Section 6

Electron Tunneling - Emergence of Bandstructure

6.3 Tunneling through a double barrier structure

Gerhard Klimeck
gekco@purdue.edu

PURDUE UNIVERSITY

School of Electrical and Computer Engineering

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Video

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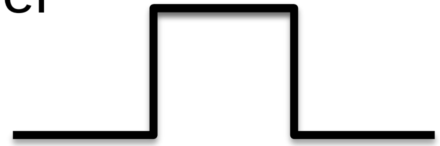
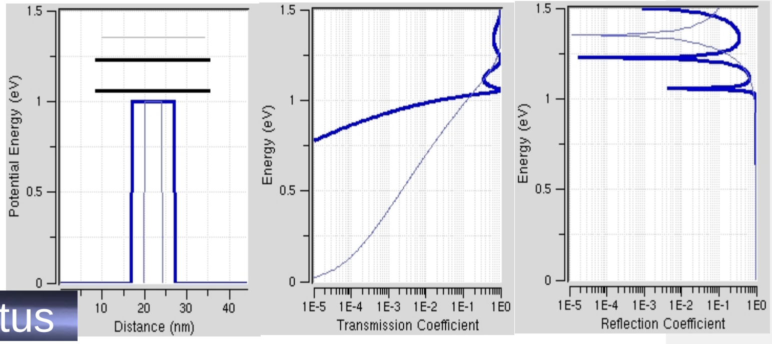
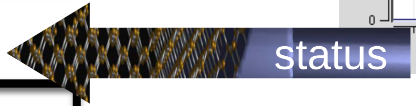
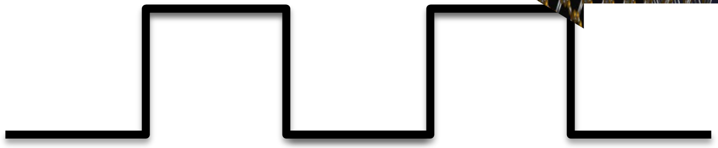
- 6.1 Transfer Matrix Method
- 6.2 Tunneling through a single barrier
  - » Analytical Solution
  - » Numerical observations







- 6.3 Tunneling through a double barrier structure

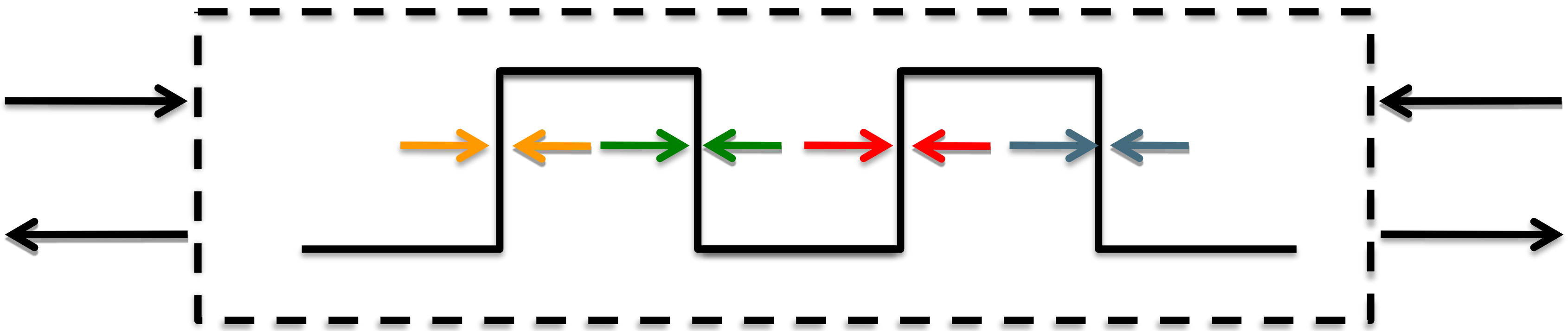


- 6.4 Tunneling through N barriers - Formation of bandstructure
- 6.5 Analytical and Numerical Solution Strategies

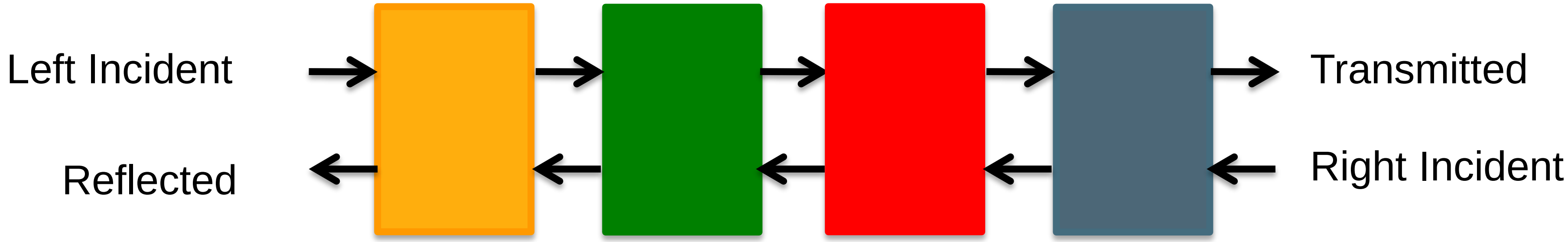
Reference:
piece-wise-constant-potential-barrier tool http://nanohub.org/tools/pcpbt

Double Barrier Transmission: Scattering Matrix approach

Define our system : Double barrier



One matrix each for each interface: 4 S-matrices

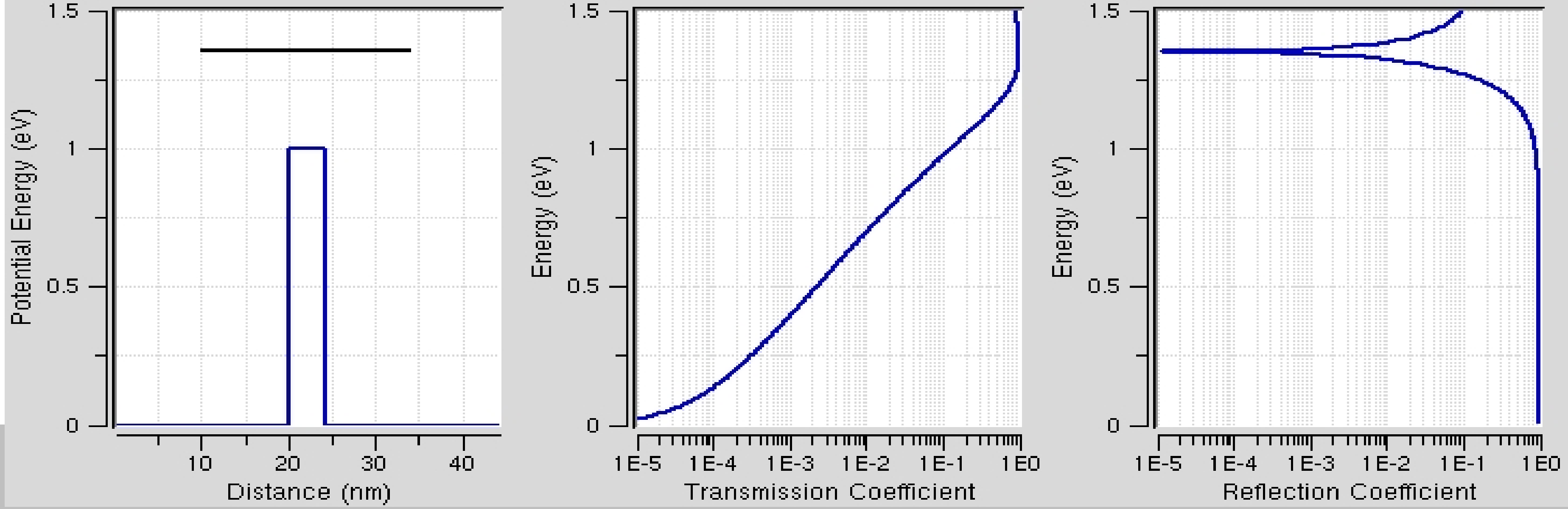


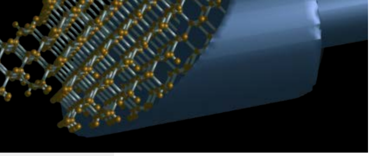
No particles lost!
Typically Left Incident wave is normalized to one.
Right incident is assumed to be zero.

Also this problem is analytically solvable! => Homework assignment

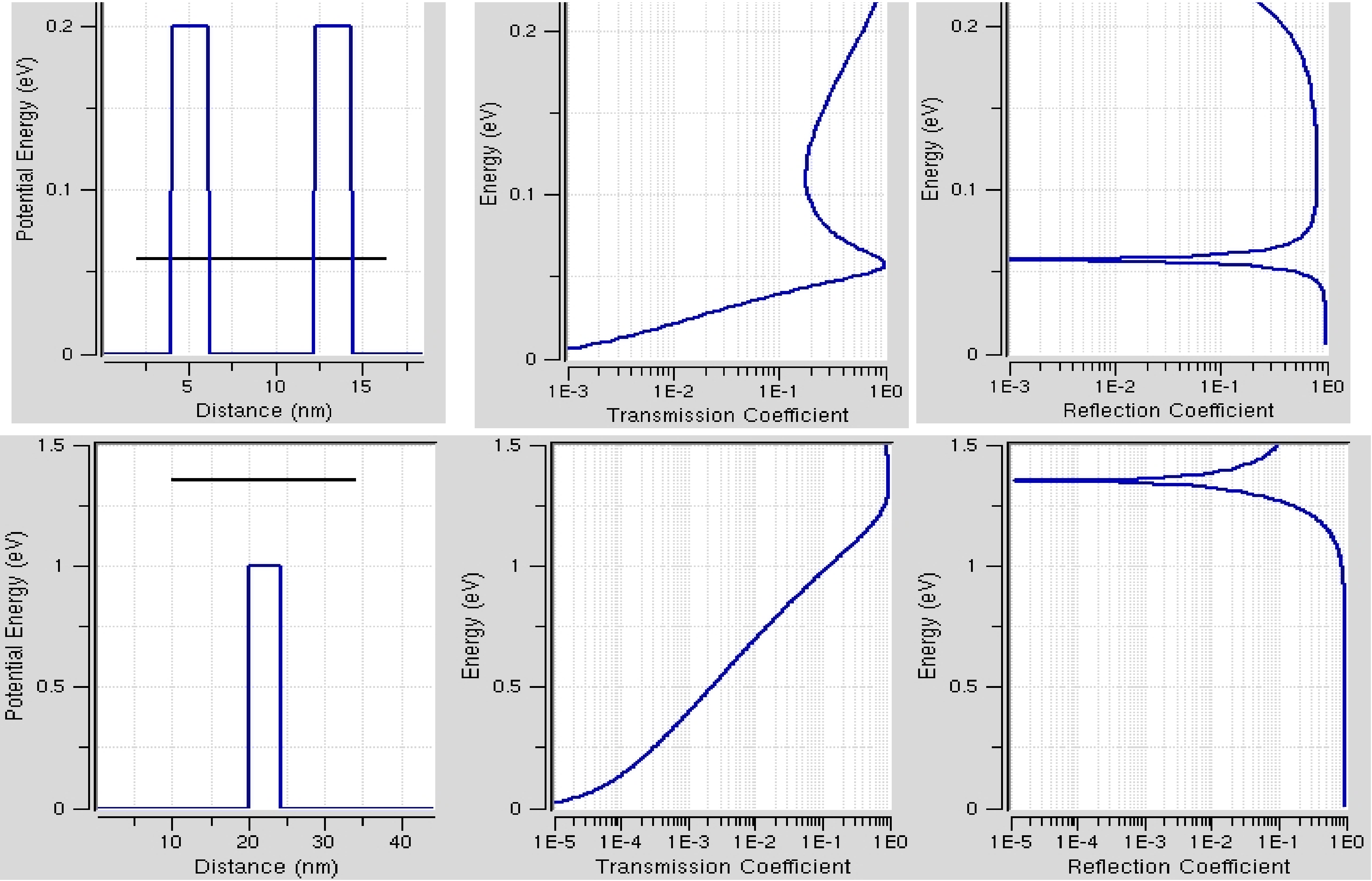
Reminder: Single barrier

- Transmission is finite under the barrier – tunneling!
- Transmission above the barrier is not perfect unity!
- Quasi-bound state above the barrier.
  Transmission goes to one.

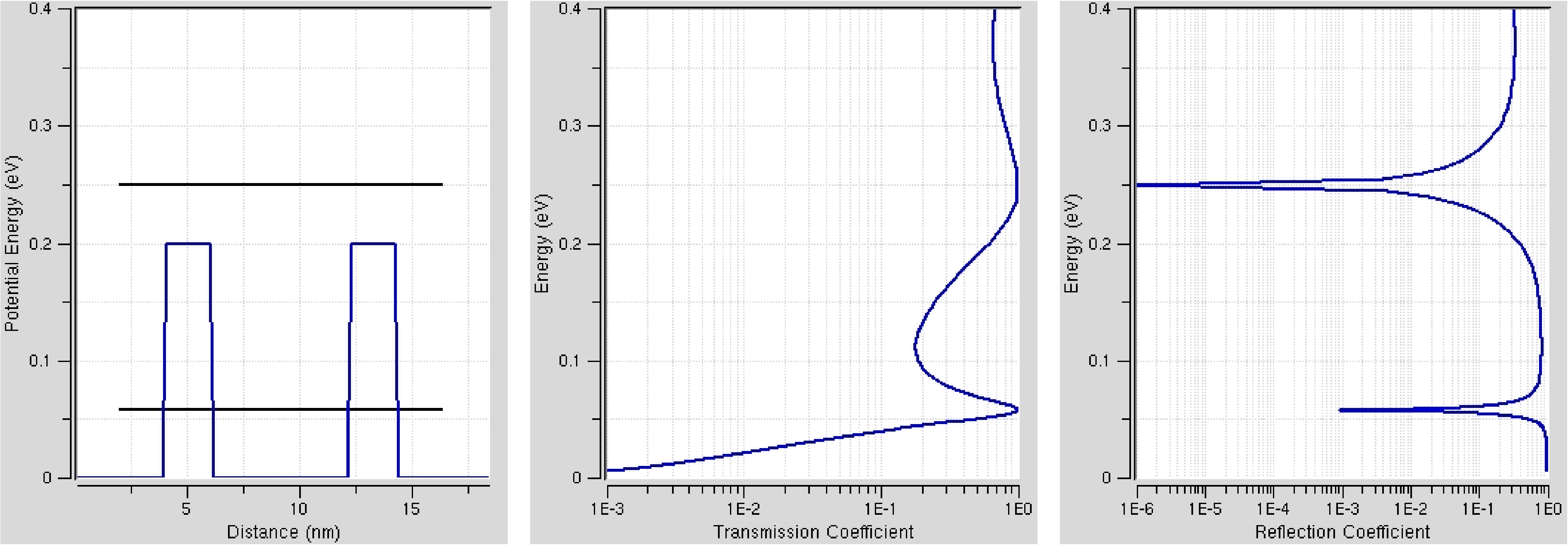




- Double barriers allow a transmission probability of one / unity for discrete energies
- (reflection probability of zero) for some energies below the barrier height.
- This is in sharp contrast to the single barrier case
- Cannot be predicted by classical physics.

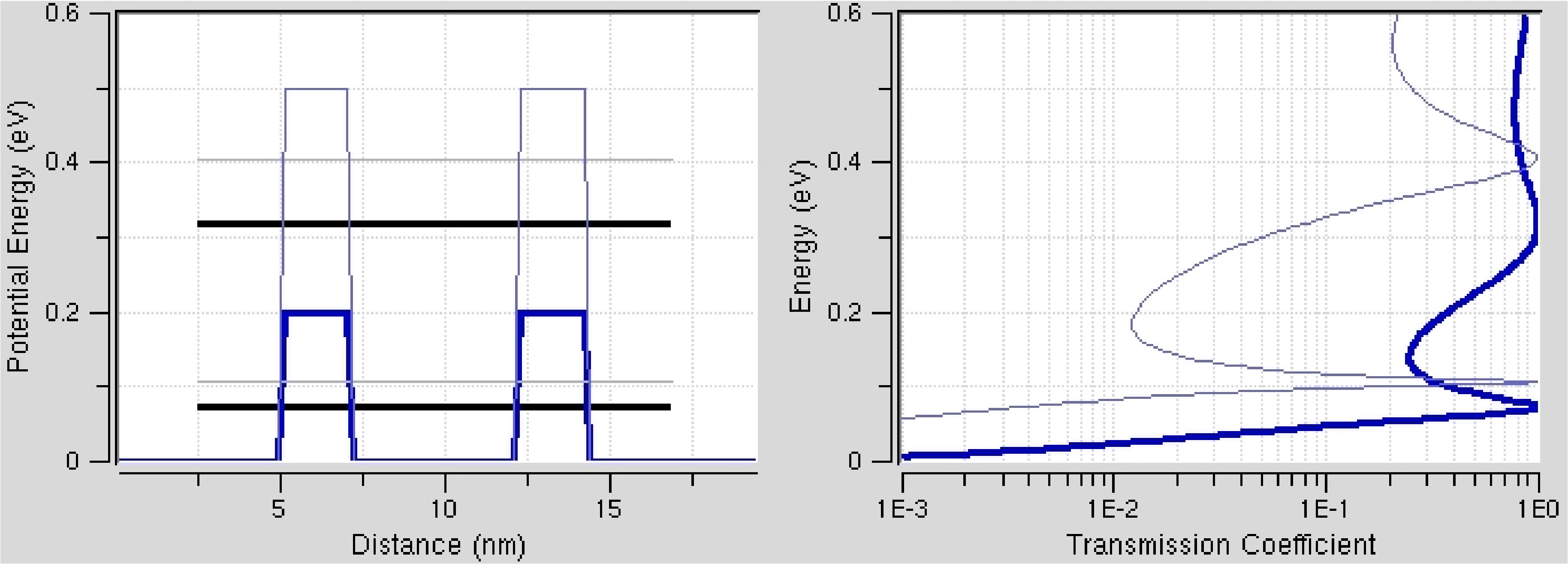


Double barrier: Quasi-bound states



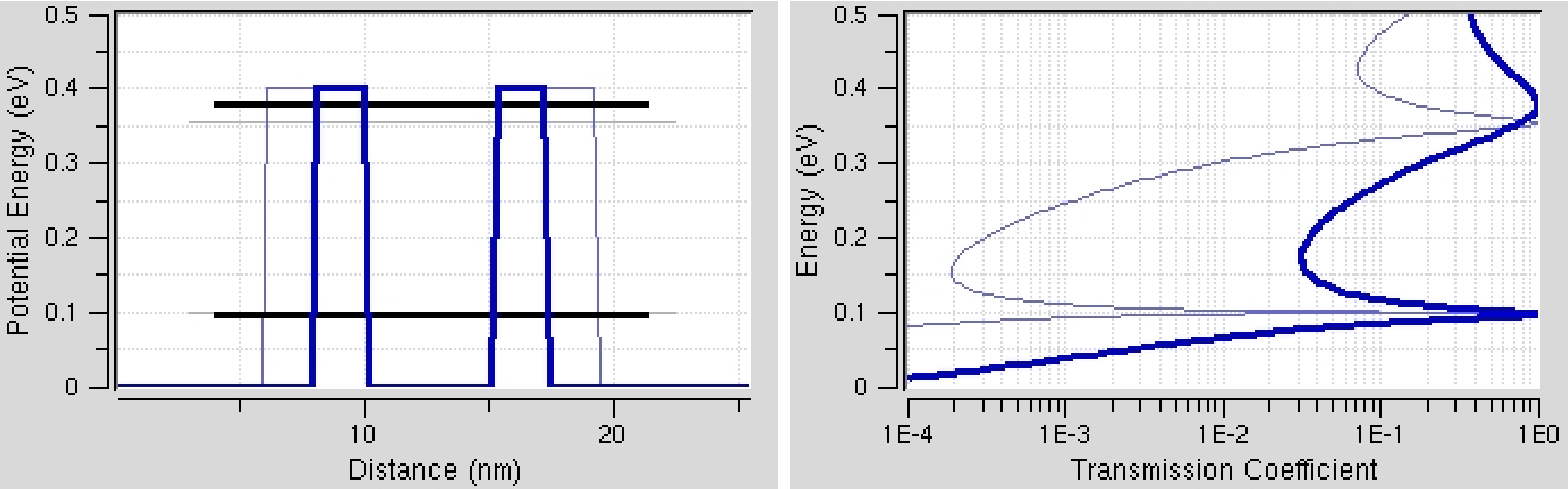
- In addition to states inside the well, there could be states above the barrier height.
- States above the barrier height are quasi-bound or weakly bound.
- How strongly bound a state is can be seen by the width of the transmission peak.
- The transmission peak of the quasi-bound state is much broader than the peak for the state inside the well.

Effect of barrier height



- Increasing the barrier height makes the resonance sharper.
- By increasing the barrier height, the confinement in the well is made stronger, increasing the lifetime of the resonance.
- A longer lifetime corresponds to a sharper resonance.

Effect of barrier thickness



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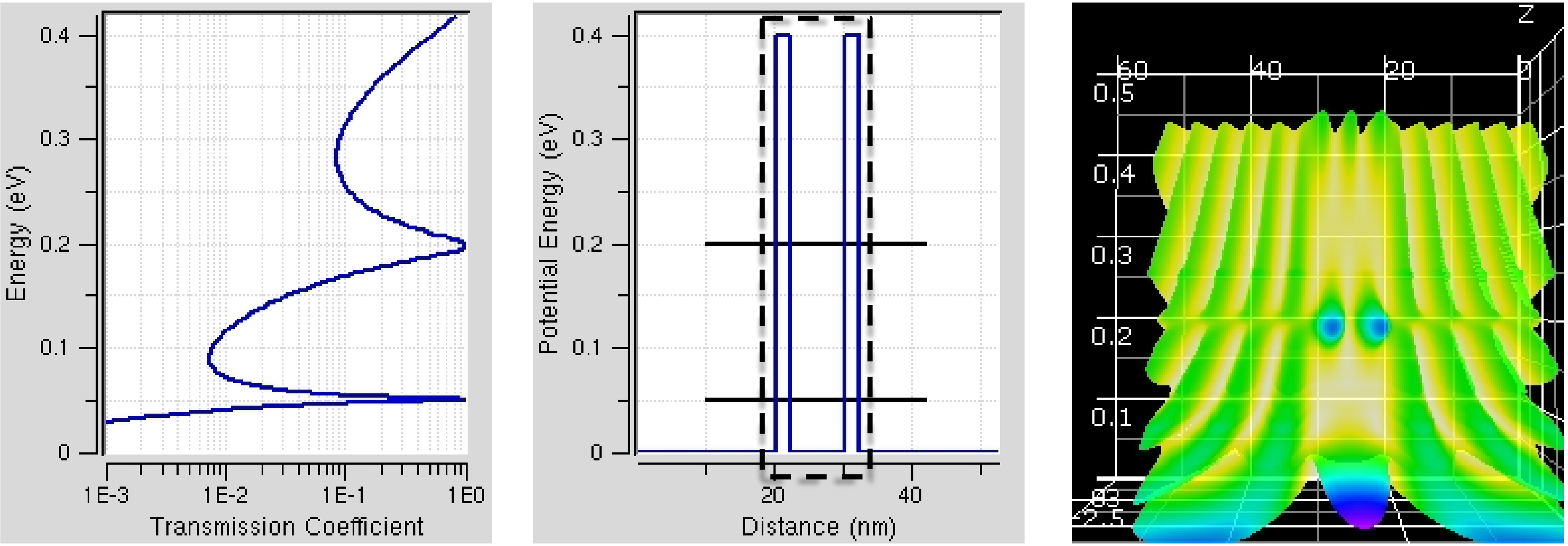
Double barrier energy levels Vs Closed system



The well region in the double barrier case can be thought of as a particle in a box.

Reminder: Particle in a Box

- The time independent Schrödinger equation is

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x)+V(x)\psi(x)=E\psi(x)\quad \text{where, } V(x)=\begin{cases}0 & 0<x<L_x\\ \infty & \text{elsewhere}\end{cases}$$

- The solution in the well is:

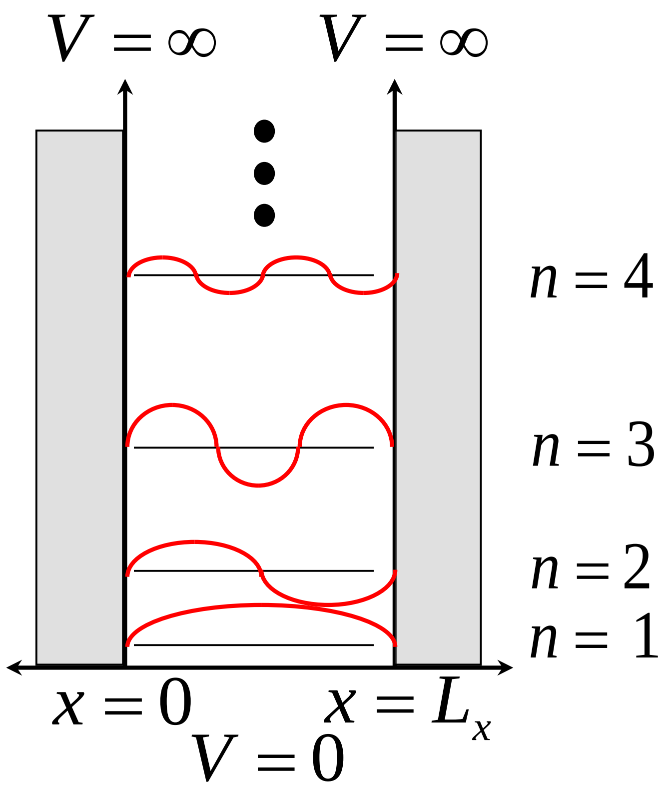
$$\psi_n(x)=A\sin\left(\frac{n\pi}{L_x}x\right),\quad n=1,2,3,\ldots$$

- Plugging the normalized wave-functions back into the Schrödinger equation we find that energy levels are quantized.

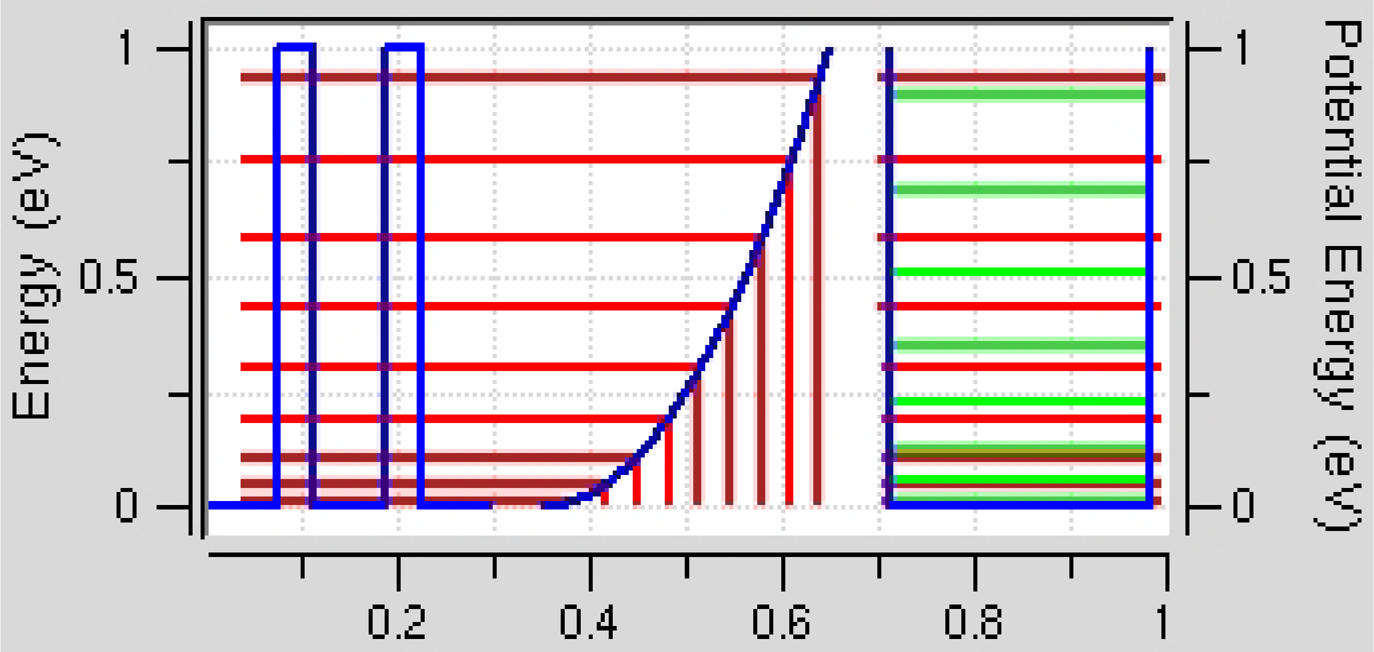
$$\psi_n(x)=\sqrt{\frac{2}{L_x}}\sin\left(\frac{n\pi}{L_x}x\right)$$

$$E_n=\frac{\hbar^2\pi^2}{2mL_x^2}n^2$$

$$n=1,2,3,\ldots,\quad 0<x<L_x$$



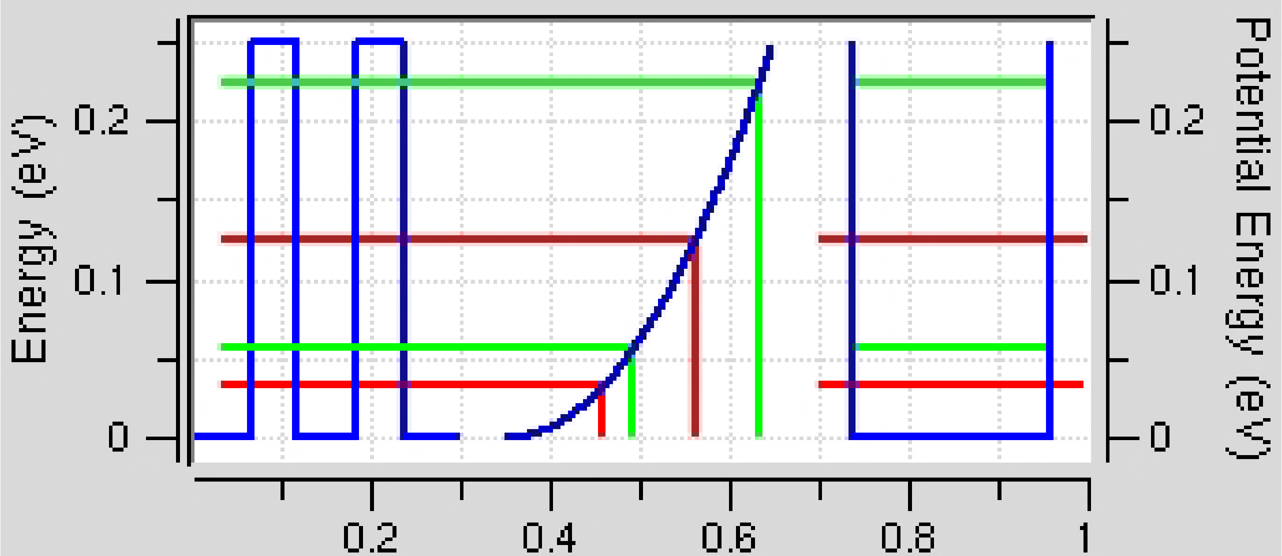
Double barrier & particle in a box



- Green: Particle in a box energies.
- Red: Double barrier energies

- Double barrier: Thick Barriers(10nm), Tall Barriers(1eV), Well(20nm).
- First few resonance energies match well with the particle in a box energies.
- The well region resembles the particle in a box setup.

Open systems Vs closed systems



- Green: Particle in a box energies.
- Red: Double barrier energies

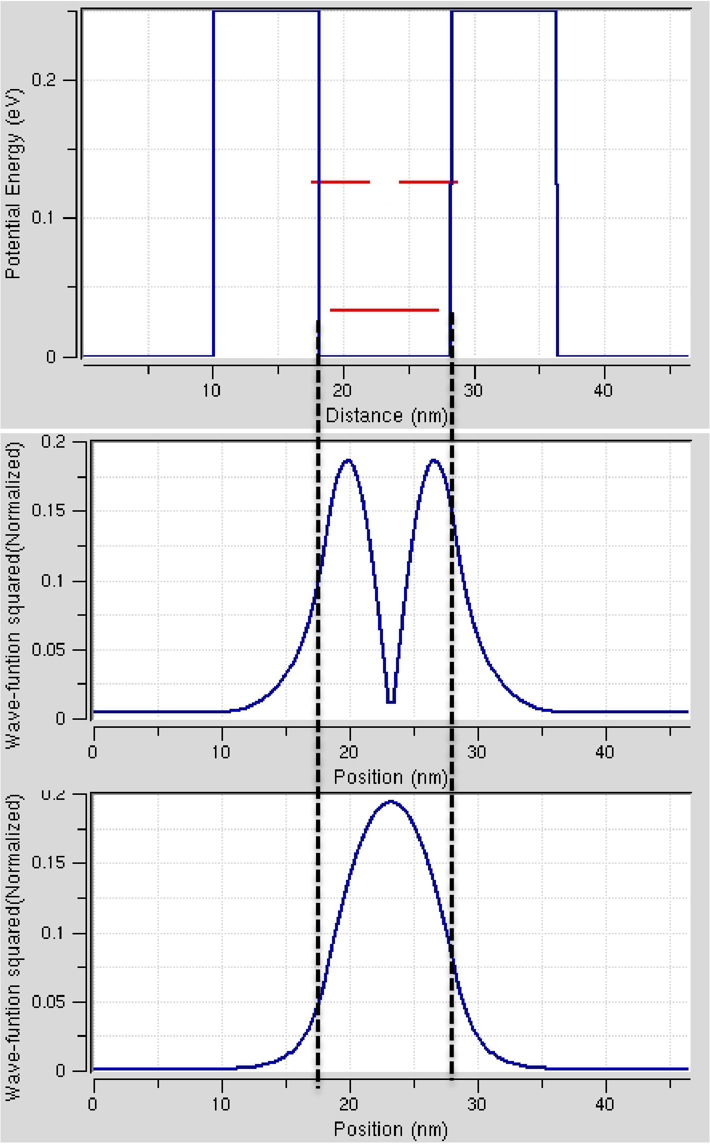
- Double barrier: Thinner Barriers(8nm), Shorter Barriers(0.25eV), Well(10nm).
- Even the first resonance energy does not match with the particle in a box energy.
- The well region does not resemble a particle in a box.
- A double barrier structure is an OPEN system, particle in a box is a CLOSED system.

Reason for deviation?

Potential profile and resonance energies using tight-binding.

First excited state wave-function amplitude using tight binding.

Ground state wave-function amplitude using tight binding.



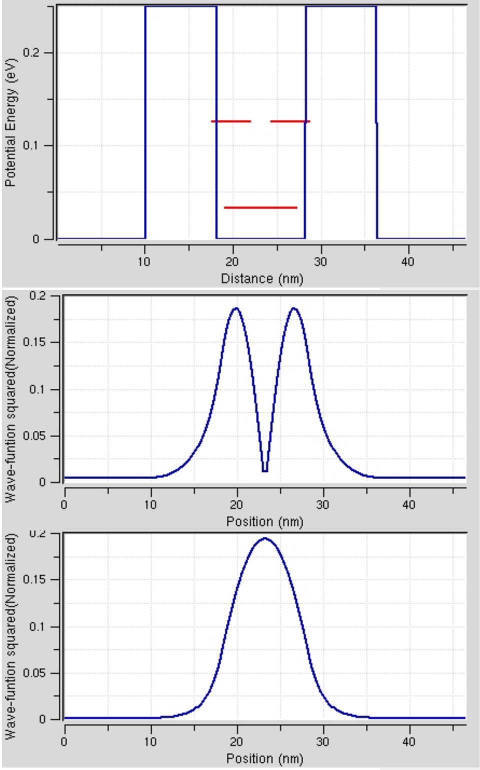
- Wave-function penetrates into the barrier region.
- The effective length of the well region is modified.
- The effective length of the well is crucial in determining the energy levels in the closed system.

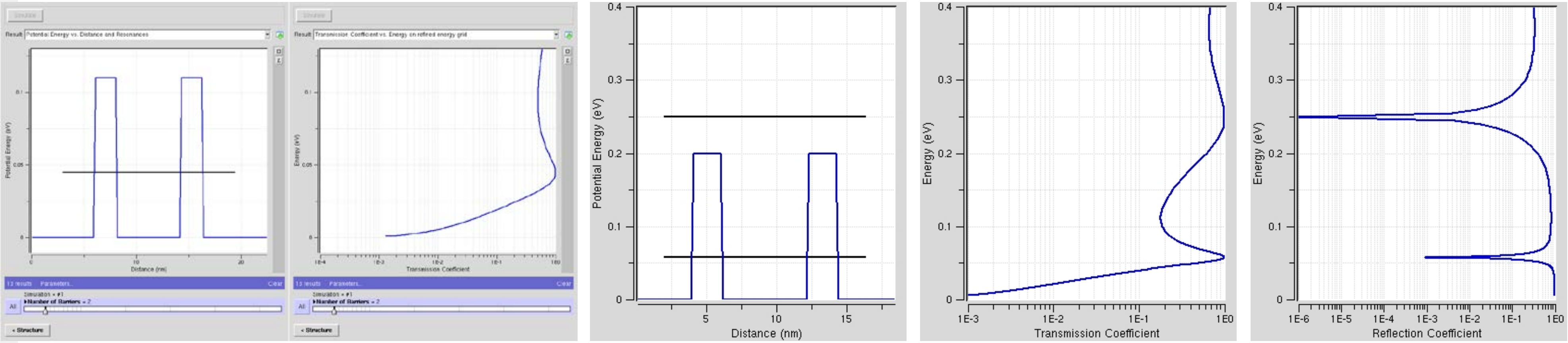
$$E_n = \frac{\hbar^2 \pi^2}{2mL_{well}^2} n^2$$

$$n = 1, 2, 3, \ldots, \quad 0 < x < L_{well}$$

Double Barrier Structures - Key Summary

- Double barrier structures can show unity transmission for energies BELOW the barrier height
  - » Resonant Tunneling
- Resonance can be associated with a quasi bound state
  - » Can relate the bound state to a particle in a box
  - » State has a finite lifetime / resonance width
- Increasing barrier heights and widths:
  - » Increases resonance lifetime / electron residence time
  - » Sharpens the resonance width





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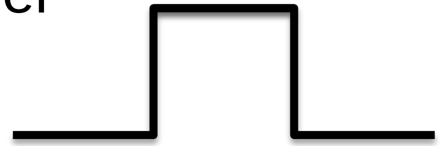
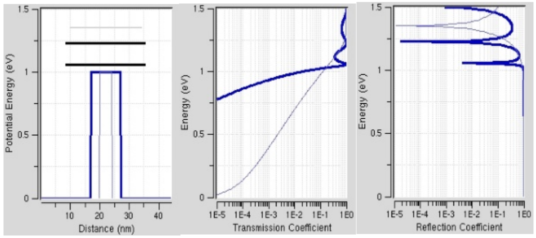
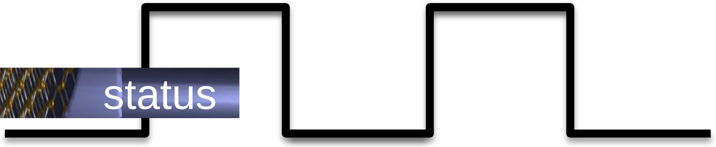
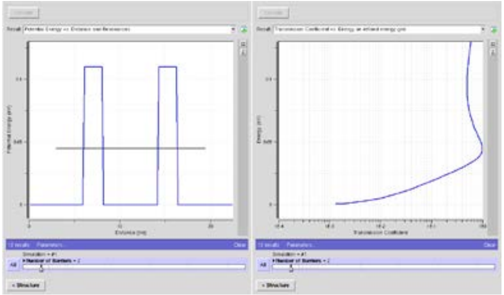




- 6.3 Tunneling through a double barrier structure
  - » Resonant Transmission
  - » Transmission Peak Width

status





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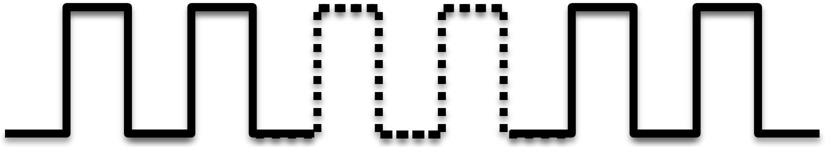

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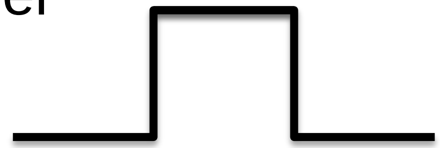

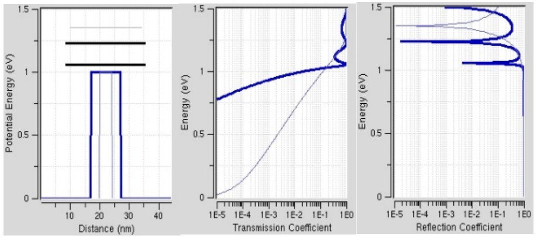

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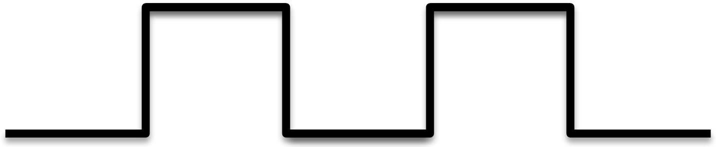

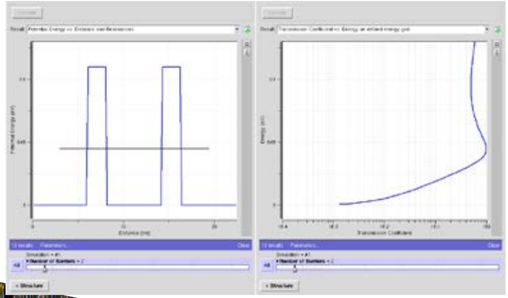

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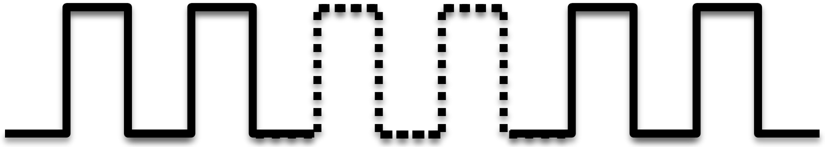

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