Section 9
Constant Energy Surfaces & Density of States

9.2 Density of States

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Section 9
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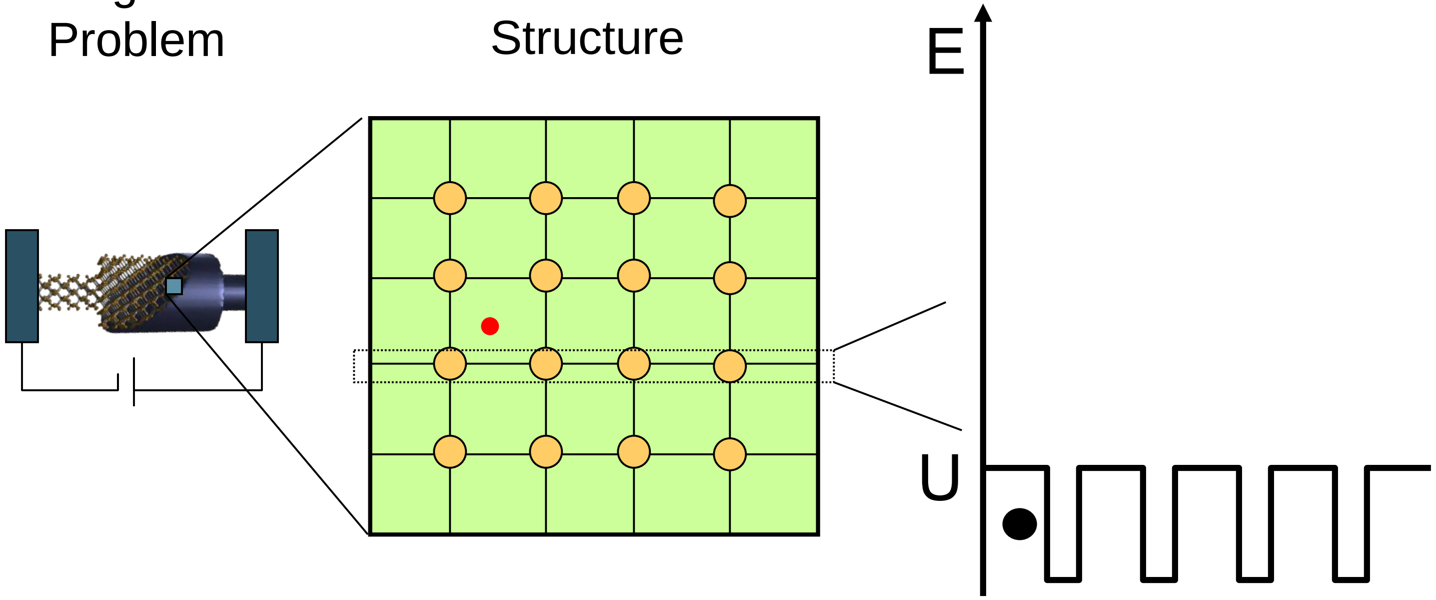
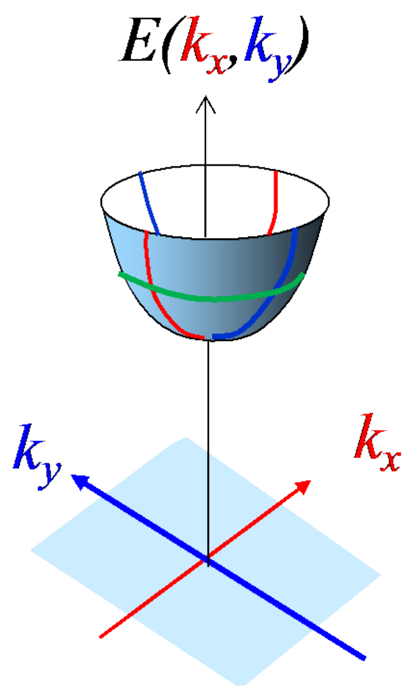
Original Problem

Periodic Structure





Section 7 – Bandstructure in 1D Periodic Potentials

Section 8 – Brillouin Zone - Reciprocal Lattice

Section 9 – Some fundamental methods

- 9.1 Constant Energy Surfaces
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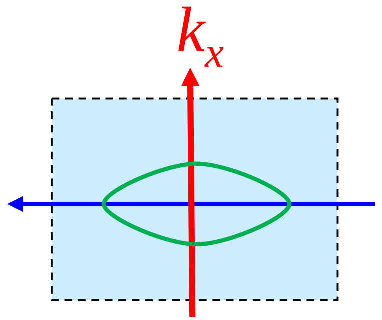
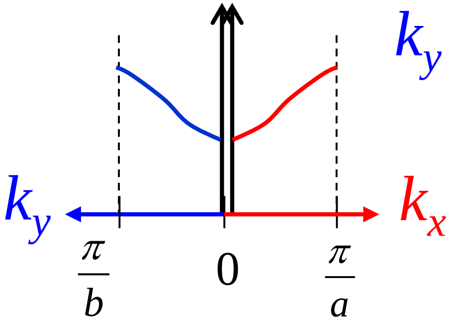
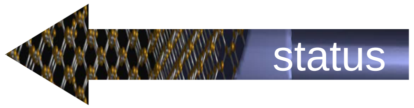
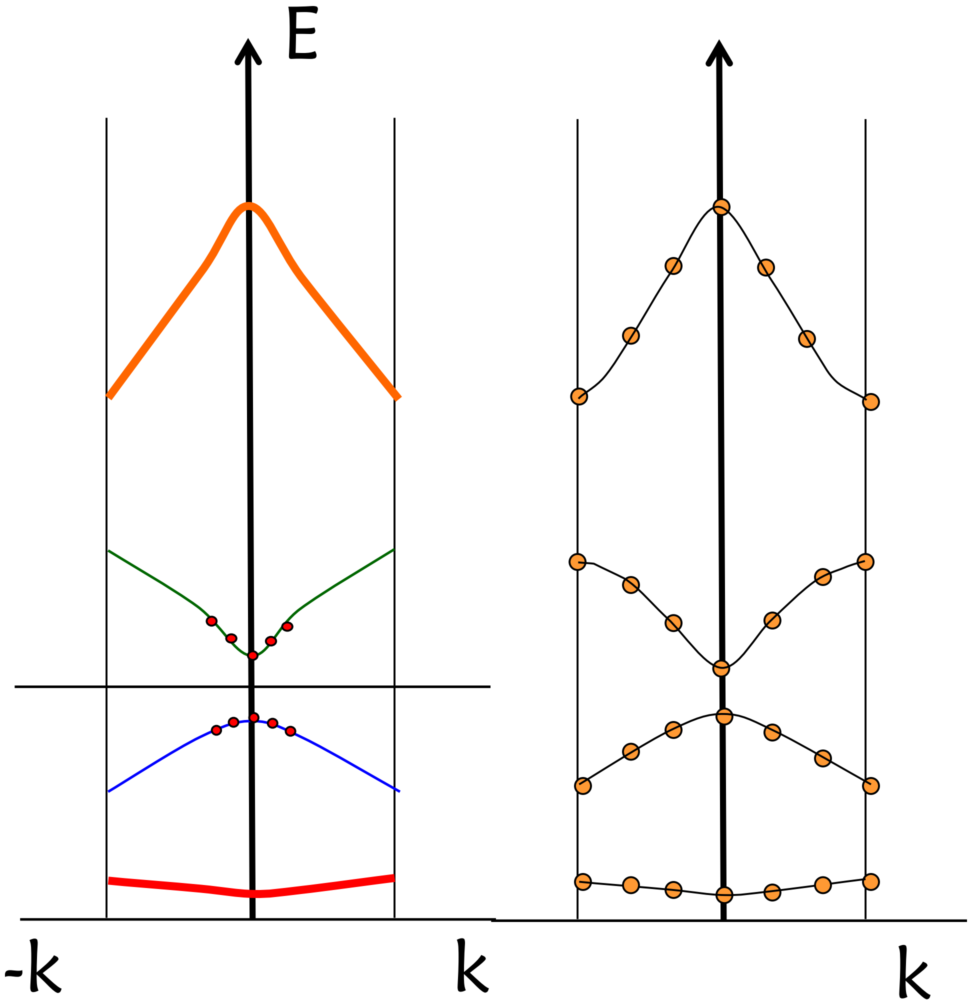






Video

Video



A single band has total of N-states

Only a fraction of states are occupied

How many states are occupied up to E?

Helpful for an answer…

How many states available per unit energy ? (Density of States - DOS)?

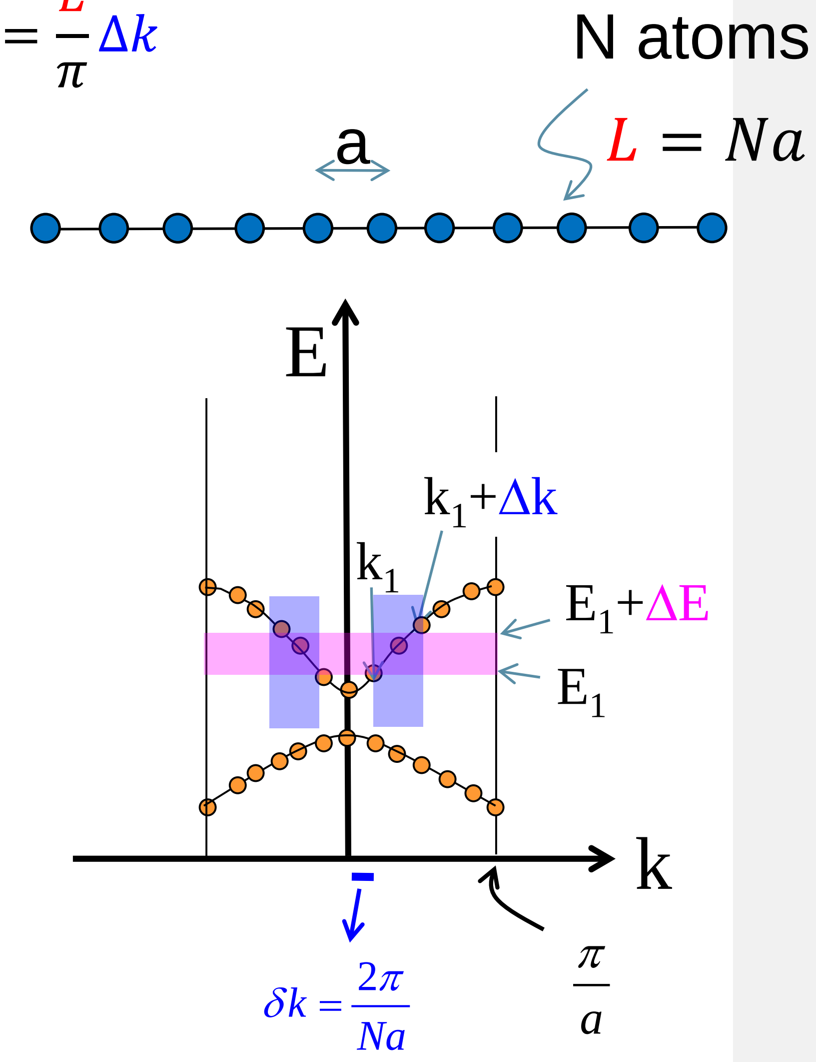
Density of States in 1-D Semiconductors

States between E_1 *and* $E_1 + \Delta E$ $= 2 \times \dfrac{\Delta k}{\delta k} = 2 \times \dfrac{\Delta k}{2\pi/Na} = 2 \times \dfrac{\Delta k}{2\pi/L} = \dfrac{L}{\pi}\Delta k$

States / Unit Energy @ E_1 = $D(E_1)$ $= \dfrac{L}{\pi}\dfrac{\Delta k}{\Delta E}$

States / Unit Energy / Volume @ E_1 = $D(E_1)$ $= \dfrac{1}{\pi}\dfrac{\Delta k}{\Delta E}$

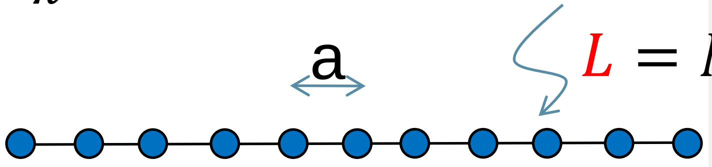
$$D(E) = \frac{1}{\pi}\frac{dk}{dE}$$



Density of States in 1-D Semiconductors

States between E_1 and $E_1 + \Delta E$ $= 2 \times \frac{\Delta k}{\delta k} = 2 \times \frac{\Delta k}{2\pi/Na} = 2 \times \frac{\Delta k}{2\pi/L} = \frac{L}{\pi}\Delta k$

N atoms



States / Unit Energy @ $E_1 = D(E_1) = \frac{L}{\pi}\frac{\Delta k}{\Delta E}$

States / Unit Energy / Volume @ $E_1 = D(E_1) = \frac{1}{\pi}\frac{\Delta k}{\Delta E}$

$$D(E) = \frac{1}{\pi}\frac{dk}{dE}$$

$$D(E) = \frac{1}{\pi}\frac{dk}{dE}$$

$$D_{1D}(E) = \frac{1}{\pi}\sqrt{\frac{m^*}{2\hbar^2 (E - E_0)}}$$

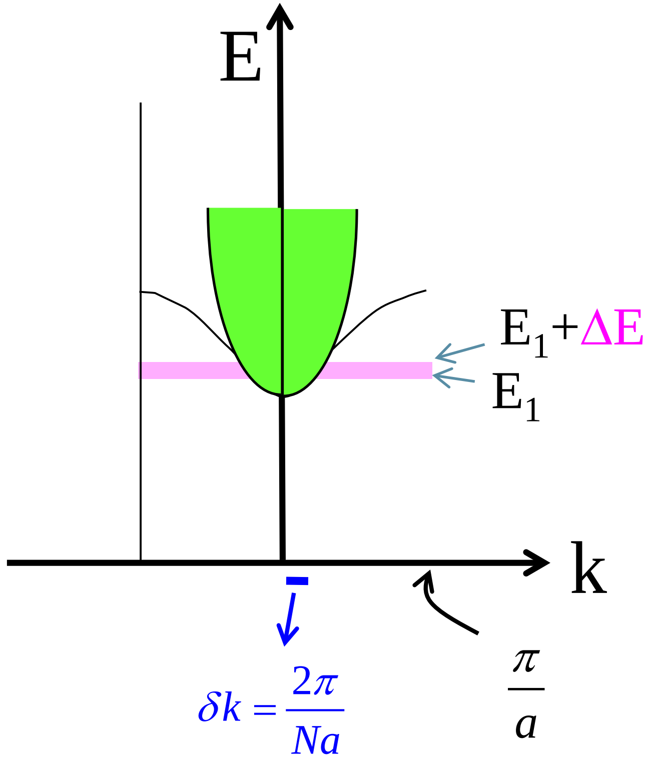
$$D_{1D}(E) \propto \frac{1}{\sqrt{E}} = E^{-1/2}$$

Assume:

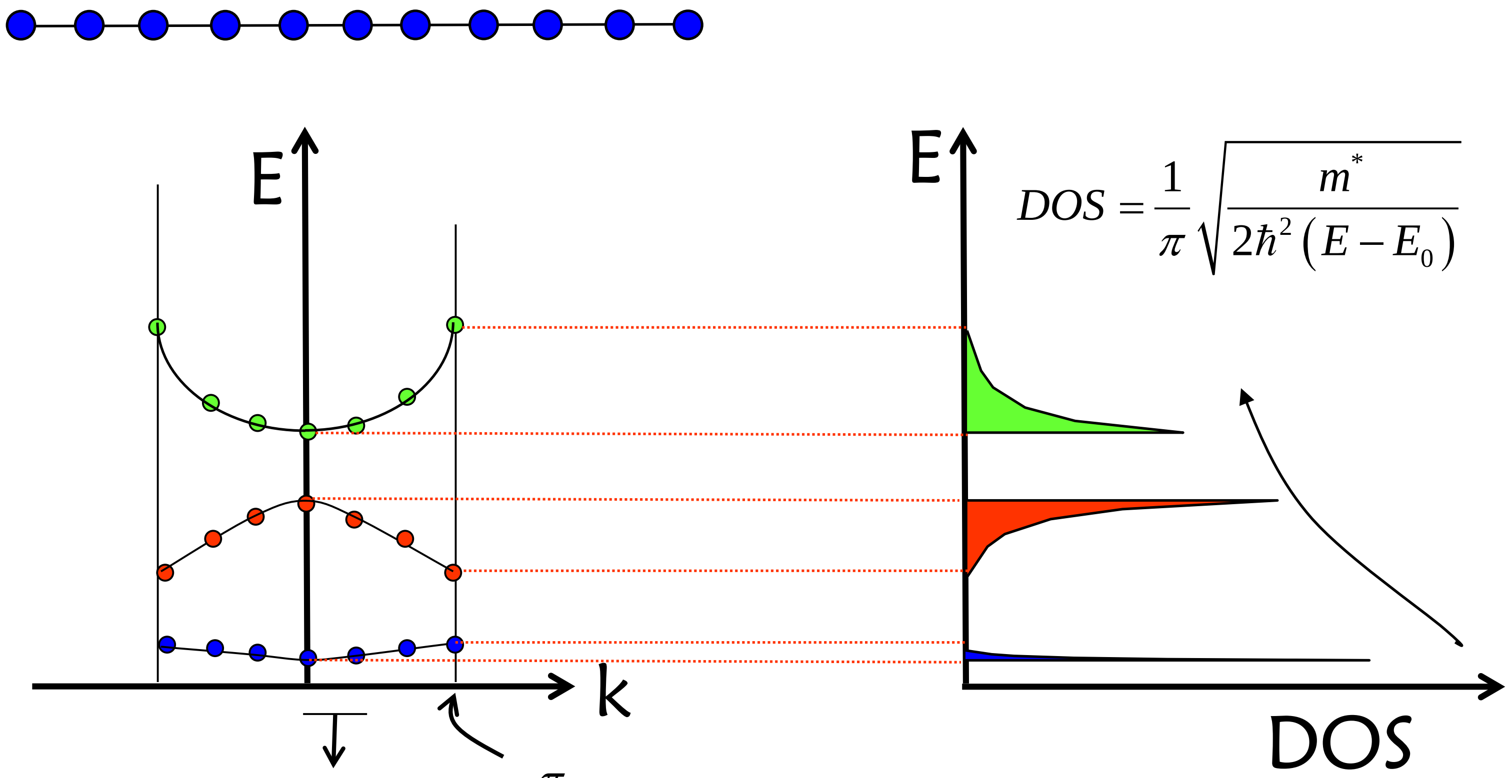
$$E = E_0 + \frac{\hbar^2 k^2}{2m^*}$$

$$k = \sqrt{\frac{2m^*(E - E_0)}{\hbar^2}}$$

$$\frac{dk}{dE} = \sqrt{\frac{m^*}{2\hbar^2 (E - E_0)}}$$



1D-DOS



$$DOS = \frac{1}{\pi}\sqrt{\frac{m^*}{2\hbar^2(E-E_0)}}$$

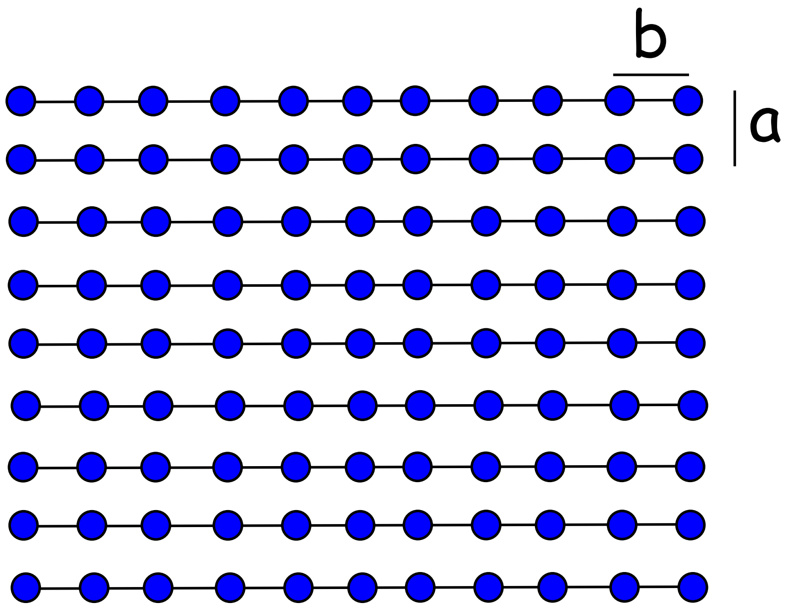
$$\Delta k = \frac{2\pi}{Na}$$

$$\frac{\pi}{a}$$

$$D_{1D}(E) \propto \frac{1}{\sqrt{E}} = E^{-1/2}$$

Conservation of DOS

(total area must be the same for each band)



Show that 2D DOS is a constant independent of energy!

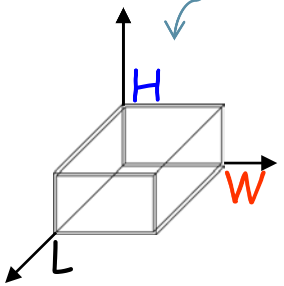
$D_{1D}(E) \propto \frac{1}{\sqrt{E}} = E^{-1/2}$ $D_{2D}(E) \propto E^{0}$

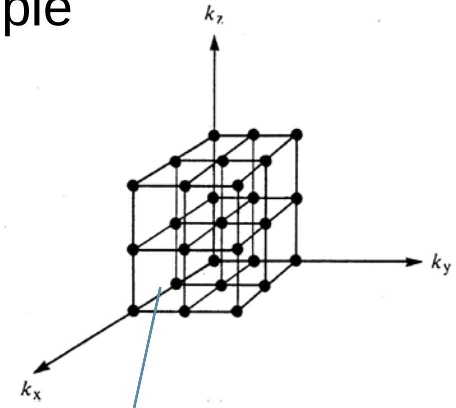
3D-DOS

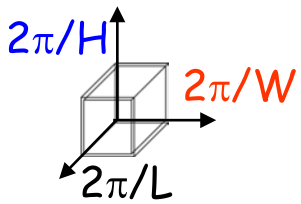
States between $E_1+\Delta E$ & E_1

$$=\frac{\frac{4}{3}\pi\left(k+dk\right)^3-\frac{4}{3}\pi k^3}{\frac{2\pi}{L}\,\frac{2\pi}{W}\,\frac{2\pi}{H}}=\frac{V}{2\pi^2}k^2\Delta k$$

Macroscopic Sample





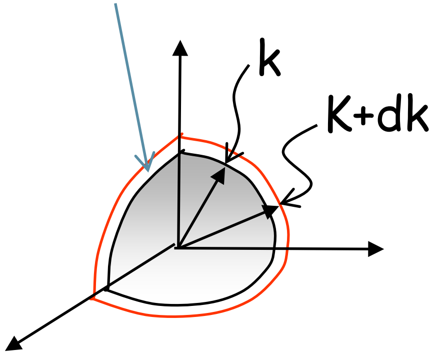


States/unit energy @ E $=\frac{V}{2\pi^2}k^2\frac{\Delta k}{dE}$

$$E-E_0=\frac{\hbar^2k^2}{2m^*}\Rightarrow k=\sqrt{\frac{2m^*\left(E-E_0\right)}{\hbar^2}}\Rightarrow\frac{dk}{dE}=\sqrt{\frac{m^*}{2\hbar^2\left(E-E_0\right)}}$$

States/unit energy/unit volume @ E_1

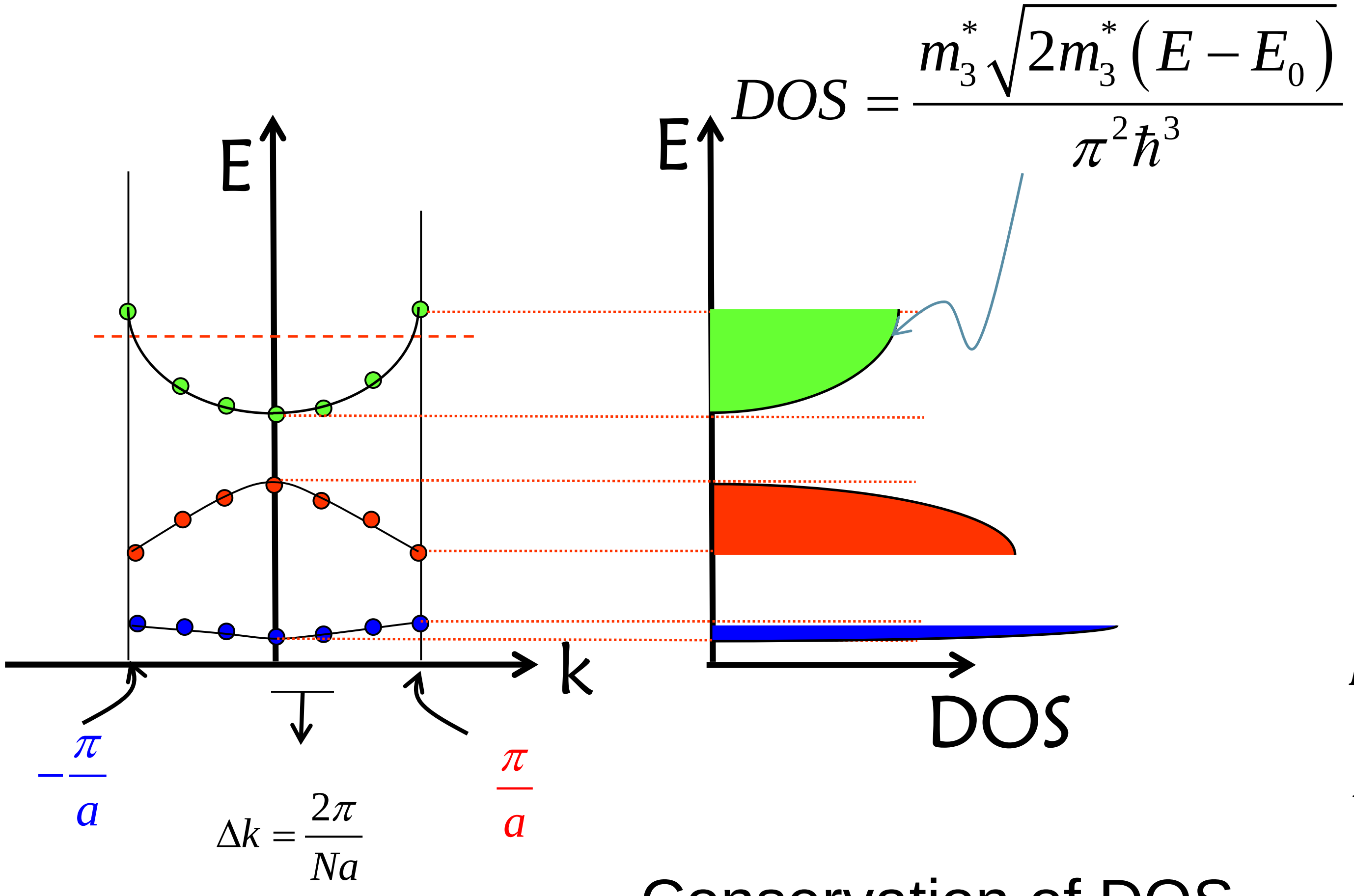
$$DOS=\frac{m^*}{2\pi^2\hbar^3}\sqrt{2m^*\left(E-E_0\right)}$$



$$D_{1D}(E)\propto\frac{1}{\sqrt{E}}=E^{-1/2}$$

$$D_{2D}(E)\propto E^0$$

$$D_{3D}(E)\propto\sqrt{E}=E^{1/2}$$



$D_{1D}(E) \propto \frac{1}{\sqrt{E}} = E^{-1/2}$

$D_{2D}(E) \propto E^0$

$D_{3D}(E) \propto \sqrt{E} = E^{1/2}$

Conservation of DOS

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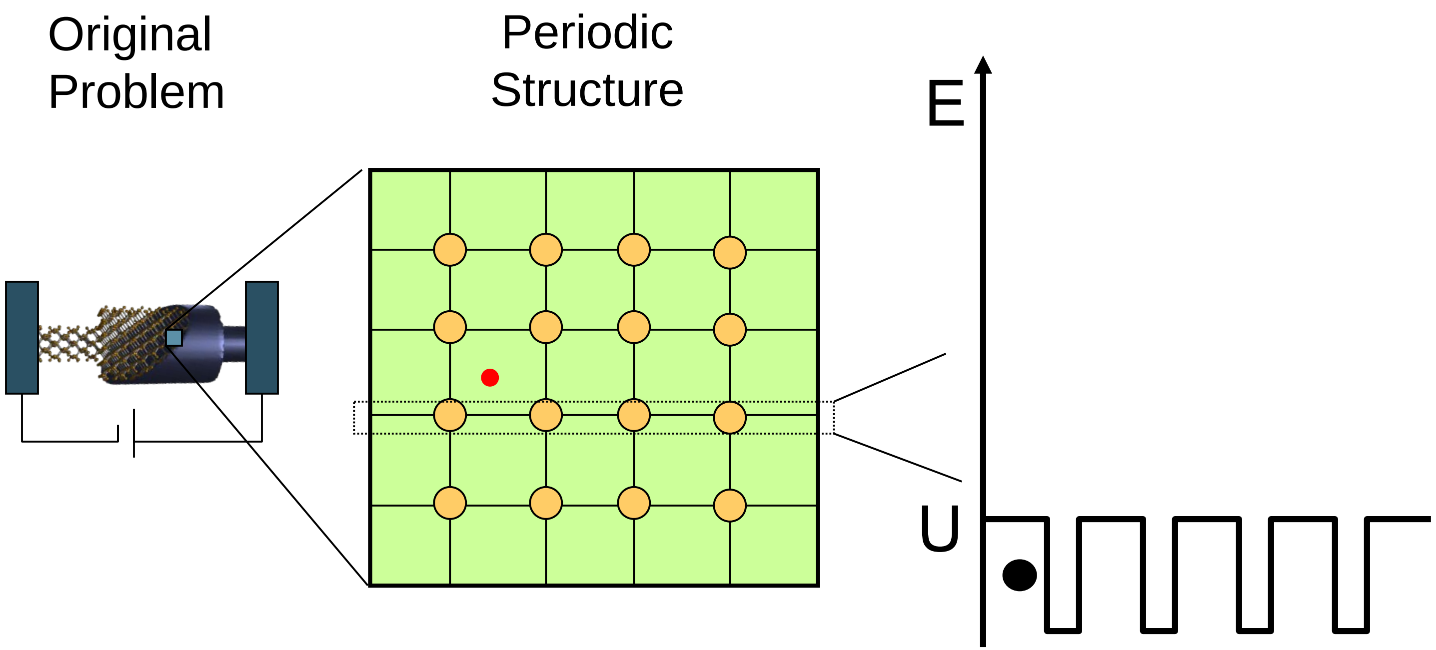

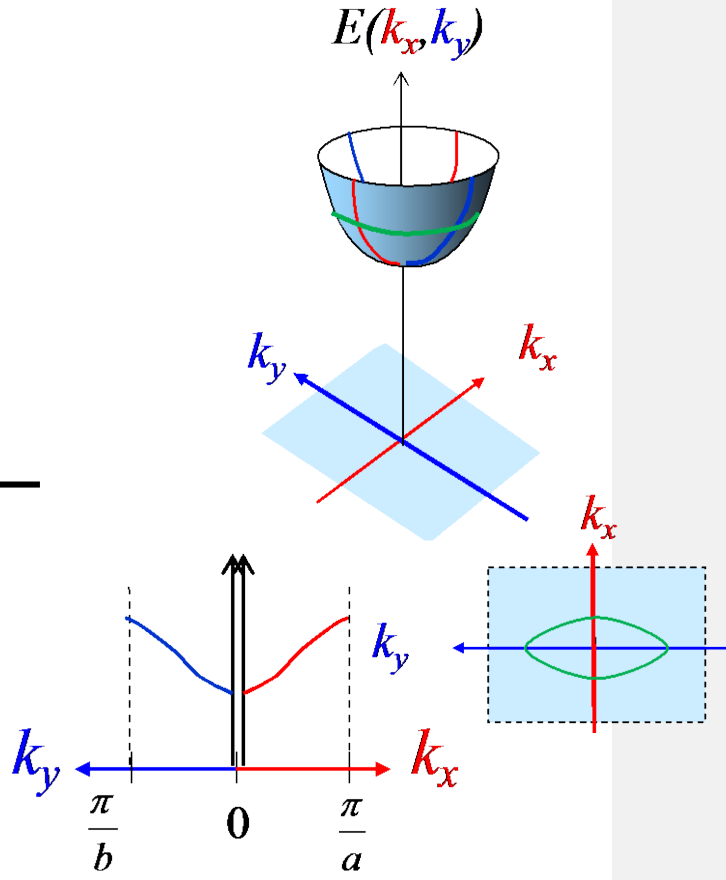

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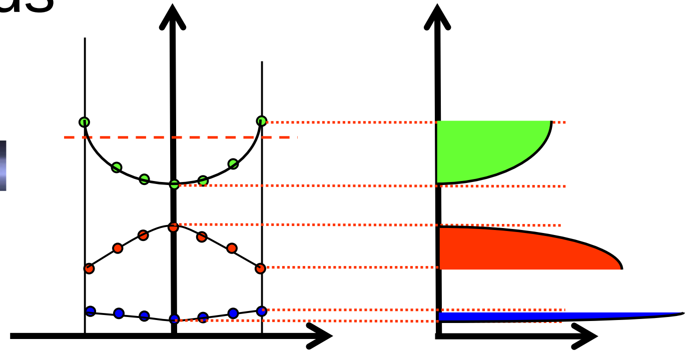

$$D_{1D}(E) \propto \frac{1}{\sqrt{E}} = E^{-1/2}$$

$$D_{2D}(E) \propto E^{0}$$

$$D_{3D}(E) \propto \sqrt{E} = E^{1/2}$$

Video

Video