Section 13
Band Diagrams

Gerhard Klimeck

gekco@purdue.edu

PURDUE UNIVERSITY

School of Electrical and Computer Engineering

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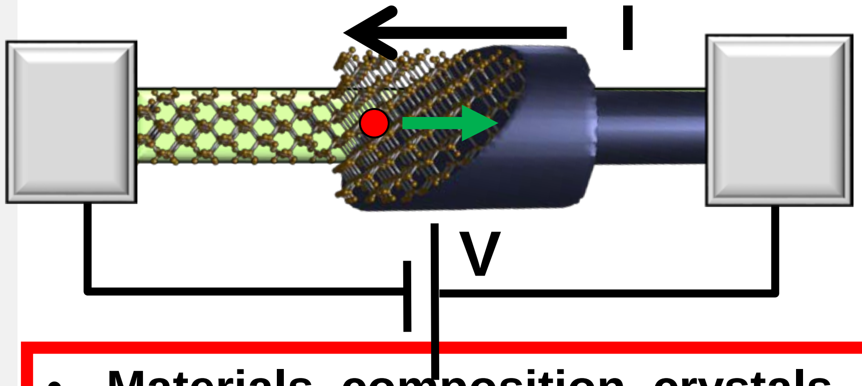
Band Diagrams



$$I = G \times V$$
$$= q \times n \times v \times A$$

charge density velocity area

- **Materials, composition, crystals**
- Tabulated for “known” bulk materials
- At nm-scale properties change with geometry => theory

⇒ **Quantum Mechanics Mechanics**
- Concepts of density of states and masses

⇒ **Equilibrium Statistical Mechanics**
- Occupation factors

Transport with scattering, non-equilibrium Stat. Mech.
- Drift-diffusion equation with recombination-generation

Understanding transport in concrete devices
- Diodes, BJT/HBT, MOS

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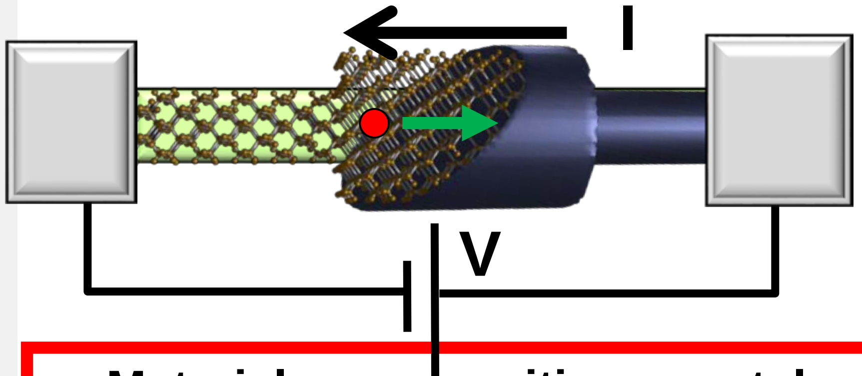

q x n
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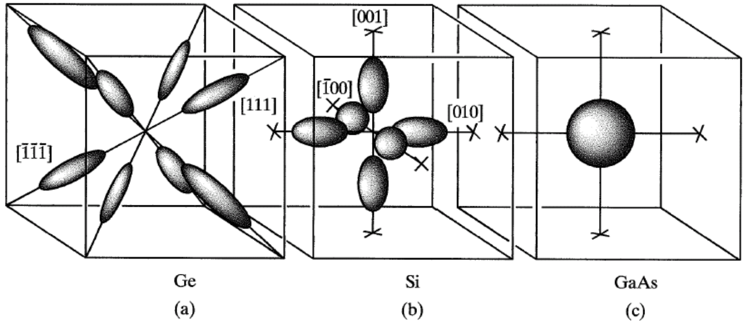

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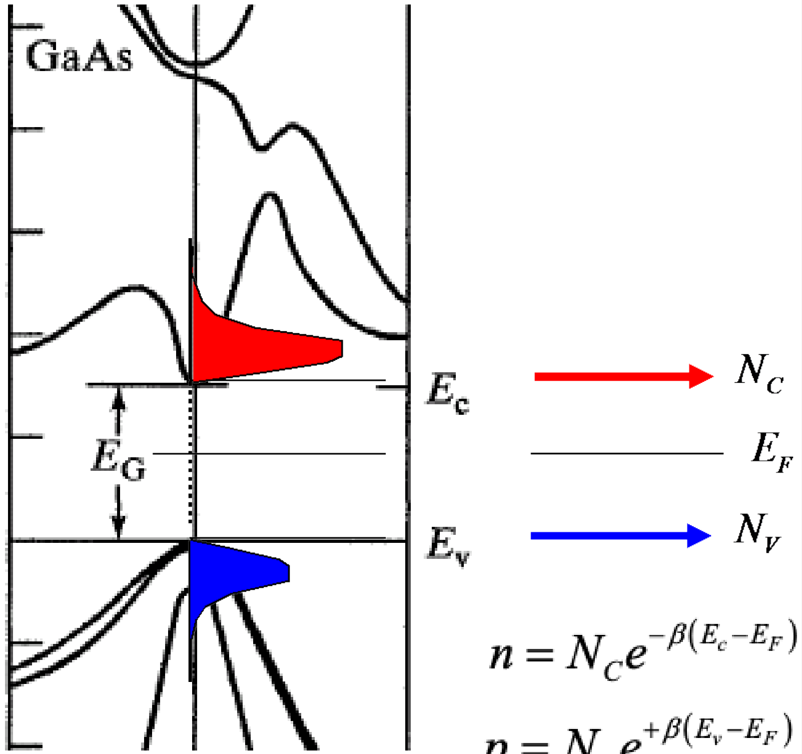

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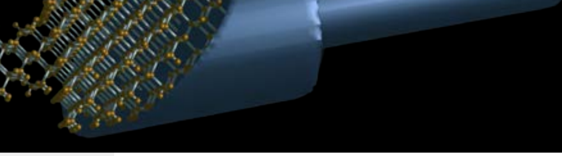

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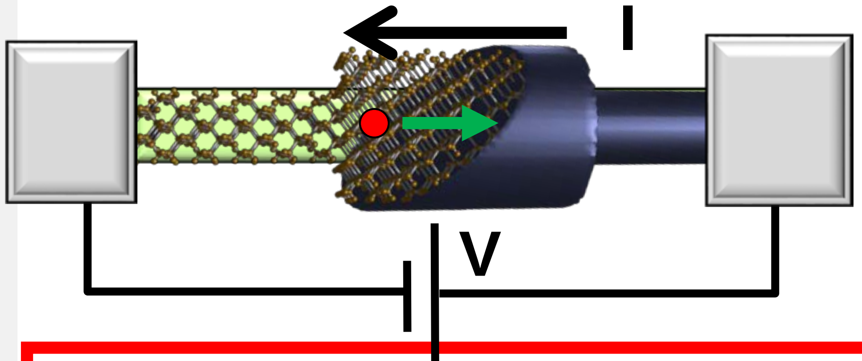

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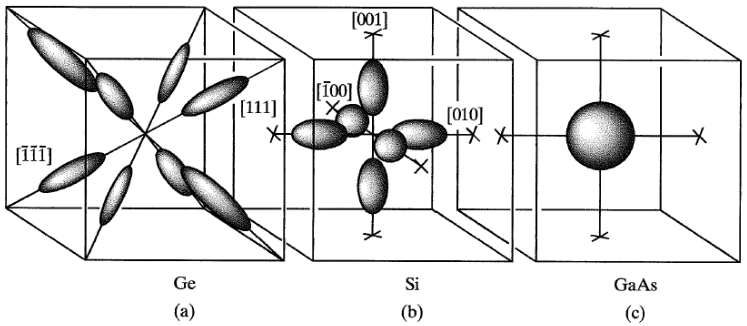

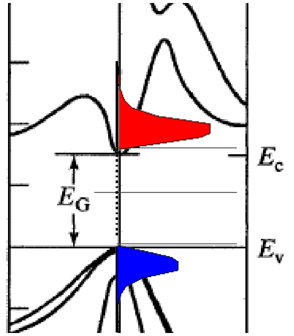

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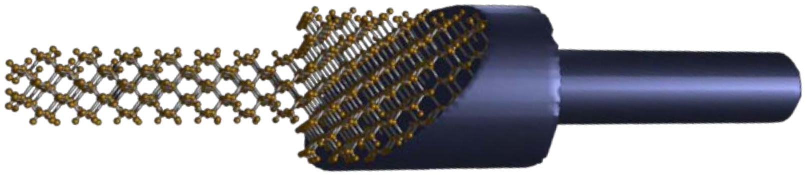

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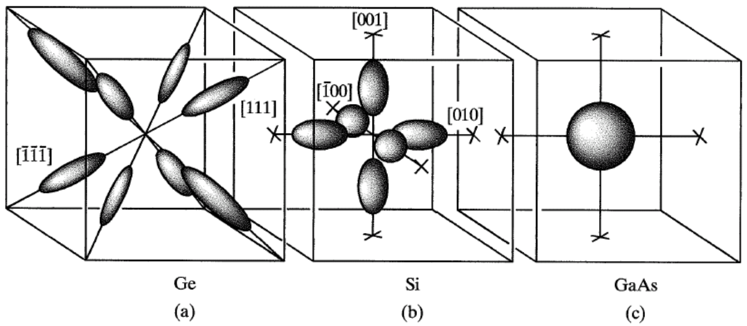

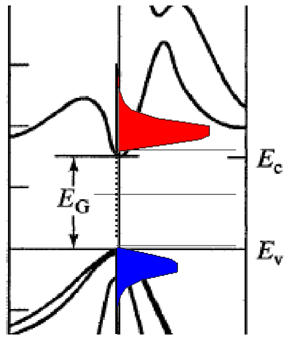

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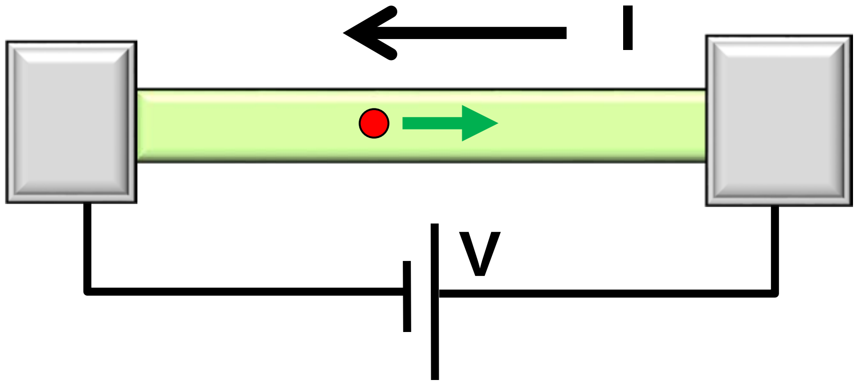

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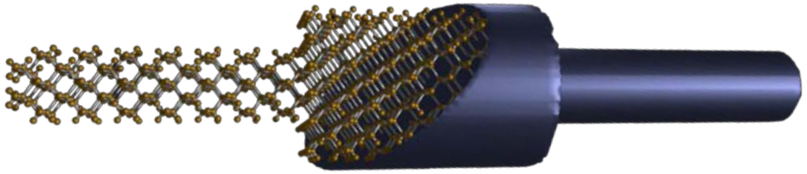

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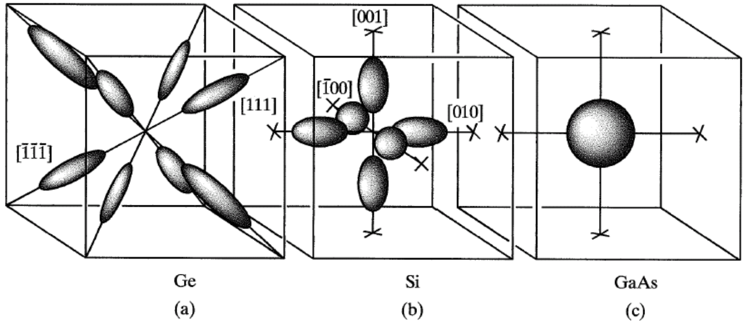

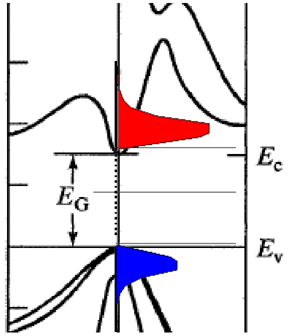

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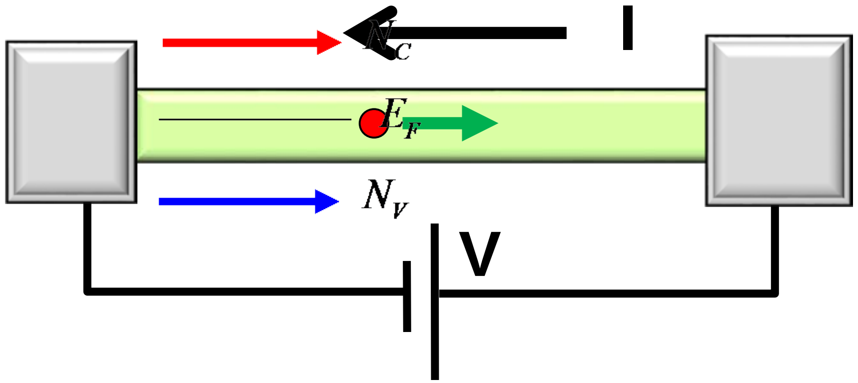

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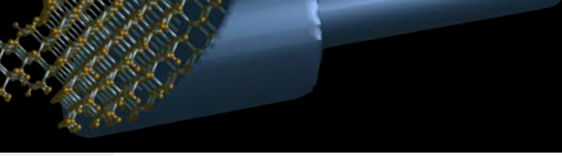

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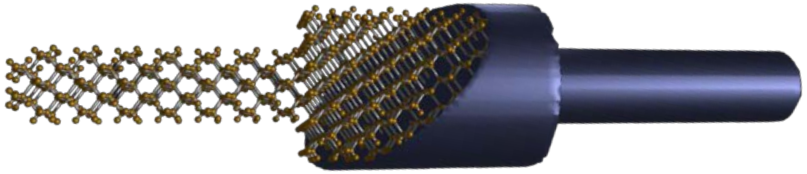

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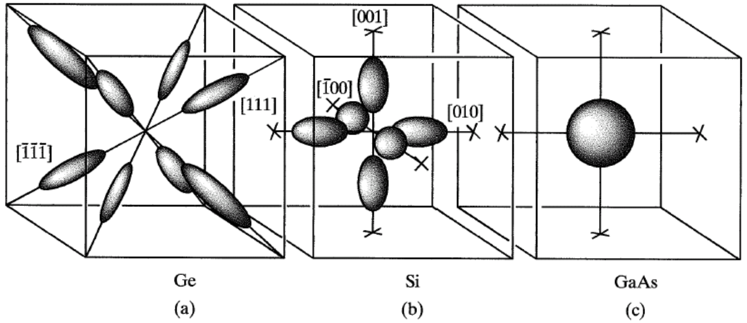

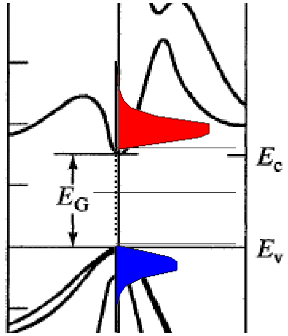

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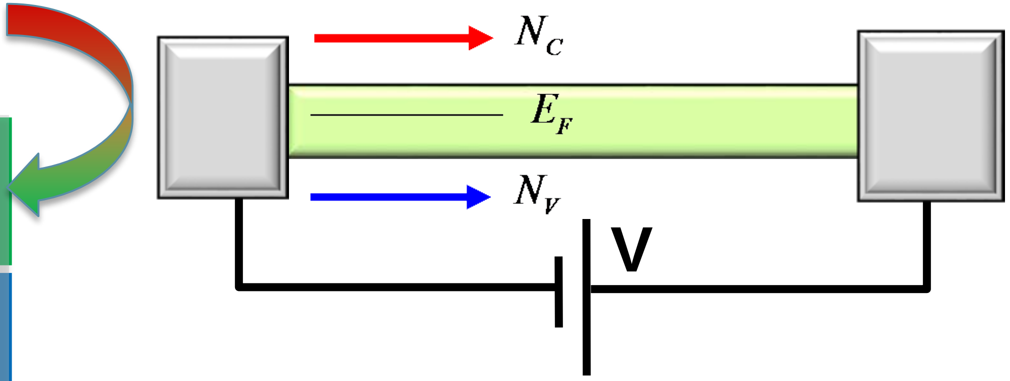
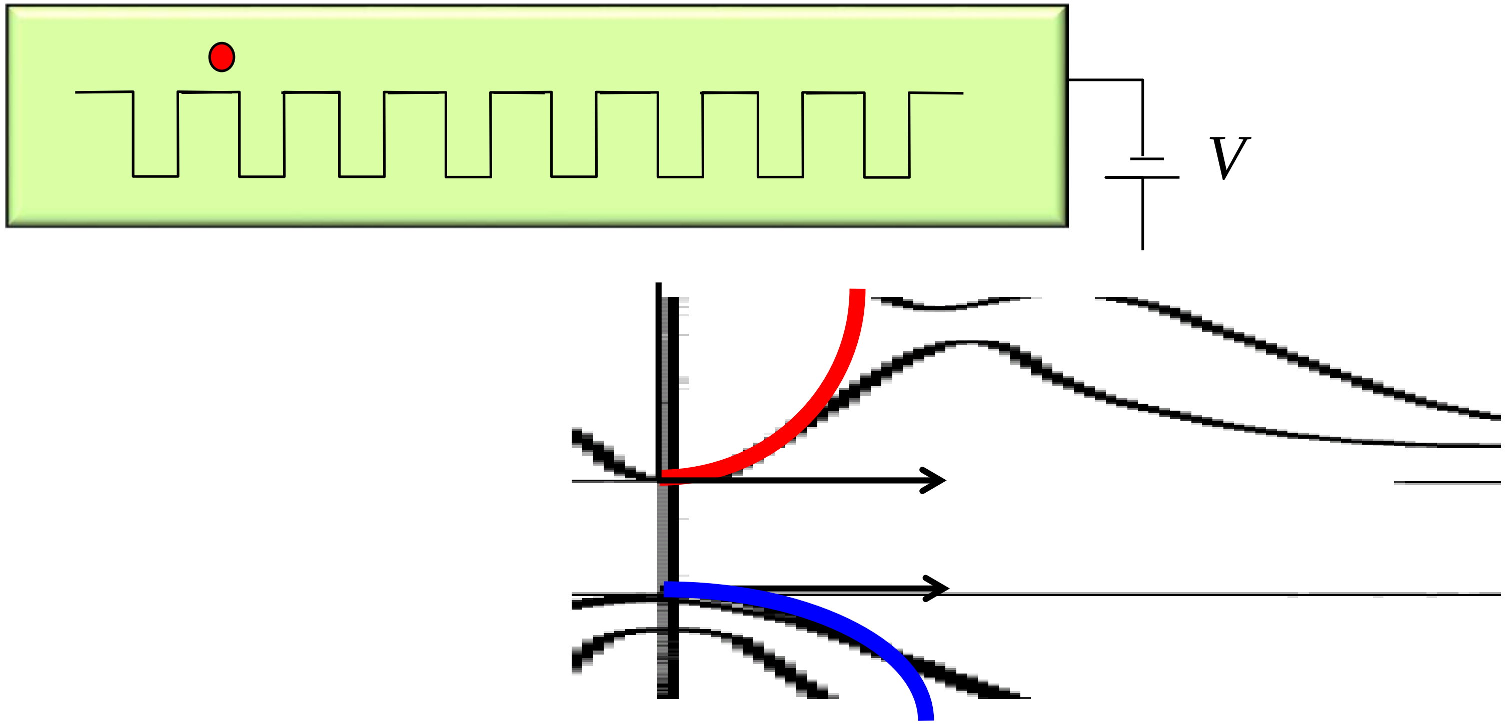


E-k Diagram



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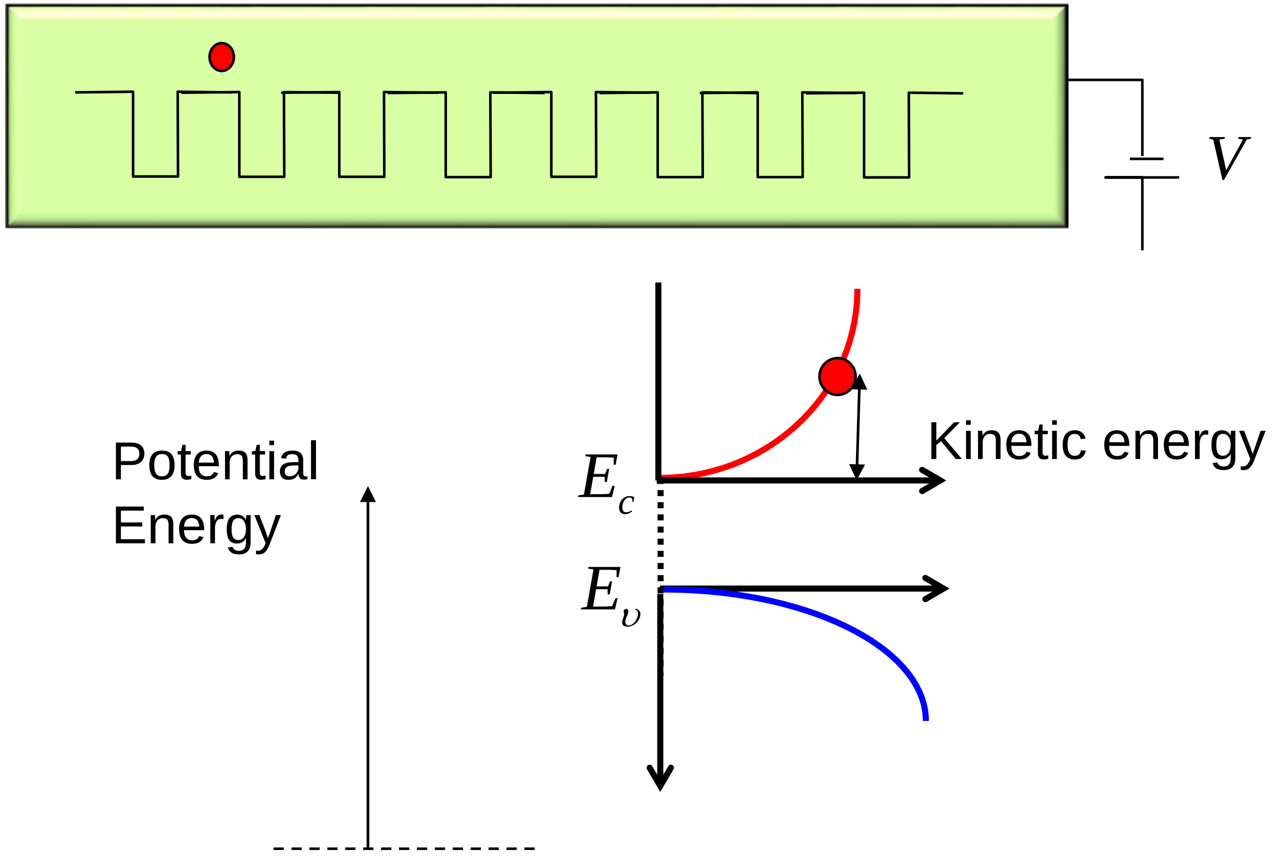
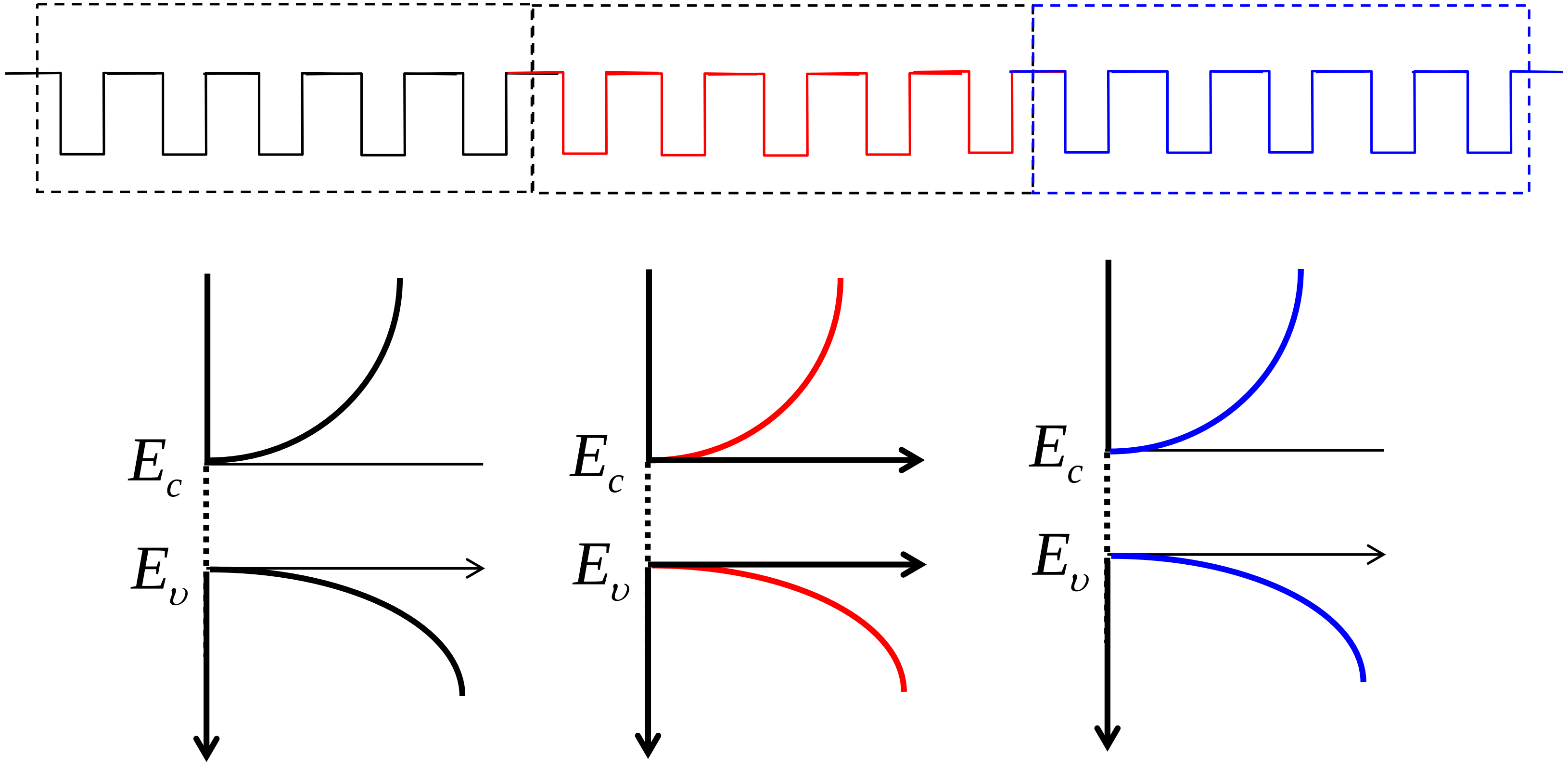
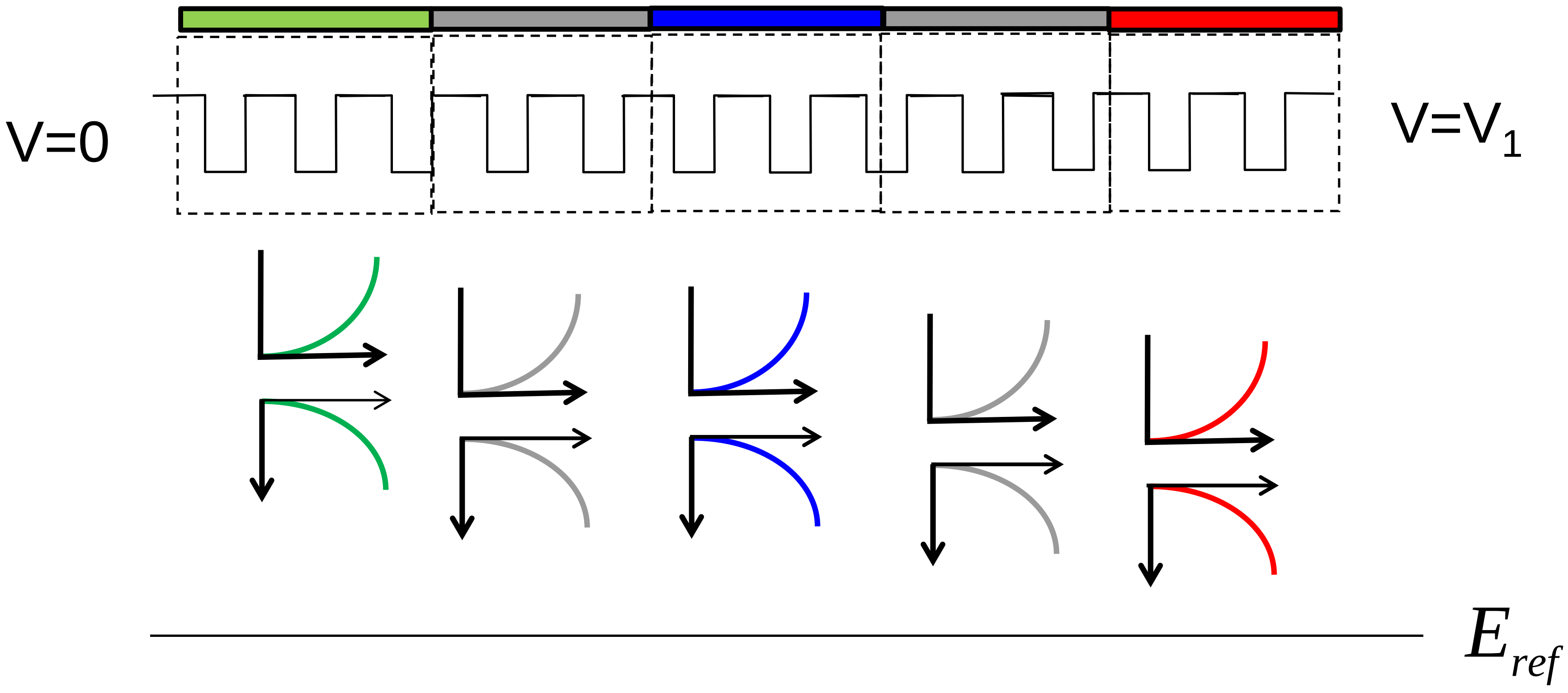


$$P.E. = E_c - E_{ref} = -qV$$

Position Resolved E-k Diagram

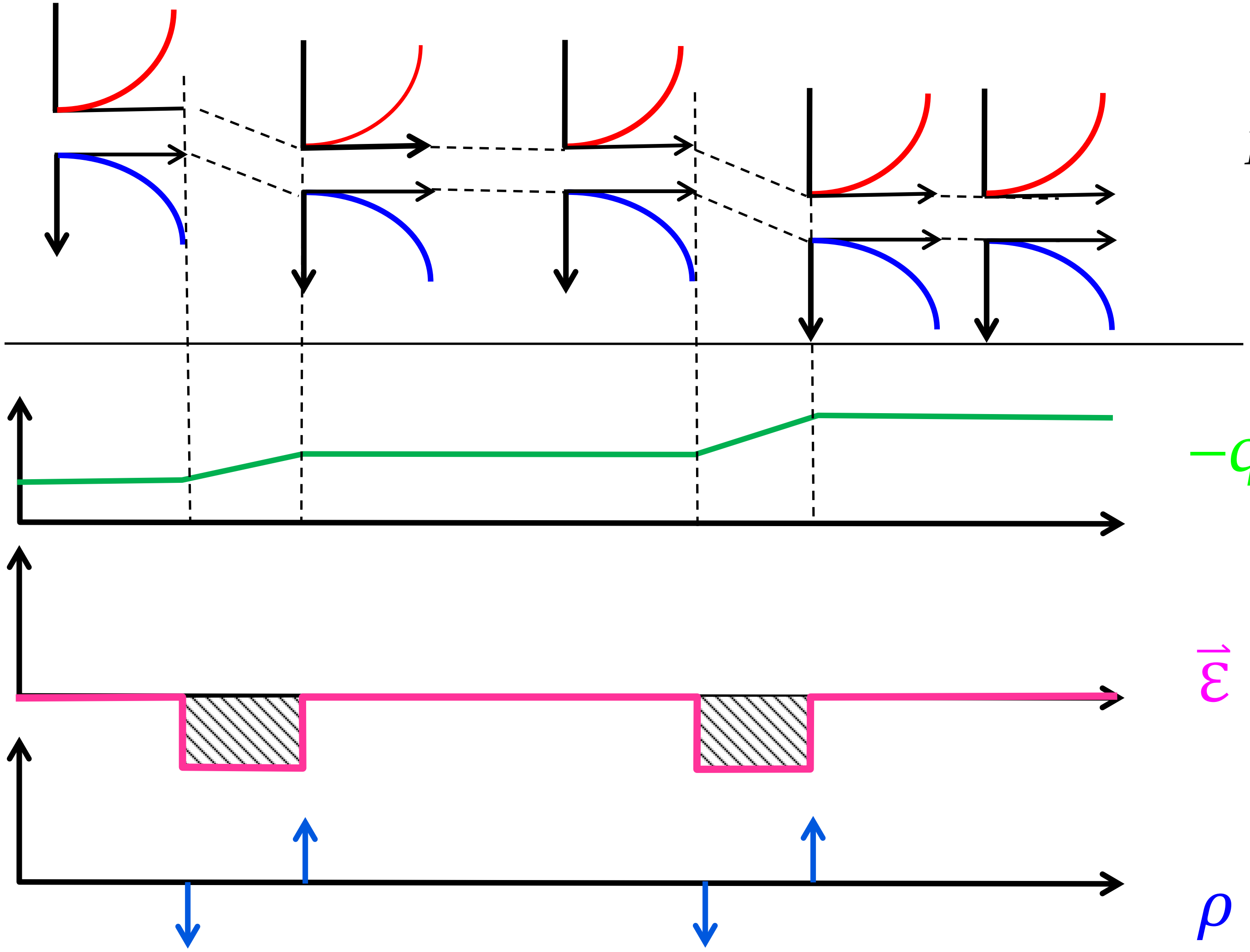


Position Resolved E-k Diagram with Applied Potential



$$P.E. = E_c - E_{ref} = -qV(x)$$

Position Resolved E-k Diagram with Applied Potential, Potential, Field, and Charge



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In most practical cases start from charge and derive potentials!

=> Useful to learn “graphical” integration

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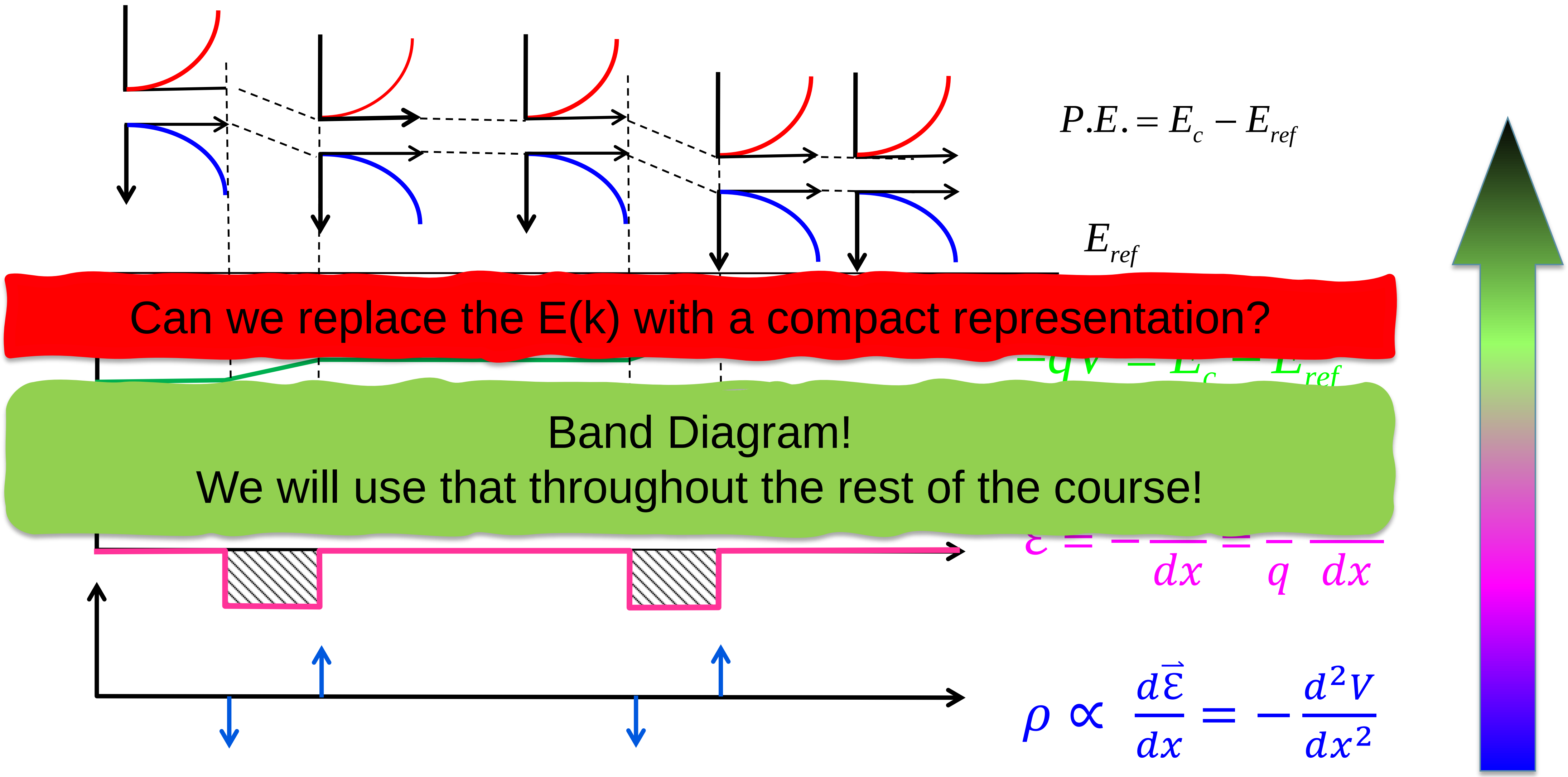

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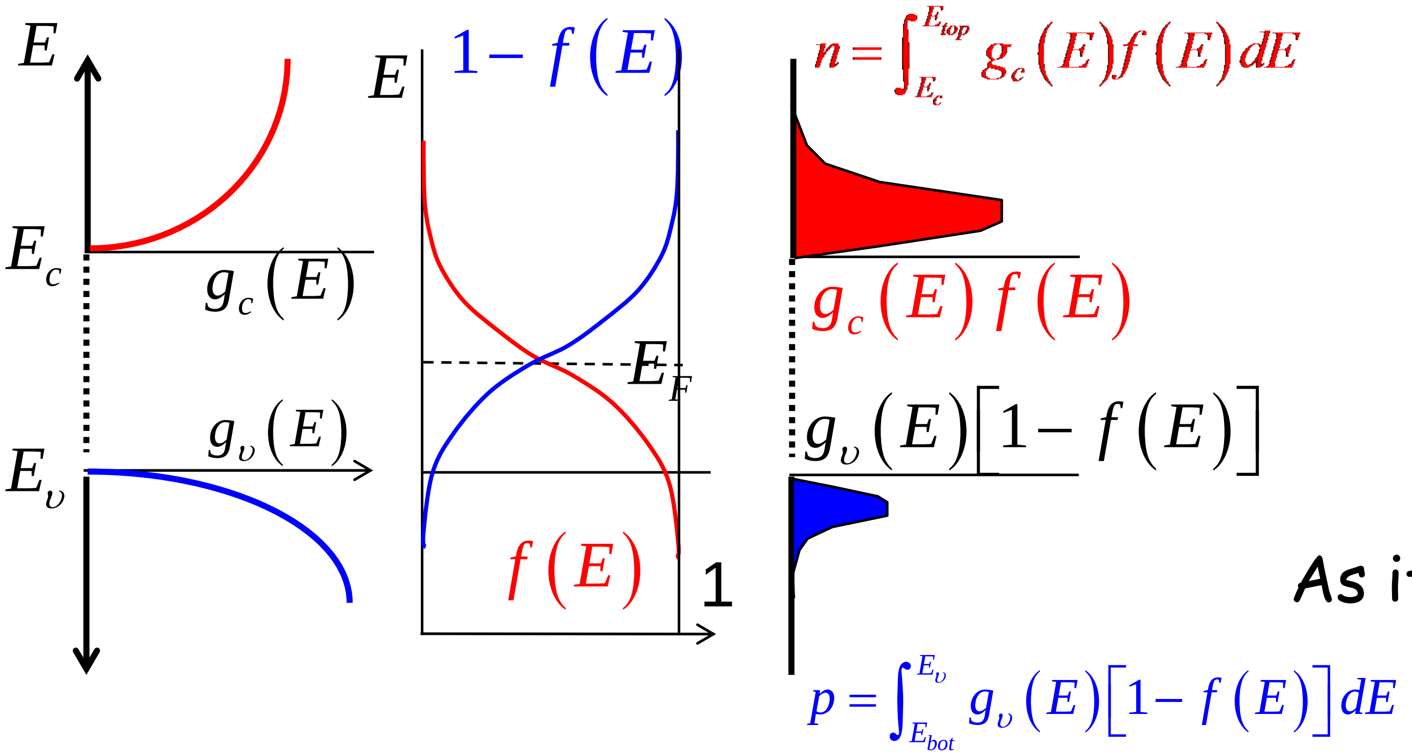
Carrier Distribution => Effective Density of States

Can we replace the E(k) with a compact representation?

DOS F-D Charge Distribution

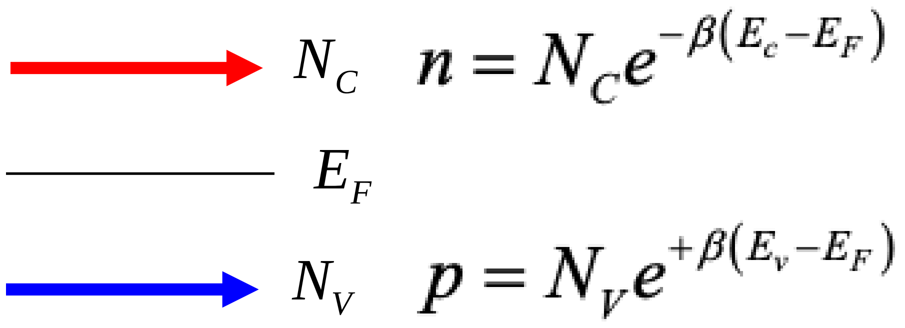


$$n=\int_{E_c}^{E_{top}} g_c(E)f(E)\,dE$$

$$p=\int_{E_{bot}}^{E_\upsilon} g_\upsilon(E)\left[1-f(E)\right]dE$$

No energy Distribution!

Delta Function Charge



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As if all states are at a single level E_C

Carrier Distribution => Effective Density of States

Why does this work?

- n_i is of the order of $10^{10}/cm^3$ in $10^{22}/cm^3$ atoms!
- Each atom has ~10-20 electrons

=> ~1 in 10^{13} electrons is mobile

- Typical doped devices have 10^{18} mobile electrons

=> ~1 in 10^{5} electrons is mobile

=> Do not include the coulomb interactions of individual free electrons

$\Rightarrow$ Consider system to be in local equilibrium

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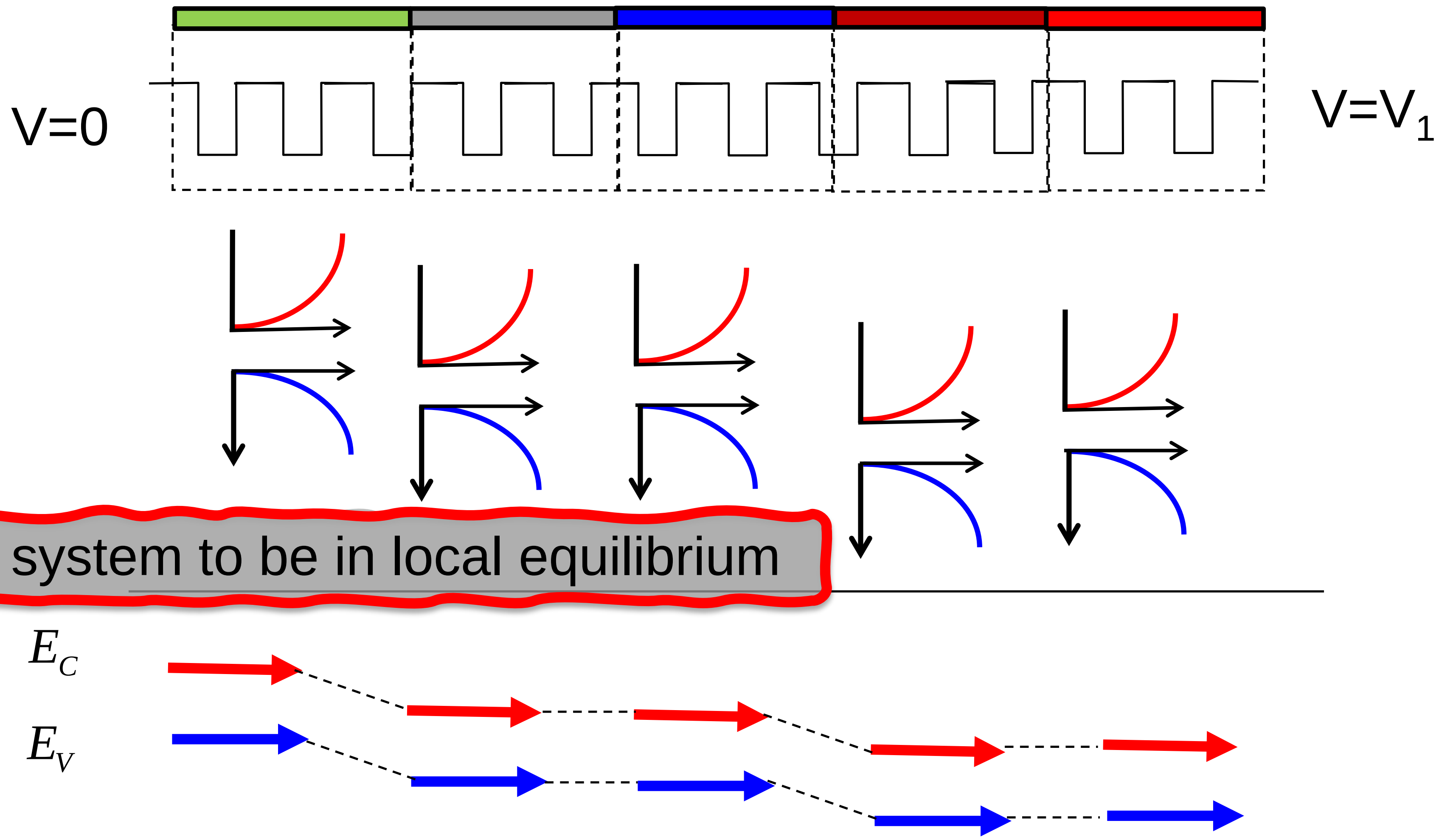
E-k Diagram vs. Band-diagram



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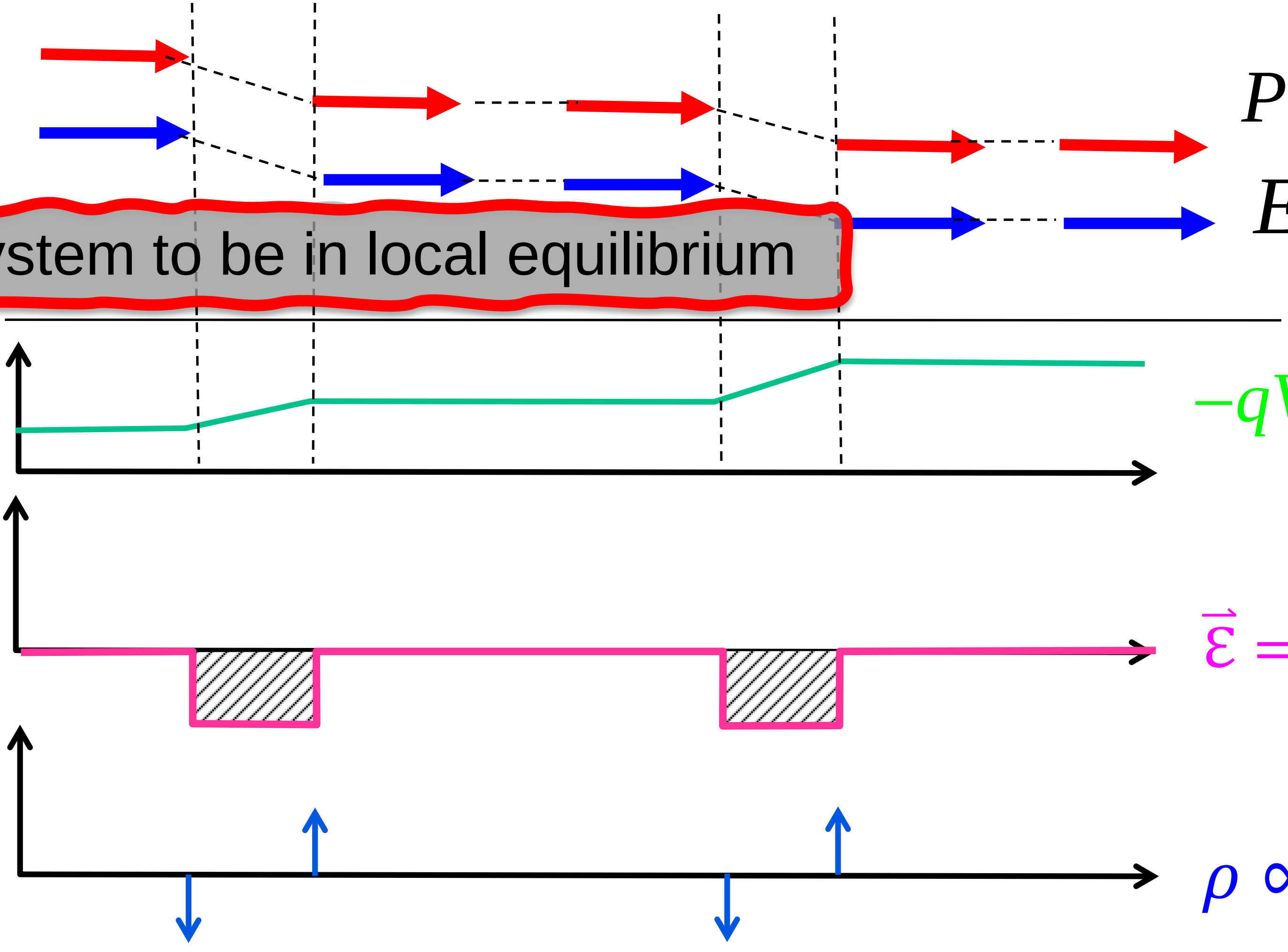
All quantum mechanics is now hidden in a single point per band!

Potential, Field and Charge

⇒ Consider system to be in local equilibrium



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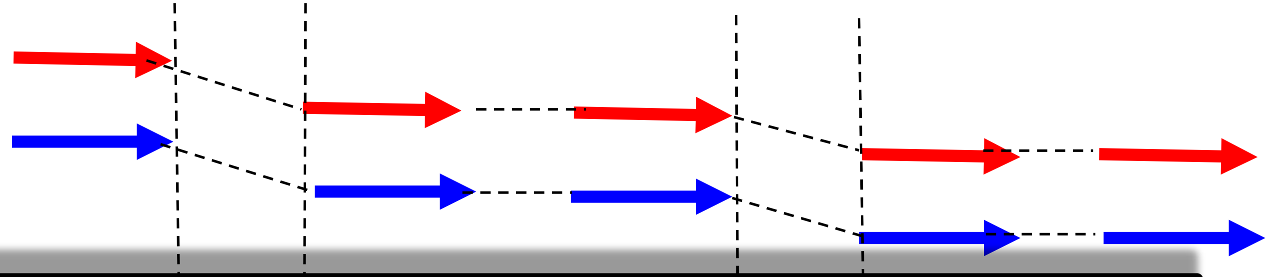

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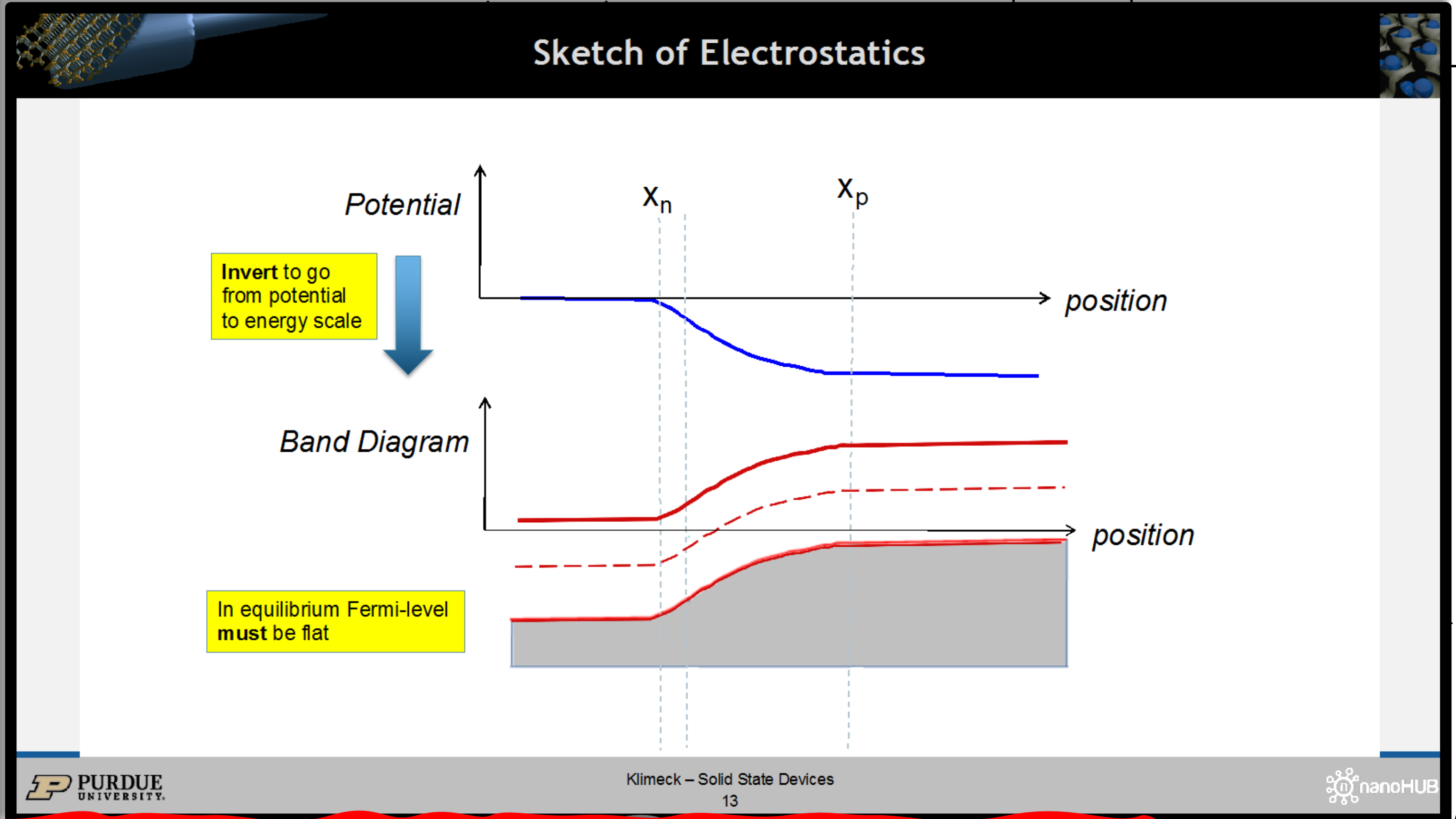

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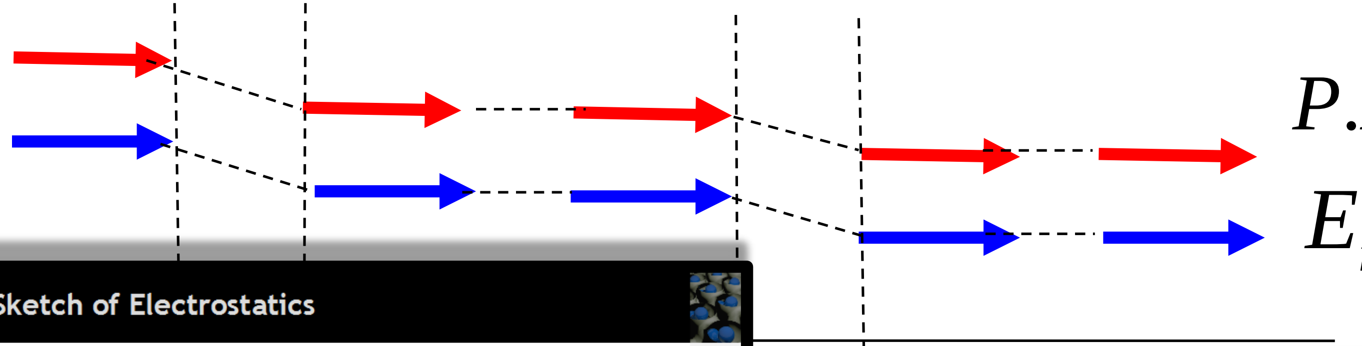

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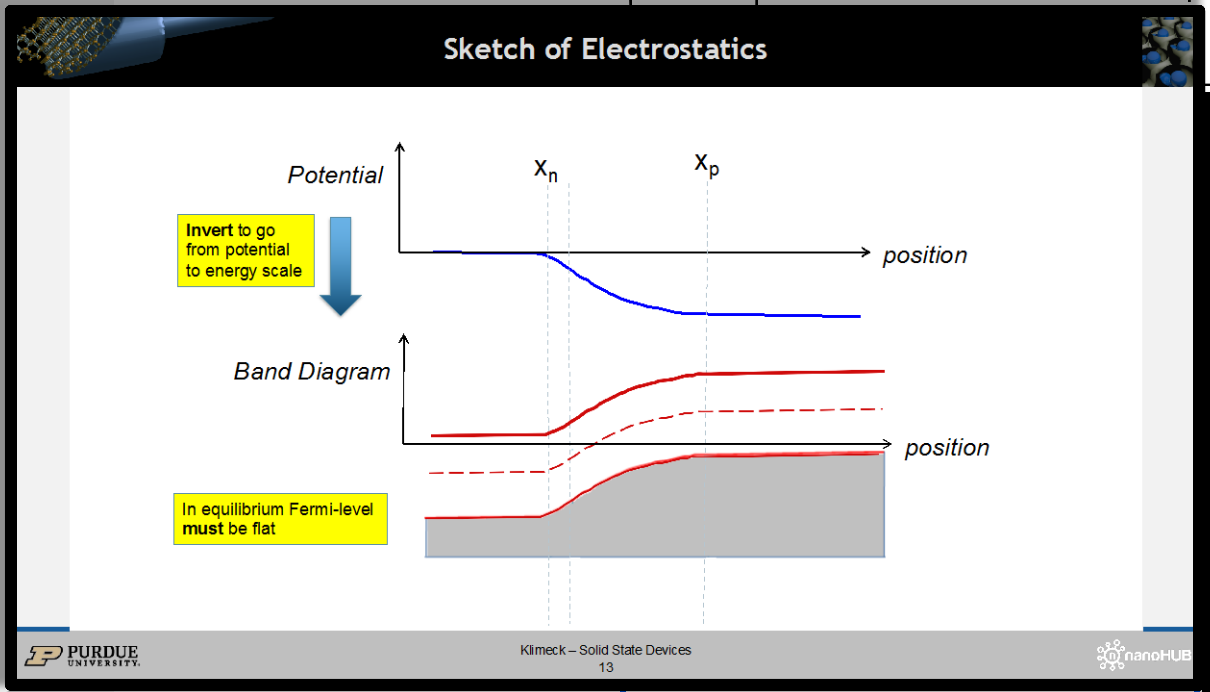

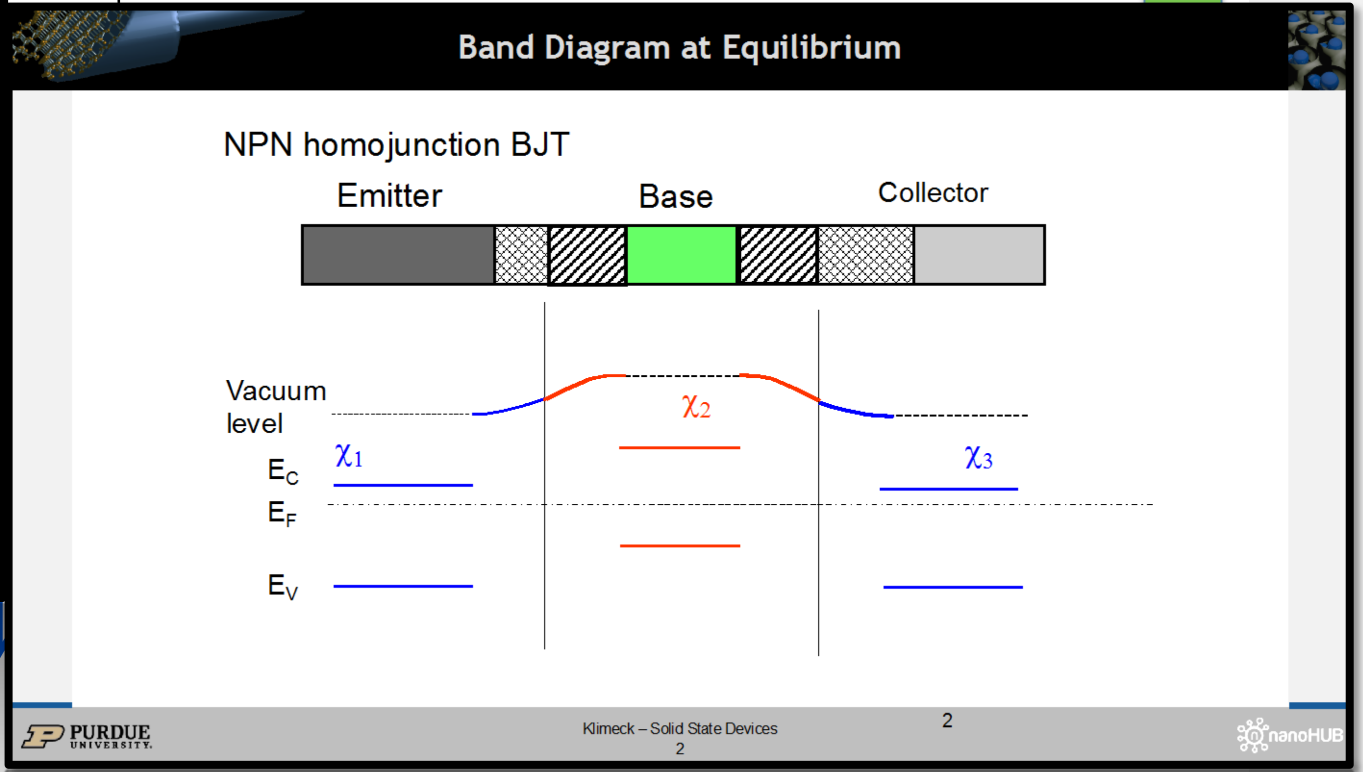

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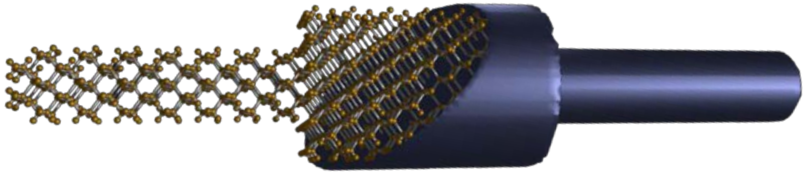

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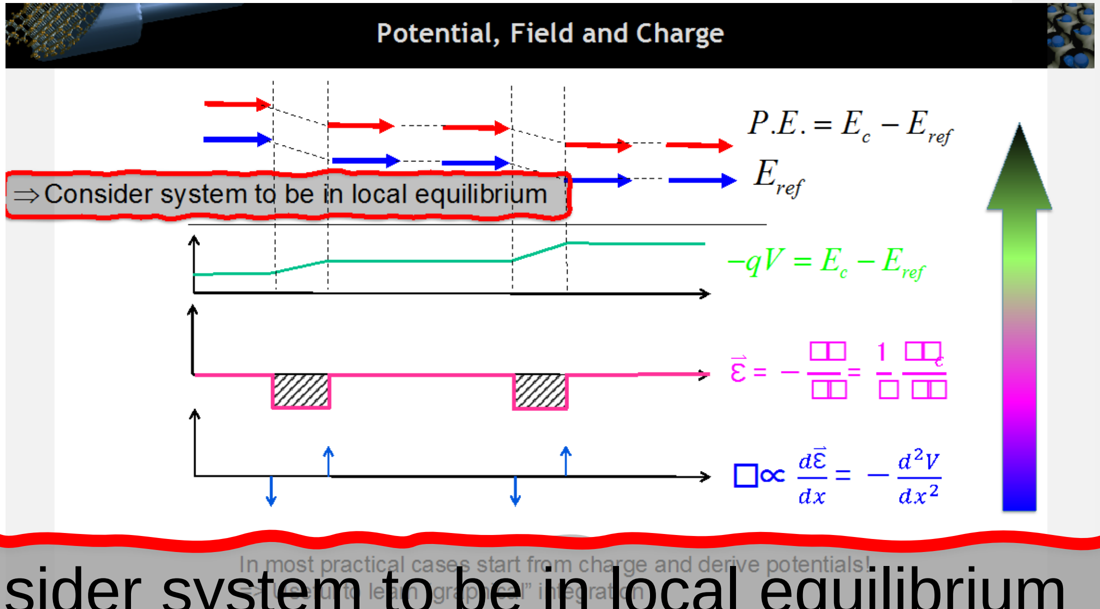

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