Section 10
Bandstructure in Real Materials (Si, Ge, GaAs)
10.3 Density of States Effective Mass

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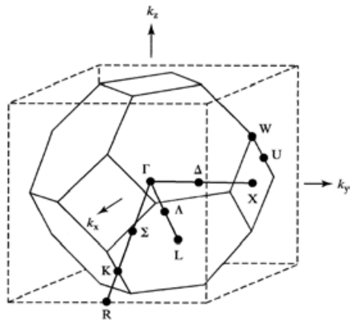
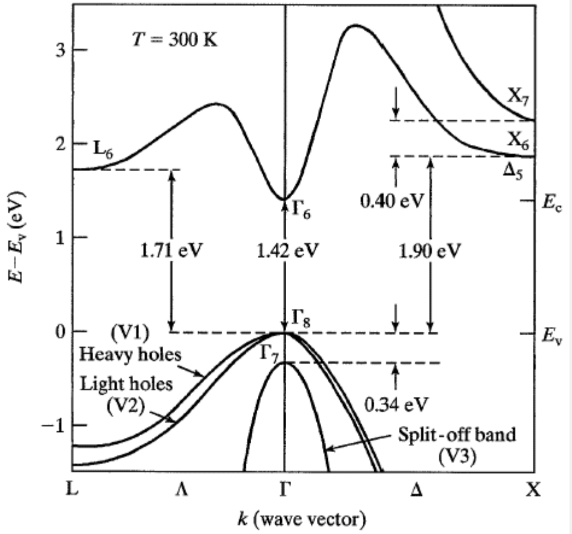
One Video Segment

- 10.1 E(k) diagrams in specific crystal directions

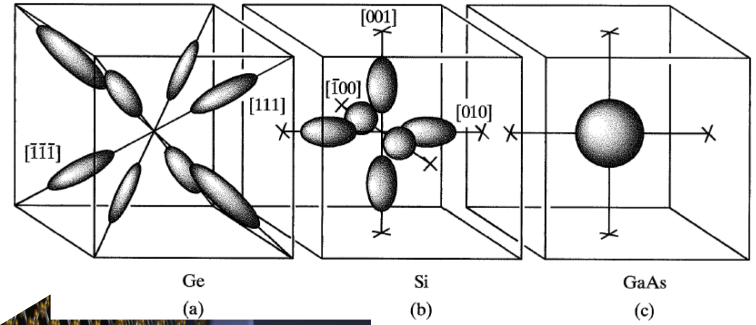


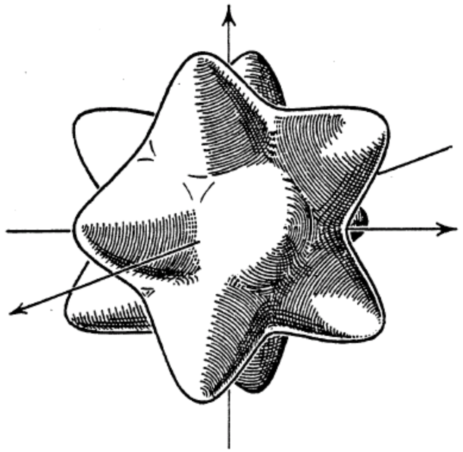


One Video Segment

- 10.2 Constant Energy Surfaces – Effective Mass Tensor

$$\frac{1}{m_{ij}} = \frac{1}{\hbar^2}\frac{\partial^2 E}{\partial k_i \partial k_j}$$





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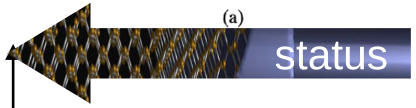
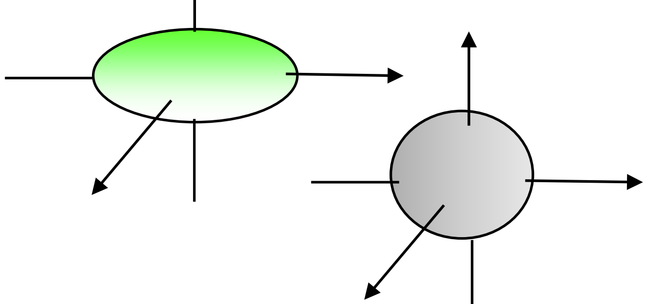
- 10.3 Density of States Effective Mass



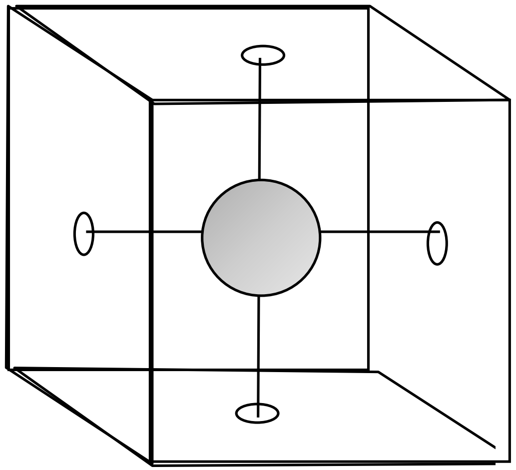


Reference: Vol. 6, Ch. 3

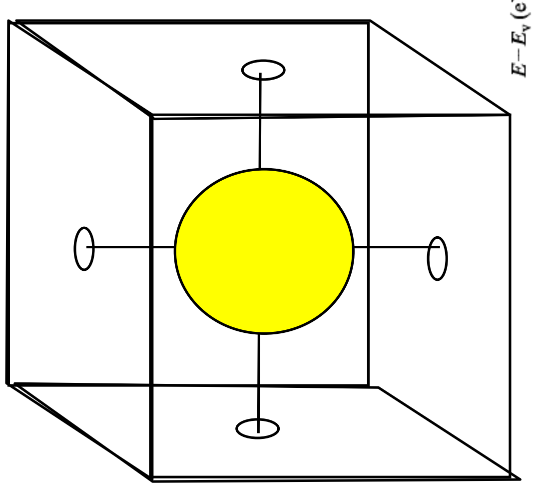
Density of States of GaAs: Conduction/Valence Bands

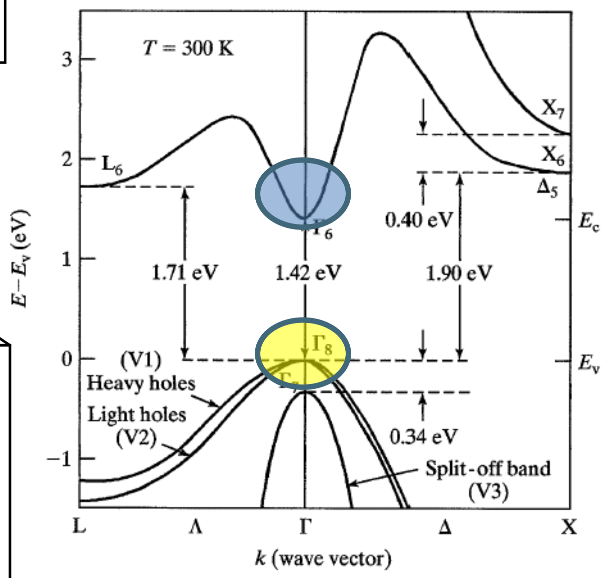
$$D_{3D}(E) \propto \sqrt{E}$$

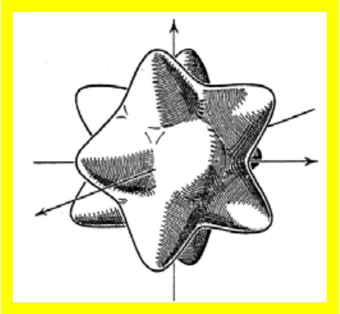
$$g_c(E) = \frac{m_n^* \sqrt{2m_n^*(E - E_c)}}{2\pi^2\hbar^3}$$



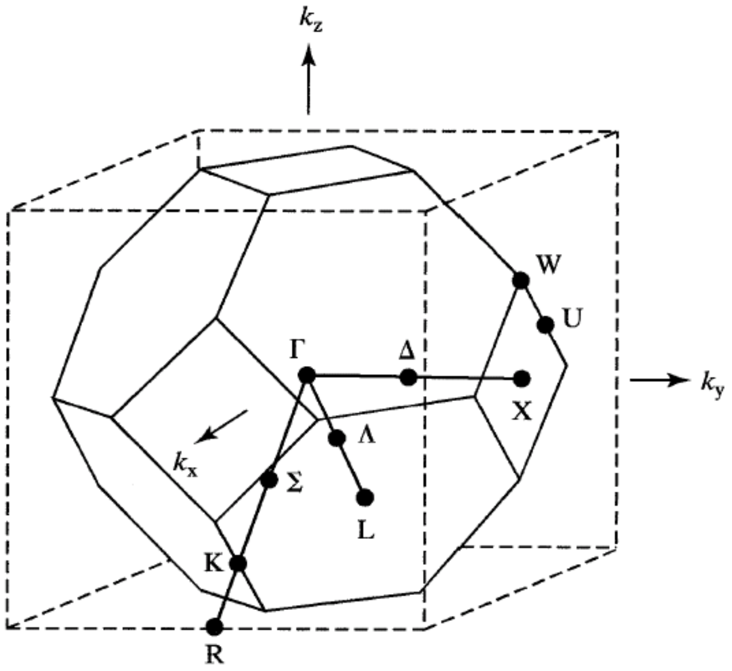
$$g_\upsilon(E) = \begin{cases} \dfrac{m_{hh}^* \sqrt{2m_{hh}^*(E - E_\upsilon)}}{2\pi^2\hbar^3} \\ \\ \dfrac{m_{lh}^* \sqrt{2m_{lh}^*(E - E_\upsilon)}}{2\pi^2\hbar^3} \end{cases}$$

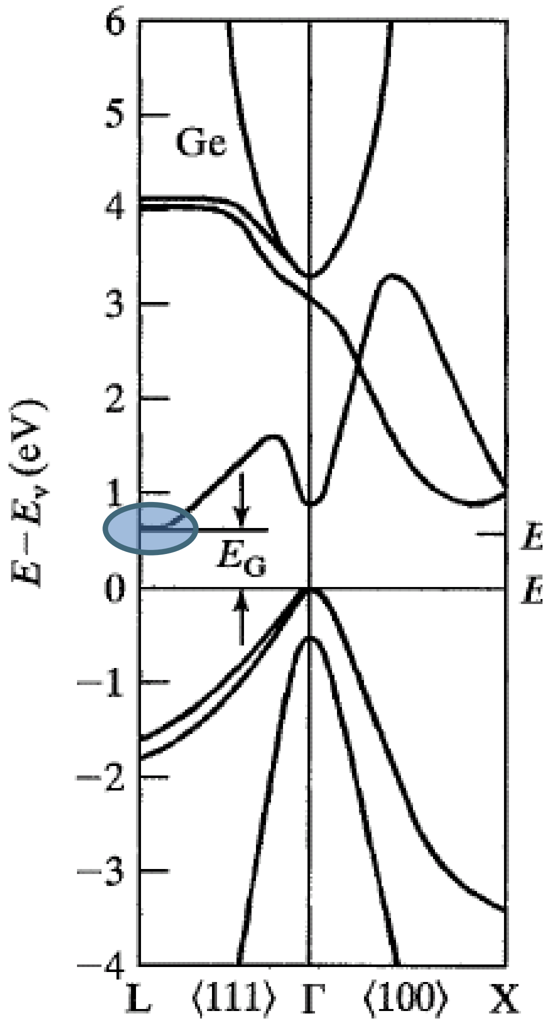


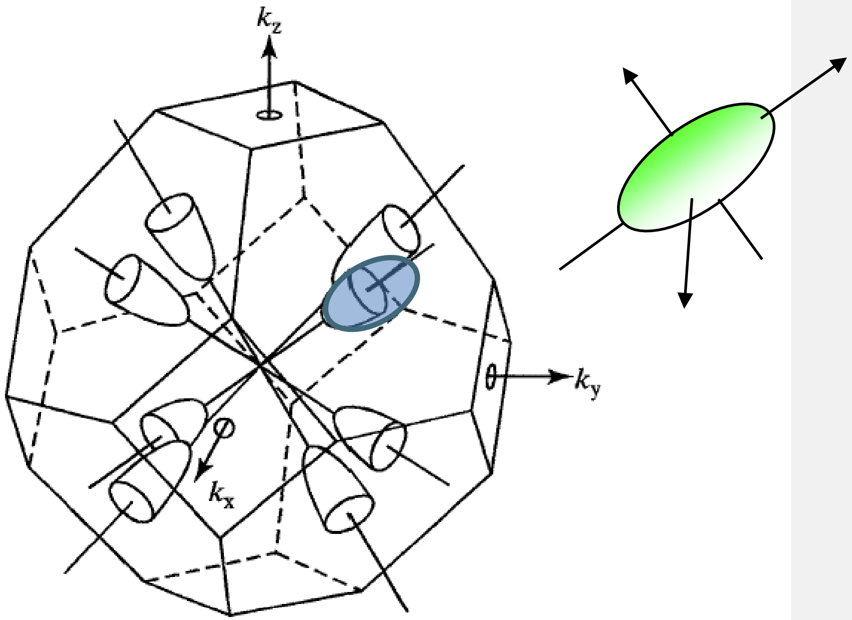




Four valleys (8 halves) inside BZ for Germanium



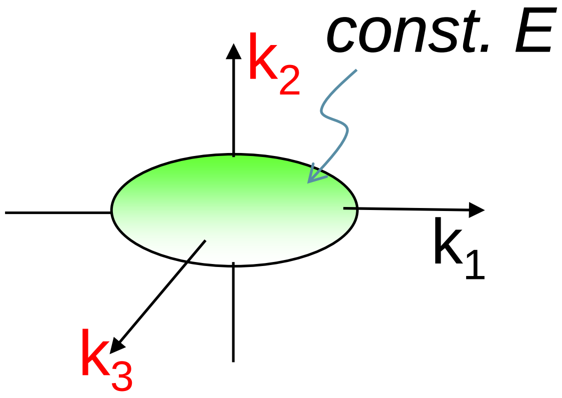




Ellipsoidal Bands and DOS Effective Mass

$$E - E_C = \frac{\hbar^2 k_1^{\,2}}{2m_l^*} + \frac{\hbar^2 k_2^{\,2}}{2m_t^*} + \frac{\hbar^2 k_3^{\,2}}{2m_t^*}$$

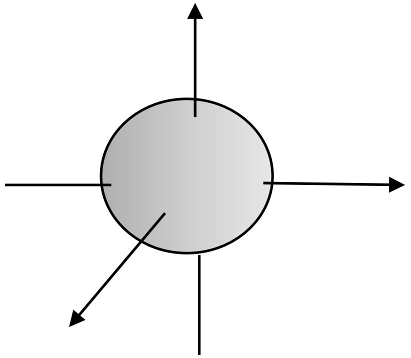
E=const. ellipsoid



$$1 = \frac{k_1^{\,2}}{\left[\frac{2m_l^*\left(E - E_C\right)}{\hbar^2}\right]} + \frac{k_2^{\,2}}{\left[\frac{2m_t^*\left(E - E_C\right)}{\hbar^2}\right]} + \frac{k_3^{\,2}}{\left[\frac{2m_t^*\left(E - E_C\right)}{\hbar^2}\right]}$$

α^2 β^2

Transform into …



$$\mathcal{V}_k = N_{el}\left(\frac{4}{3}\pi\alpha\beta^2\right) \equiv \frac{4}{3}\pi k_{eff}^3$$

$$N_{el}\frac{4}{3}\pi\sqrt{\frac{2m_l^*\left(E - E_c\right)}{\hbar^2}}\sqrt{\frac{2m_t^*\left(E - E_c\right)}{\hbar^2}}\sqrt{\frac{2m_t^*\left(E - E_c\right)}{\hbar^2}} \equiv \frac{4}{3}\pi\left[\sqrt{\frac{2m_{eff}^*\left(E - E_c\right)}{\hbar^2}}\right]^3$$

$$m_{eff}^* = N_{el}^{2/3}\left(m_l^* m_t^{*2}\right)^{1/3}$$

N_{el} is number of equivalent ellipsoids

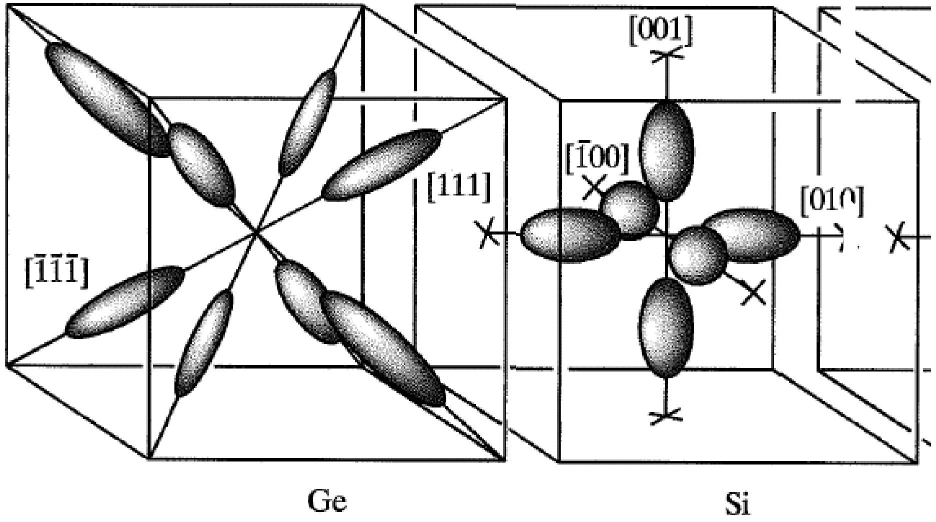
DOS Effective Mass for Conduction Band

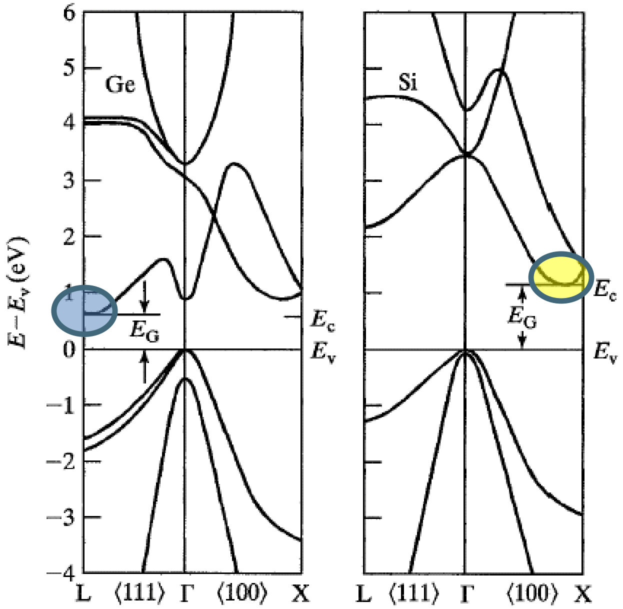
$$m_{eff}^{*} = 4^{2/3}\left(m_l^{*} m_t^{*2}\right)^{1/3}$$

$$g_c(E) = \frac{m_{eff}^{*}\sqrt{2m_{eff}^{*}\left(E - E_c\right)}}{2\pi^2\hbar^3}$$

$$m_{eff}^{*} = 6^{2/3}\left(m_l^{*} m_t^{*2}\right)^{1/3}$$

$$g_c(E) = \frac{m_{eff}^{*}\sqrt{2m_{eff}^{*}\left(E - E_c\right)}}{2\pi^2\hbar^3}$$





Video

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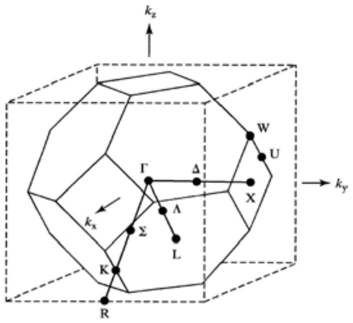
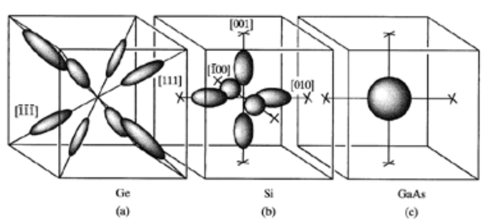


$$\frac{1}{m_{ij}} = \frac{1}{\hbar^2}\frac{\partial^2 E}{\partial k_i \partial k_j}$$

$$m_{eff}^* = 6^{2/3}\left(m_l^* m_t^{*2}\right)^{1/3}$$



1) E-k diagrams emerge from the solution of Schrödinger’s Eq.
2) E-k diagrams at which energy an electron with a specific crystal momentum ***can*** reside.
3) E-k diagrams depend on the underlying crystal symmetry
4) Only a fraction of the available states are occupied. The number of available states change with energy.
5) DOS of g(E) describes the number of available states in a small energy window around energy E
6) DOS is an important and useful characteristic of a material