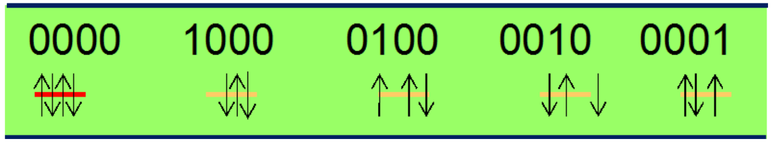

$$N_V e^{-(E_F - E_V)/k_BT} - N_C e^{-(E_C - E_F)/k_BT} + \frac{N_D}{1 + 2e^{(E_F - E_D)/k_BT}} - \frac{N_A}{1 + 4e^{(E_A - E_F)/k_BT}} = 0$$

- 14.3
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Section 14
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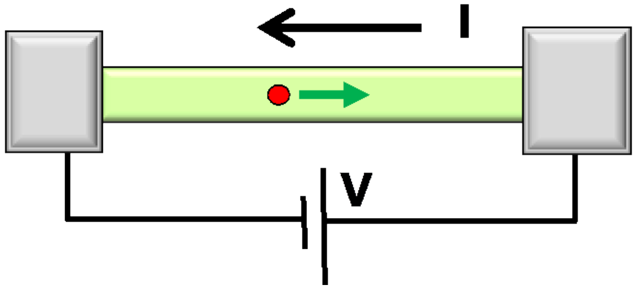

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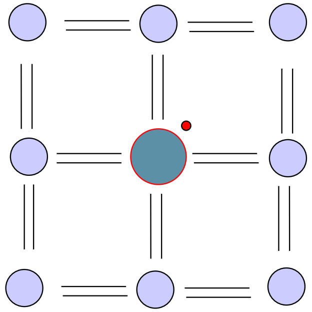
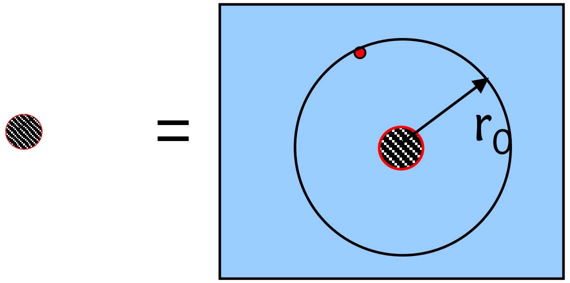
charge density velocity area

- 14.1 Basic concepts of donors and acceptors





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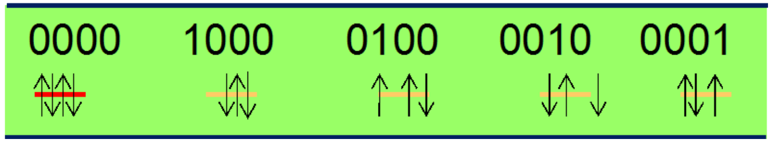
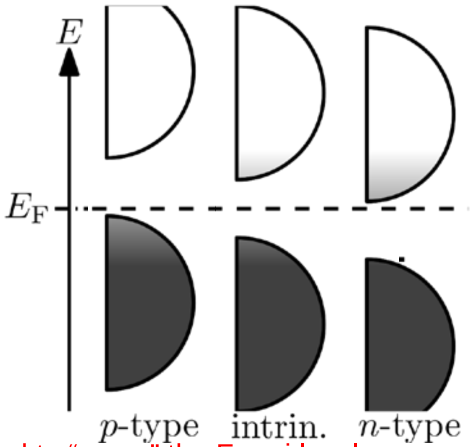


$$N_V e^{-(E_F-E_V)/k_BT} - N_C e^{-(E_C-E_F)/k_BT} + \frac{N_D}{1+2e^{(E_F-E_D)/k_BT}} - \frac{N_A}{1+4e^{(E_A-E_F)/k_BT}} = 0$$

- 14.3 Temperature dependence of carrier concentration
- 14.4



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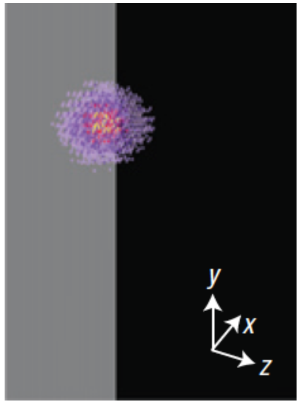

Video

Video

Video

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