Section 11
Bandstructure Measurements
11.2 Effective mass measurements

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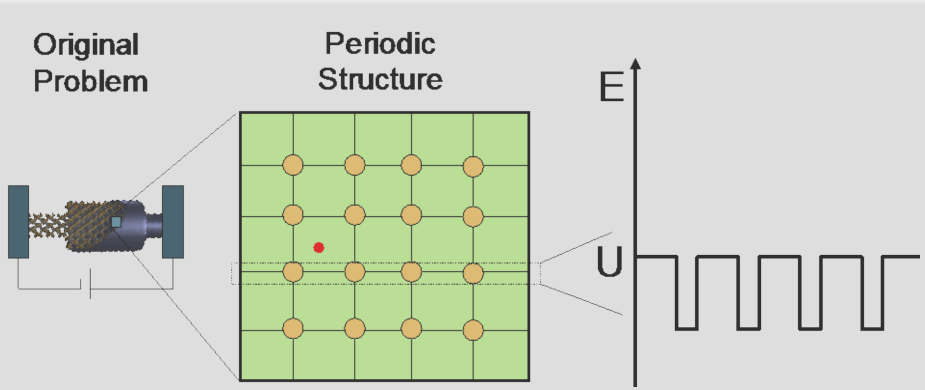
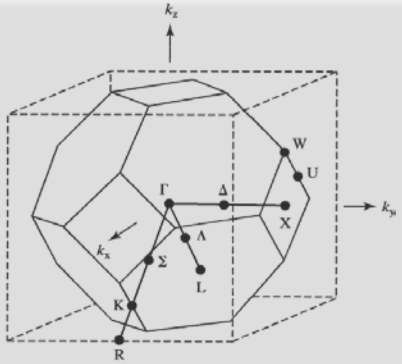
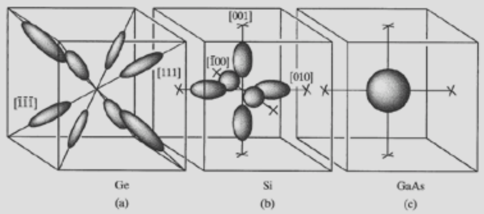
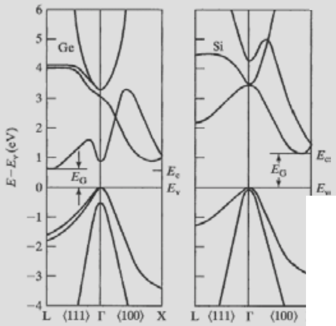




$$\frac{1}{m_{ij}} = \frac{1}{\hbar^2}\frac{\partial^2 E}{\partial k_i \partial k_j}$$





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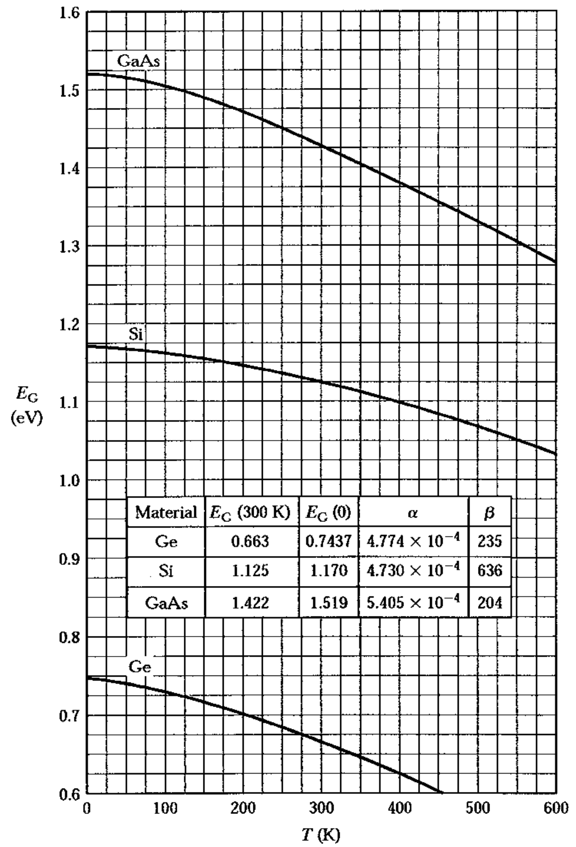

Material	E_G (300 K)	E_G (0)	α	β
Ge	0.663	0.7437	4.774×10^{-4}	235
Si	1.125	1.170	4.730×10^{-4}	636
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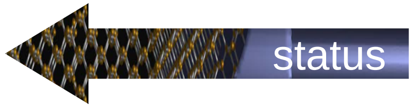
Bandstructure Measurements **test and validate the theories**

Video Segment

- 11.1 Bandgap measurements

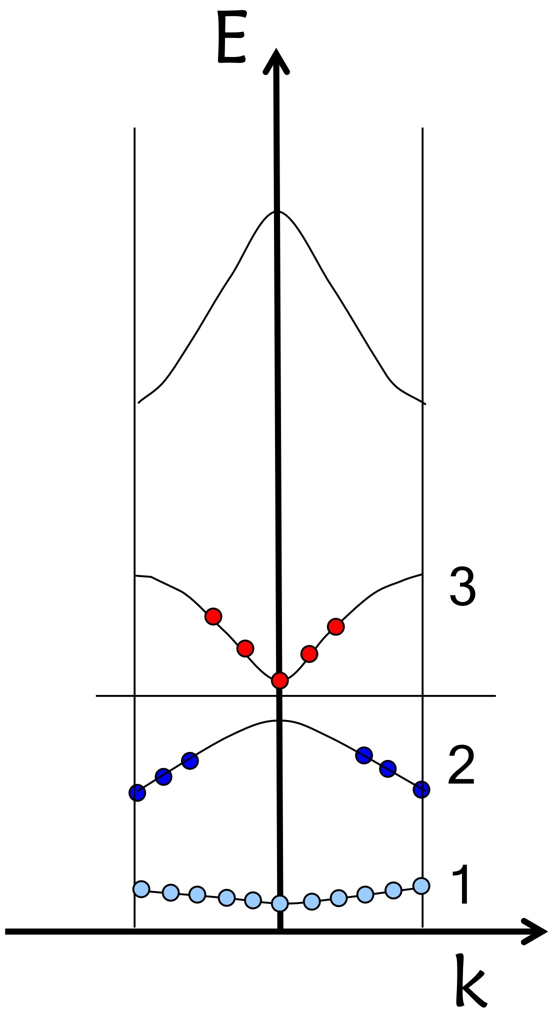
Video Segment

- 11.2 Effective mass measurements



Reference: Vol. 6, Ch. 3

11.2 Effective mass measurements



Important things to remember:

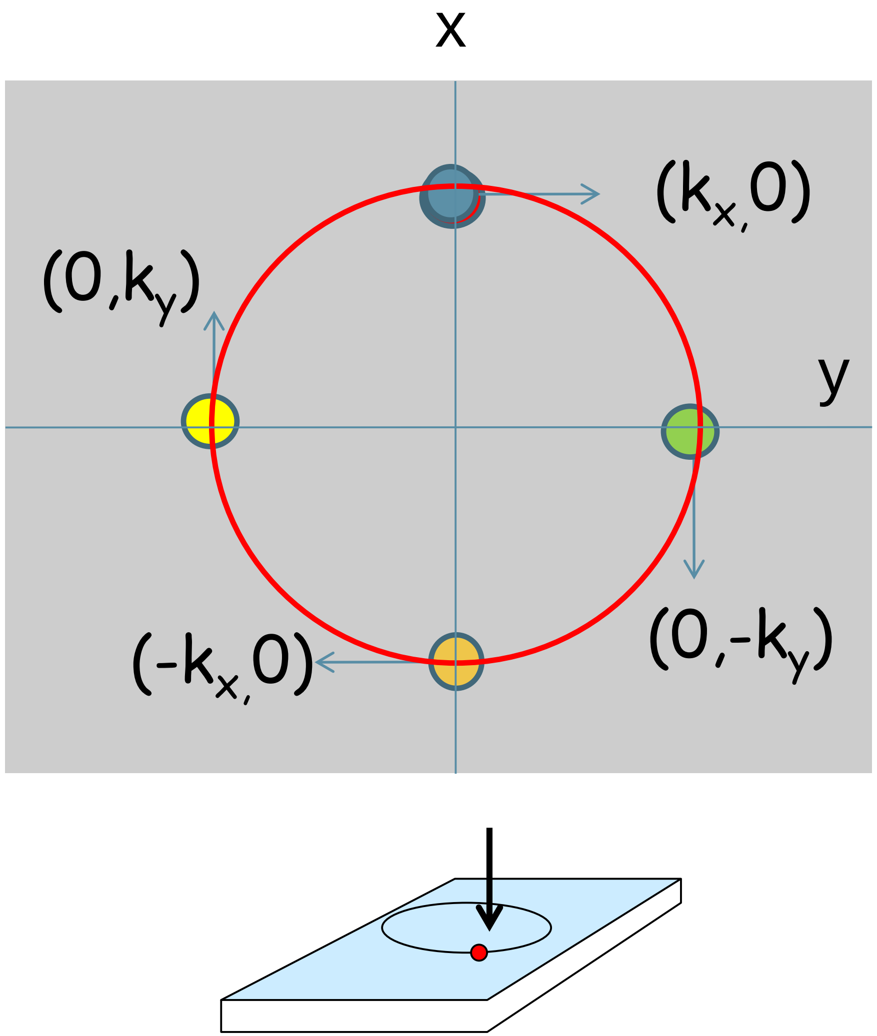
- Full bands do not conduct – electrons have no space to go
- Empty bands to not conduct – there are no electrons to go around

Question:

- We are interested in the top-most valence band holes and the bottom-most electron states
- We want to figure out the slope of the bands
- How can we probe just one particular species of electrons/holes?
- We do not want to transfer them from one band to the next!

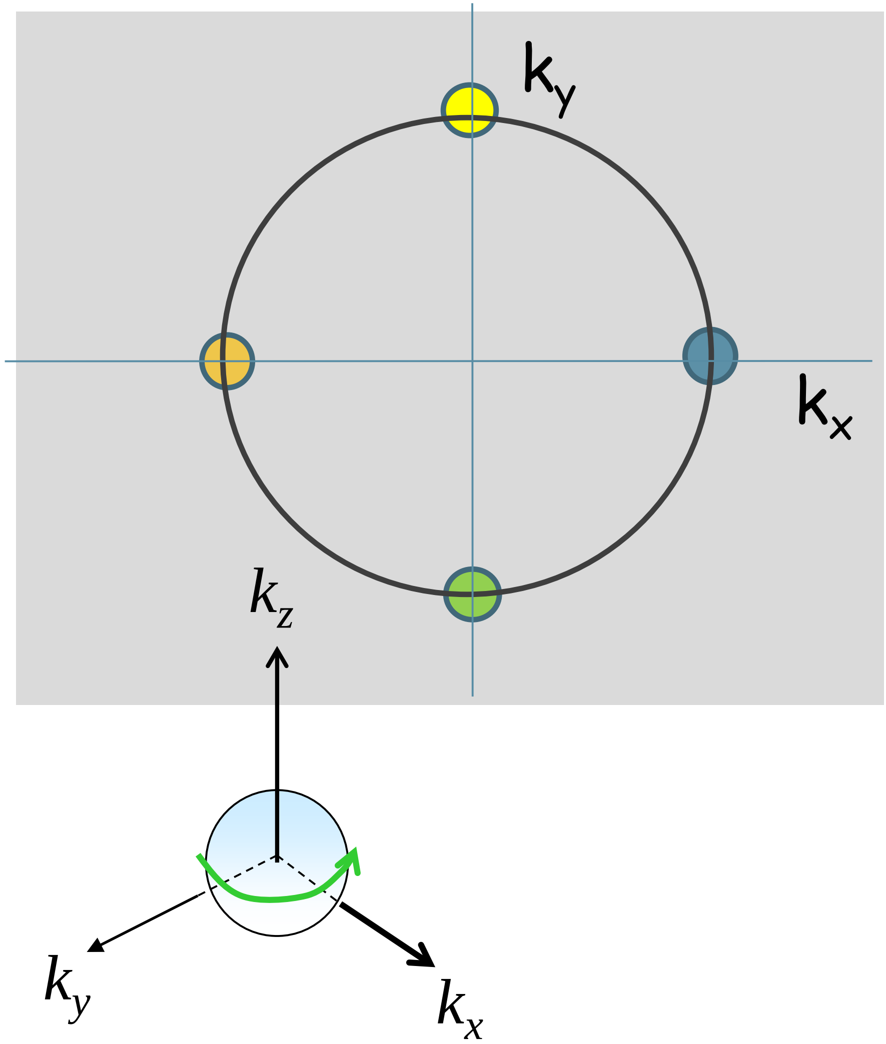
=> can we rotate the electrons around in a single band?

Motion in Real Space and Phase Space



Energy=constant.

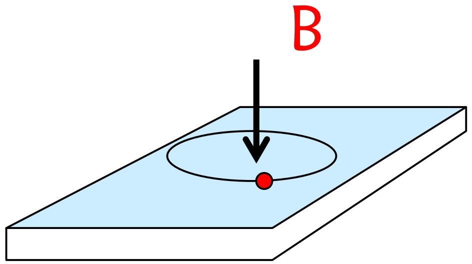
Liquid He temperature …



Derive the Cyclotron Formula

$$m^* = \frac{qB_0}{2\pi\nu_0}$$

For an particle in (x-y) plane with B-field in z-direction, the Lorentz force is …



$$\frac{m^* \upsilon^2}{r_0} = q\upsilon \times B_z = q\upsilon B_z$$

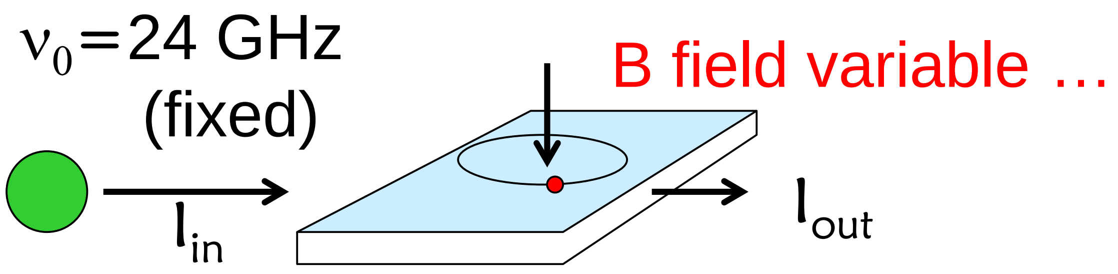
$$\upsilon = \frac{qB_0 r_0}{m^*}$$

$$\tau = \frac{2\pi r_0}{\upsilon} = \frac{2\pi m^*}{qB_0}$$

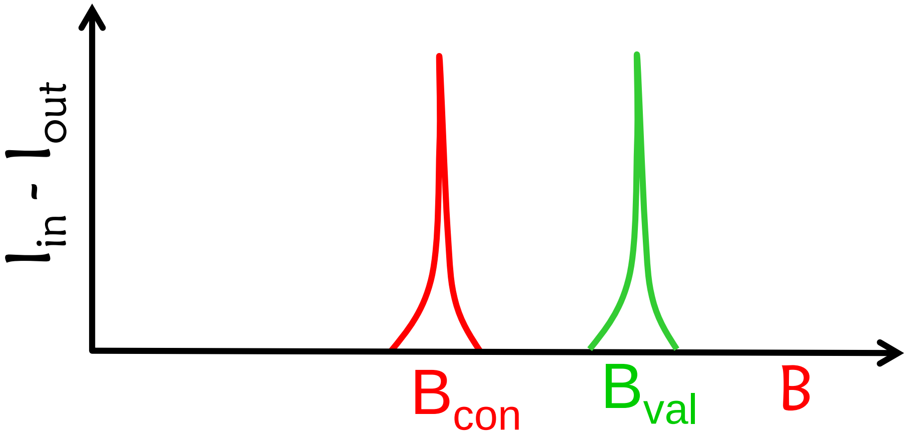
$$\nu_0 \equiv \frac{1}{\tau} = \frac{qB_0}{2\pi m^*}$$

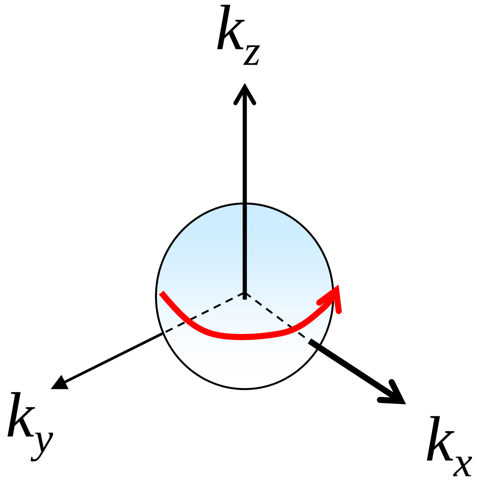
$$\omega_0 = 2\pi\nu_0 = \frac{qB_0}{m^*}$$

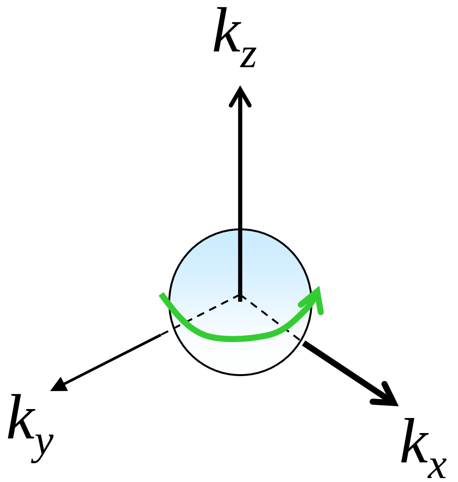
Measurement of Effective Mass



$$\nu_0 = \frac{qB_0}{2\pi m^*} \qquad m^* = \frac{qB_0}{2\pi \nu_0}$$

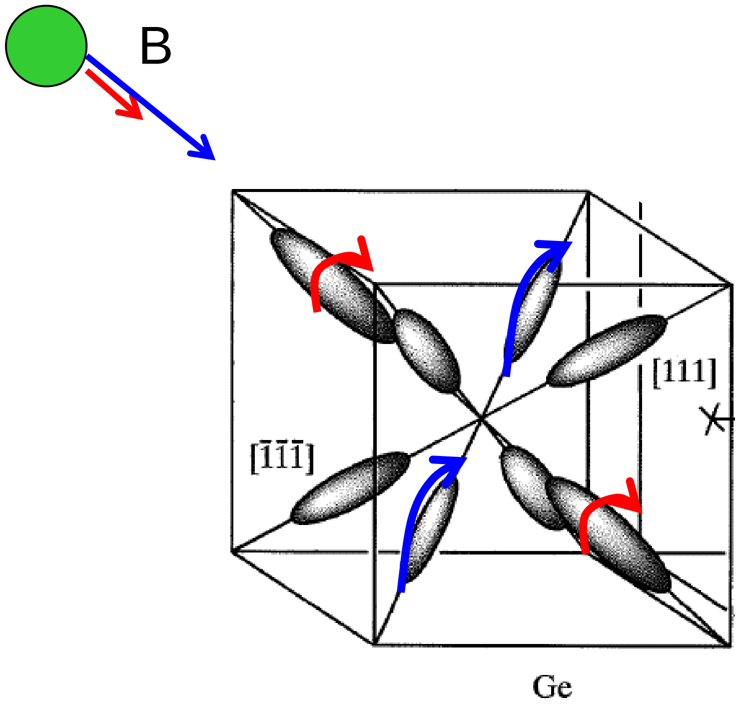






Measurement "easy" for spherical surfaces

Effective mass in Ge



$[\bar{1}\bar{1}\bar{1}]$ $[\bar{1}\bar{1}1]$ $[\bar{1}1\bar{1}]$ $[1\bar{1}\bar{1}]$

$[1\bar{1}\bar{1}]$ $[1\bar{1}\bar{1}]$ $[\bar{1}1\bar{1}]$ $[111]$

4 symmetry angles between B field and the ellipsoids …

(in Si, there would 6 equivalent valleys, 3 specific directions)

Cyclotron Formula for Multiple Valleys

Show that $$\frac{1}{m_c^{\,2}} = \frac{\cos^2\theta}{m_t^{\,2}} + \frac{\sin^2\theta}{m_l m_t}$$

Experiment looks at three θ,
and measures three m_c
From this expression you derive m_t, and m_l

The Lorentz force on electrons in a B-field

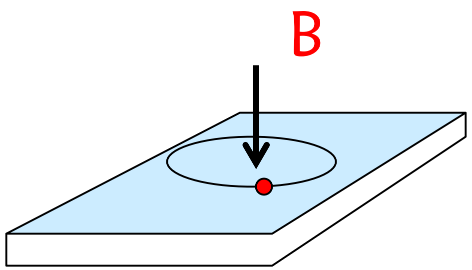
$$F = q\upsilon \times B = [M]\frac{d\upsilon}{dt}$$

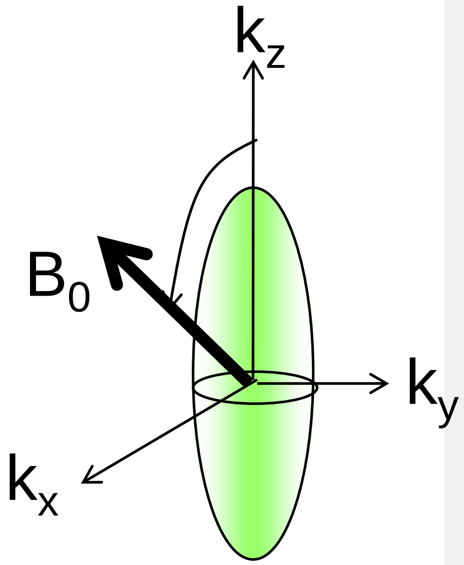
In other words,

$$F_x = q(\upsilon_y B_z - \upsilon_z B_y) = m_t^* \frac{d\upsilon_x}{dt}$$

$$F_y = q(\upsilon_z B_x - \upsilon_x B_z) = m_t^* \frac{d\upsilon_y}{dt}$$

$$F_z = q(\upsilon_x B_y - \upsilon_y B_x) = m_l^* \frac{d\upsilon_z}{dt}$$



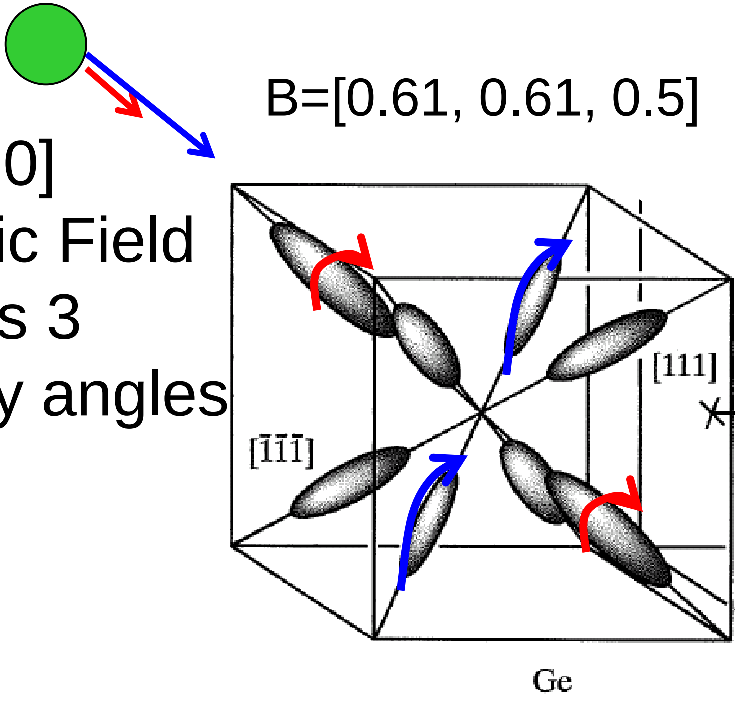


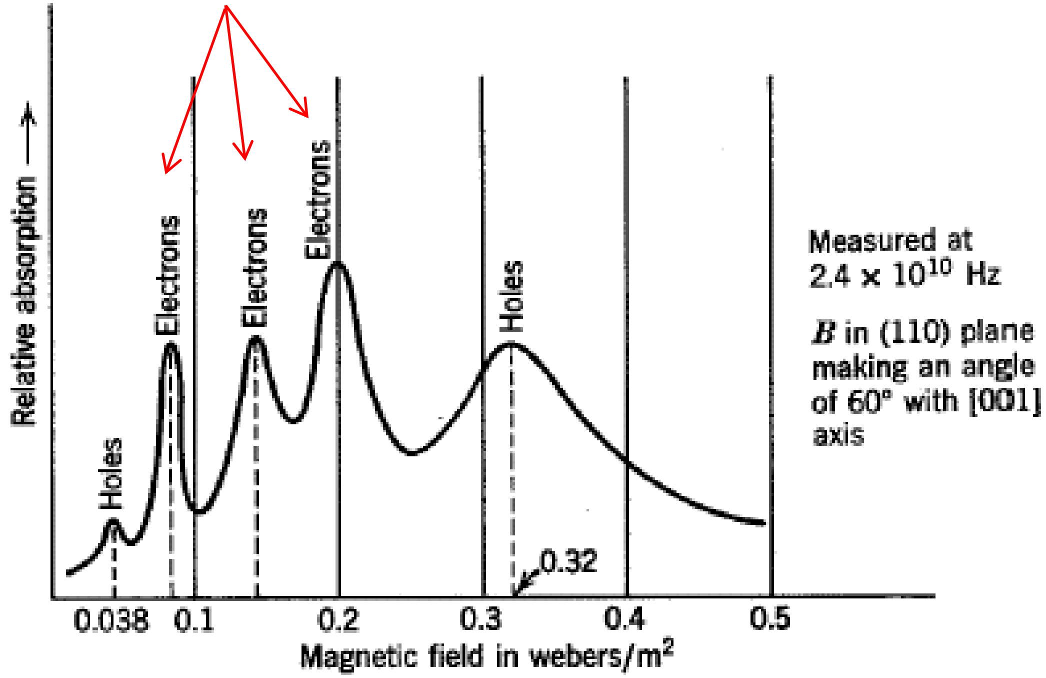
$$B_x = B_0\cos(\theta),\ B_y = 0,\ B_z = B_0\sin(\theta),$$

Measurement of Effective Mass

[110]
Magnetic Field
Picks 3
symmetry angles

B=[0.61, 0.61, 0.5]





$$\frac{1}{m_c^{\,2}} = \frac{\cos^2\theta}{m_t^{\,2}} + \frac{\sin^2\theta}{m_l m_t}$$

Three peaks B_1, B_2, B_3

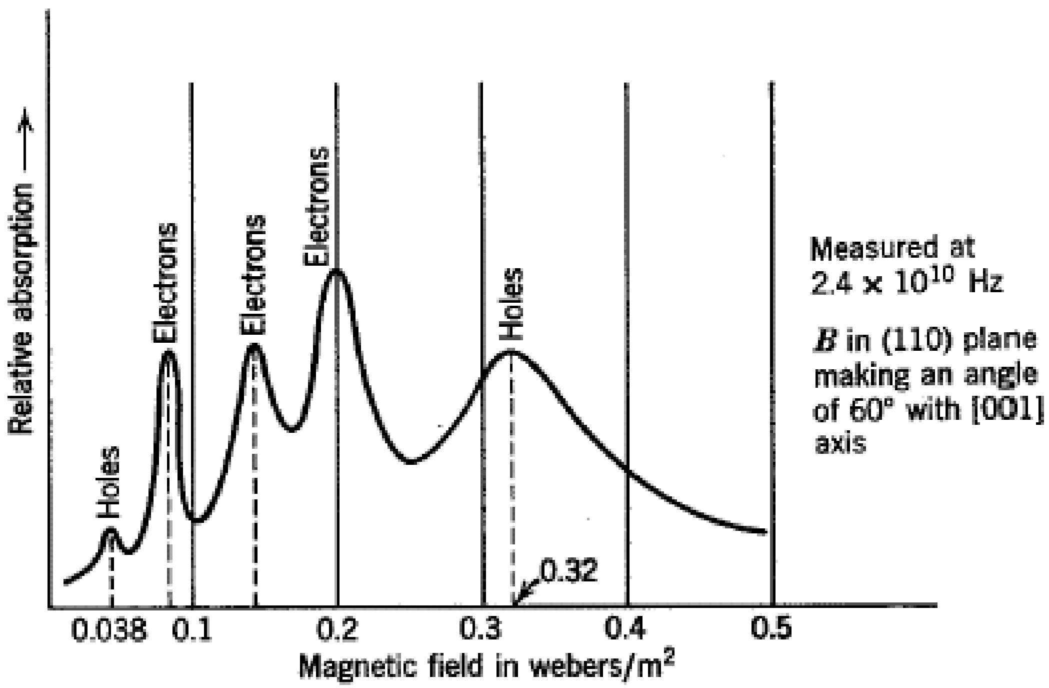
$$m_c = \frac{qB_1}{2\pi\nu_0}$$

Three masses m_{c1},m_{c2},m_{c3}

Three unique angles: 7, 65, 73

Known θ and m_c allows calculation of m_t and m_l.

Valence Band Effective Mass



Which peaks relate to valence band?
Why are there two valence band peaks?

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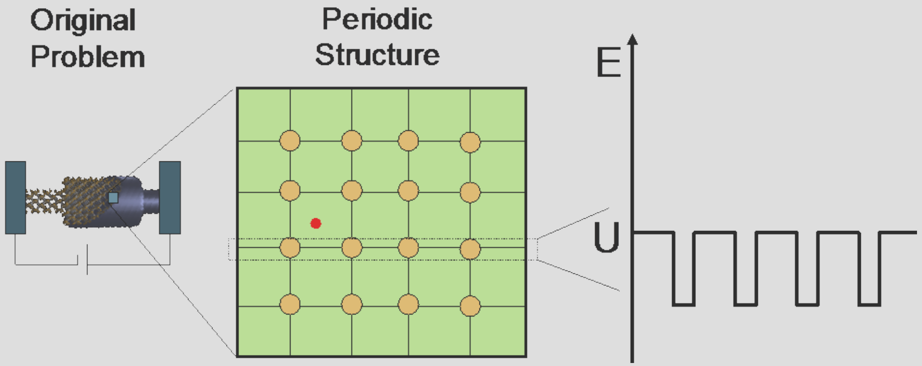

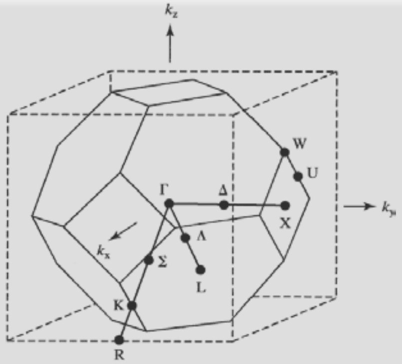

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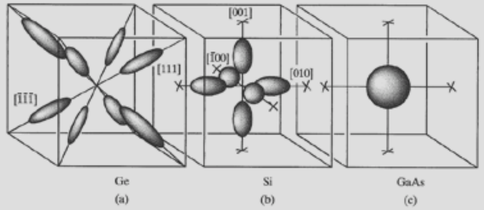

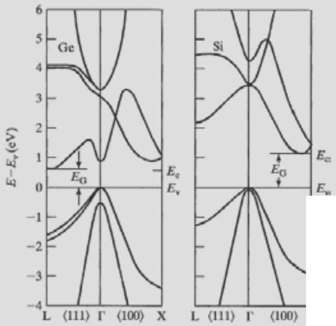

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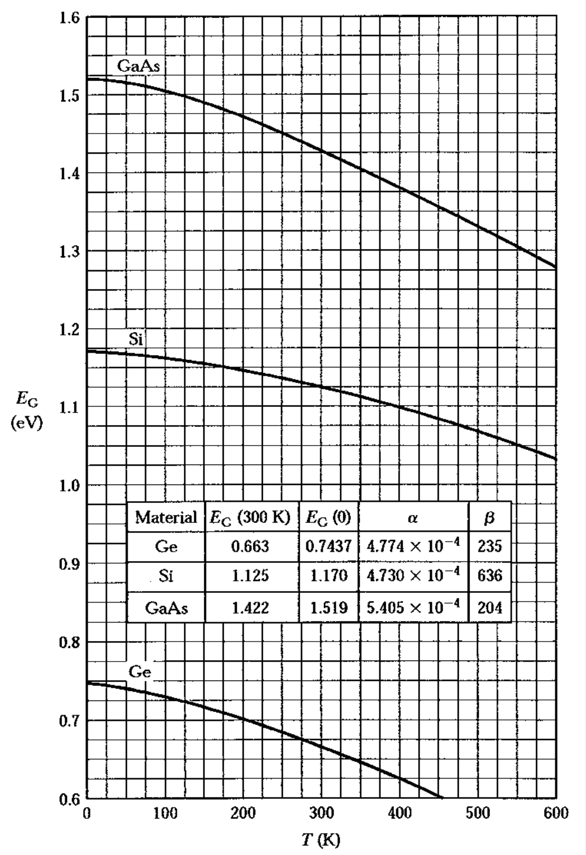

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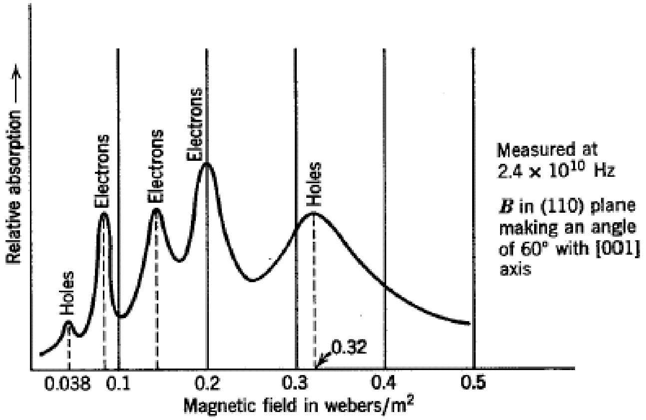

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