Section 12
Occupation of States
12.3 Intrinsic carrier concentration

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Section 12
Occupation of States

One Video Segment

- 12.1 Rules of filling electronic states
  - » Pauli exclusion
  - » Total particle conservation
  - » Total energy conservation

One Video Segment

- 12.2 Derivation of Fermi-Dirac Statistics: three techniques
  - » Microcanonical ensemble - statistics
  - » Detailed Balance – thermal equilibrium & Pauli exclusion
  - » Partition Function – statistical mechanics

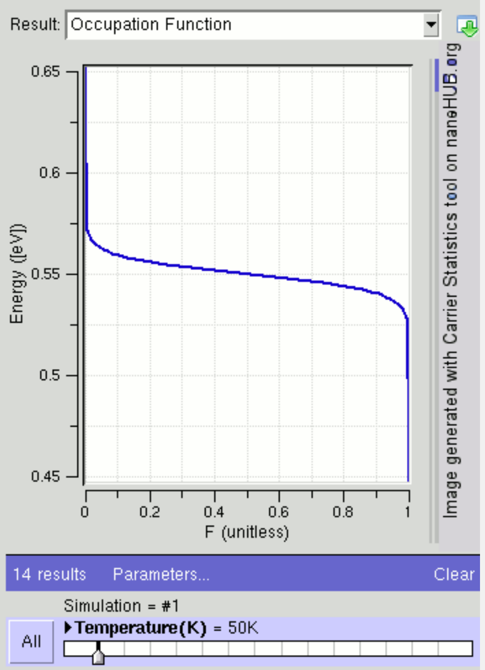
One Video Segment

- 12.3 Intrinsic carrier concentration
  - » Fermi Integral
  - » Law of mass action

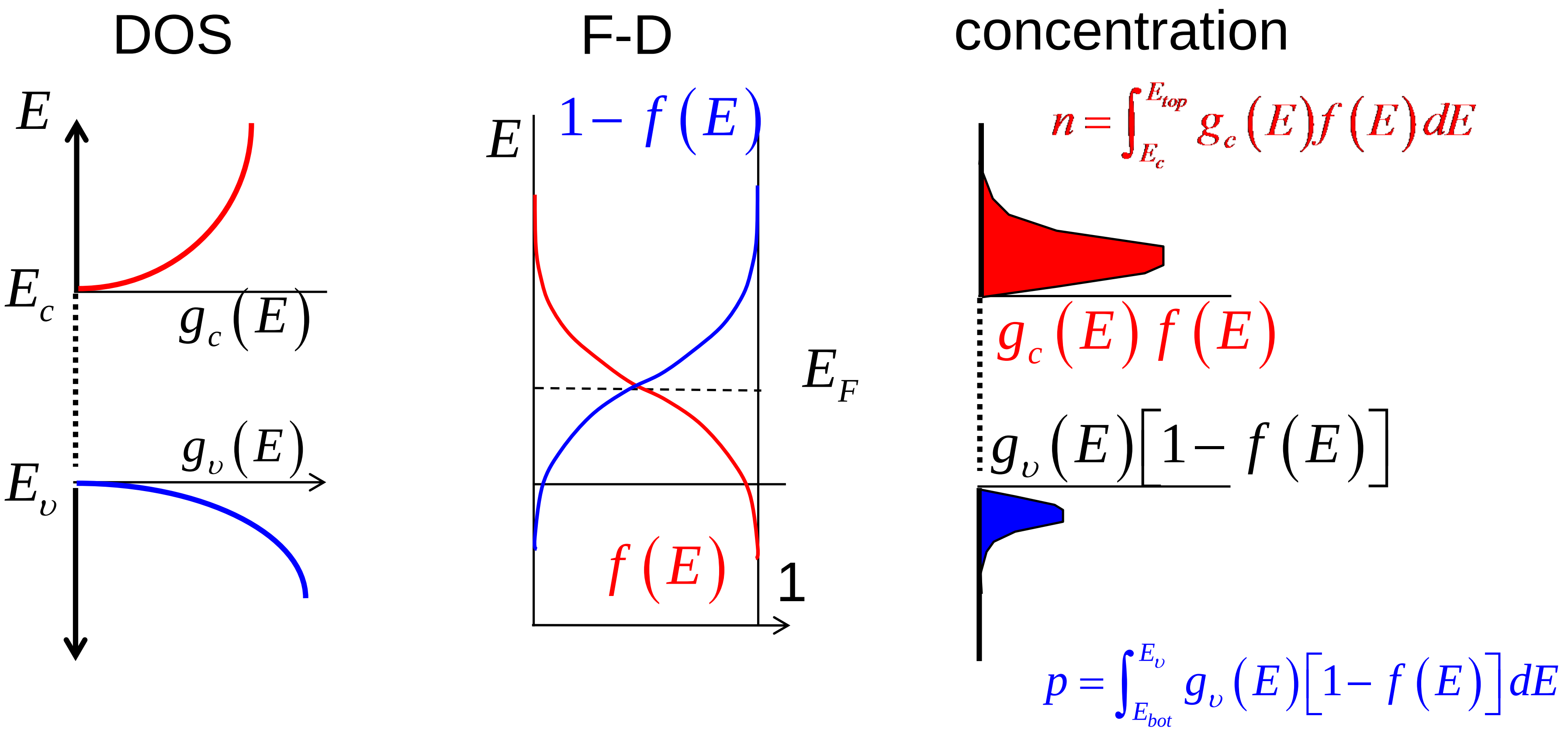
status

Carrier number =
Number of states x ***filling factor***

$$f_0(E) = \frac{1}{1+e^{\beta(E-E_F)}}$$



Reference: Vol. 6, Ch. 4


DOS
F-D
concentration
E
E_c
$g_c(E)$
$g_\upsilon(E)$
E_υ
E
$1-f(E)$
E_F
$f(E)$
1
$n=\int_{E_c}^{E_{top}} g_c(E)f(E)dE$
$g_c(E)f(E)$
$g_\upsilon(E)[1-f(E)]$
$p=\int_{E_{bot}}^{E_\upsilon} g_\upsilon(E)[1-f(E)]dE$

Electron Concentration in 3D solids

$$n = \int_{E_c}^{E_{top}} g_c(E) f(E)\, dE$$

$$= \int_{E_c}^{E_{top}} 2 \times \frac{m_n^* \sqrt{2m_n^*(E - E_C)}}{2\pi^2\hbar^3} \frac{1}{1 + e^{\beta(E - E_F)}} dE$$

Include spin factor of 2

$$\square \int_{E_c}^{\infty} \frac{m_n^* \sqrt{2m_n^*(E - E_C)}}{\pi^2\hbar^3} \frac{1}{1 + e^{\beta(E - E_c)} e^{\beta(E_c - E_F)}} dE$$

Assume wide bands

$$n = N_C \frac{2}{\sqrt{\pi}} F_{1/2}(\eta_c)$$

$$N_C \equiv 2\left(\frac{2\pi m_n^* \beta}{h^2}\right)^{3/2}$$

$$F_{1/2}(\eta) = \int_0^\infty \frac{\sqrt{\xi}\, d\xi}{1 + e^{\xi - \eta}}$$

$$\eta_c \equiv \beta(E_F - E_C)$$

Fermi Integral is Painful to Evaluate
(not analytic - lookup tables or expensive numerical)

$$n = N_C \frac{2}{\sqrt{\pi}} F_{1/2}(\eta_c) \qquad \eta_c \equiv \beta (E_F - E_C)$$

$$N_C \equiv 2\left(\frac{2\pi m_n^* \beta}{h^2}\right)^{3/2} \qquad F_{1/2}(\eta) = \int_0^\infty \frac{\sqrt{\xi}\, d\xi}{1+e^{\xi-\eta}}$$

$$n = N_C \frac{2}{\sqrt{\pi}} F_{1/2}(\eta_c) \rightarrow N_C e^{\eta_c}$$

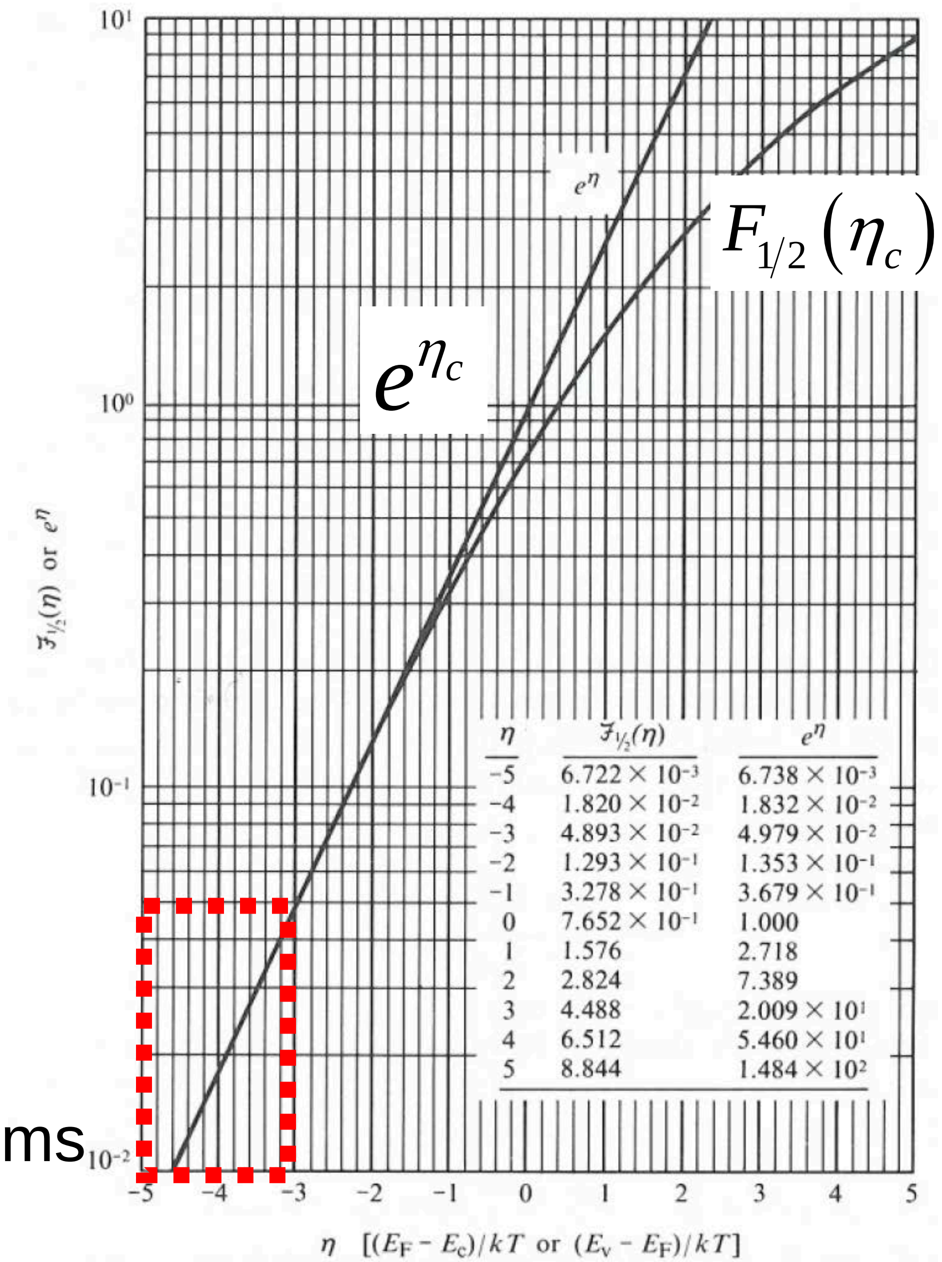
$$if \quad -\eta_c \equiv \beta(E_C - E_F) > 3$$

$$n = N_C e^{-\beta(E_c - E_F)}$$

Boltzmann Statistics!

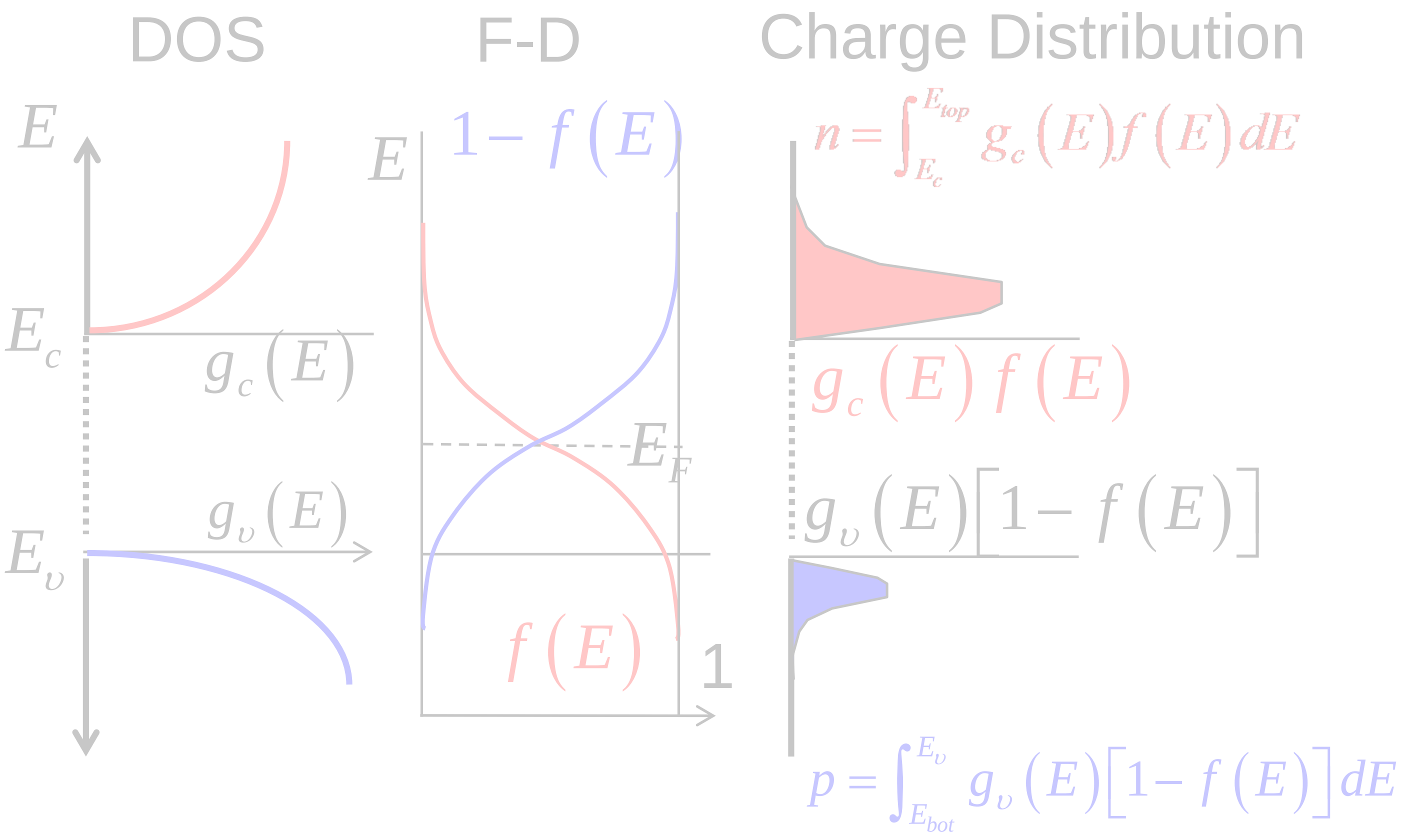
$$p = N_V e^{+\beta(E_v - E_F)}$$

For non-degenerate systems



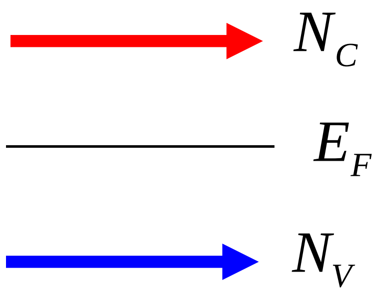
η	$\mathcal{F}_{1/2}(\eta)$	e^{η}
−5	6.722×10^{-3}	6.738×10^{-3}
−4	1.820×10^{-2}	1.832×10^{-2}
−3	4.893×10^{-2}	4.979×10^{-2}
−2	1.293×10^{-1}	1.353×10^{-1}
−1	3.278×10^{-1}	3.679×10^{-1}
0	7.652×10^{-1}	1.000
1	1.576	2.718
2	2.824	7.389
3	4.488	2.009×10^{1}
4	6.512	5.460×10^{1}
5	8.844	1.484×10^{2}

Carrier Distribution => Effective Density of States



No energy Distribution!

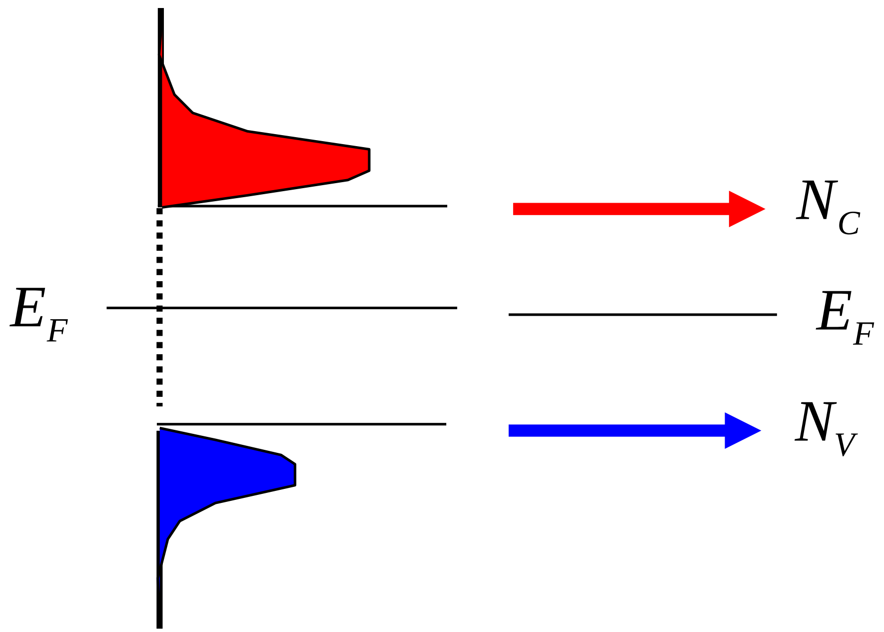
Delta Function Charge



$$n = N_C e^{-\beta(E_c - E_F)}$$

$$p = N_V e^{+\beta(E_v - E_F)}$$

As if all states are at a single level E_C



$$n = N_C e^{-\beta(E_c - E_F)}$$

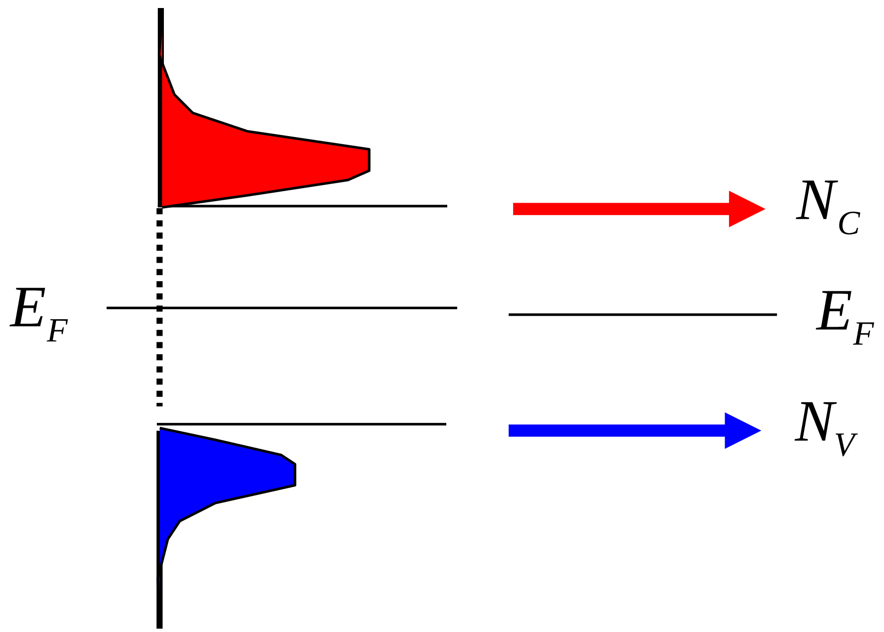
$$p = N_V e^{+\beta(E_v - E_F)}$$

$$n \times p = N_C N_V e^{-\beta(E_c - E_v)}$$

$$= N_C N_V e^{-\beta E_g}$$

Product is independent of the Fermi level!
Very useful balance equation! Will use it often

Fermi-Level for Intrinsic Semiconductors



$$n = p = n_i$$

$$n_i^2 = N_C N_V e^{-\beta E_g}$$

$$n_i = \sqrt{N_C N_V}\, e^{-\beta E_g/2}$$

$$E_F \equiv E_i$$

$$n = p \Rightarrow N_C e^{-\beta(E_c - E_i)} = N_V e^{+\beta(E_v - E_i)}$$

$$E_i = \frac{E_G}{2} + \frac{1}{2\beta} \ln \frac{N_V}{N_C}$$

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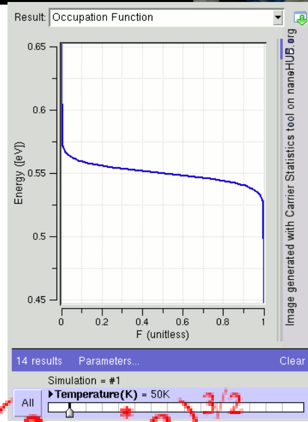
$$f_0(E) = \frac{1}{1+e^{\beta(E-E_F)}}$$



One Video Segment

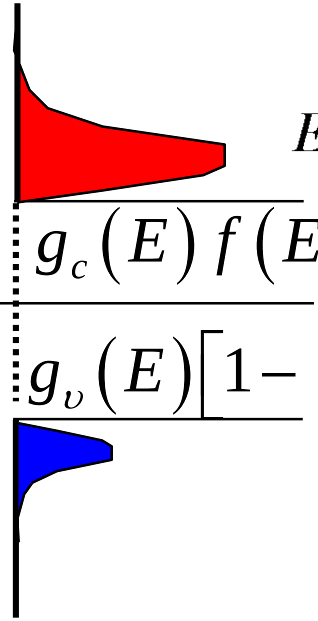
- 12.2 Derivation of Fermi-Dirac Statistics: three techniques
  - » Microcanonical ensemble - statistics
  - » Detailed Balance – thermal equilibrium & Pauli exclusion
  - » Partition Function – statistical mechanics

$$N_C \equiv 2\left(\frac{2\pi m_n \beta}{h^2}\right)^{3/2}$$

$$n = N_C \frac{2}{\sqrt{\pi}} F_{1/2}(\eta_c) \rightarrow N_C e^{-\beta(E_c-E_F)}$$

One Video Segment

- 12.3 Intrinsic carrier concentration
  - » Fermi Integral
  - » Law of mass action



$$g_c(E)\,f(E)$$

$$g_\upsilon(E)\left[1-f(E)\right]$$

$$E_i = \frac{E_G}{2} + \frac{1}{2\beta} ln \frac{N_V}{N_C}$$

$$n \times p = N_C N_V e^{-\beta(E_c-E_v)}$$
$$= N_C N_V e^{-\beta E_g}$$

⇒ **Quantum Mechanics Mechanics**
- Concepts of density of states and masses

⇒ **Equilibrium Statistical Mechanics**
- Occupation factors

What's Next?

Reference: Vol. 6, Ch. 4

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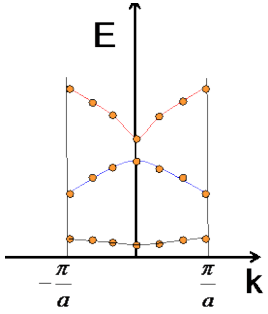

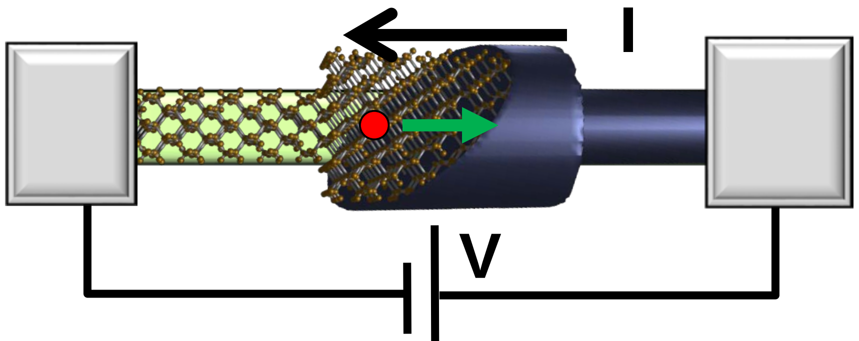

I = G × V

= q × n × v × A

charge density velocity area

$$f_0(E) = \frac{1}{1+e^{\beta(E-E_F)}}$$

$$N_C \equiv 2\left(\frac{2\pi m_n^* \beta}{h^2}\right)^{3/2}$$

$$n = N_C \frac{2}{\sqrt{\pi}} F_{1/2}(\eta_c) \rightarrow N_C e^{-\beta(E_c - E_F)}$$

$$E_i = \frac{E_G}{2} + \frac{1}{2\beta} ln \frac{N_V}{N_C}$$

$$n \times p = N_C N_V e^{-\beta(E_c - E_v)}$$

$$= N_C N_V e^{-\beta E_g}$$

- **Materials, composition, crystals**
- Tabulated for "known" bulk materials
- ~~[illegible] theory~~

~~⇒ Quantum Mechanics Mechanics~~
- Concepts of density of states and masses

⇒ **Equilibrium Statistical Mechanics**
- Occupation factors

Transport with scattering, non-equilibrium Statistical Mechanics
- Drift-diffusion equation with recombination-generation

Understanding transport in concrete devices
- Diodes, BJT/HBT, MOS