Section 6

Electron Tunneling - Emergence of Bandstructure

6.2 Tunneling through a single barrier

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Section 6
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Video

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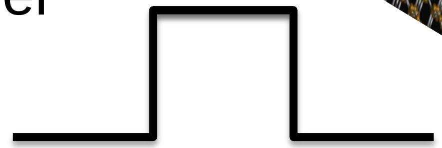
- 6.1 Transfer Matrix Method
- 6.2 Tunneling through a single barrier
  - » Analytical Solution
  - » Numerical observations



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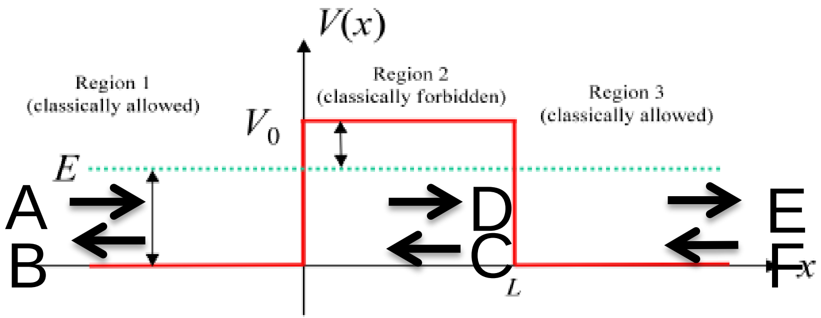
$$\begin{bmatrix} A \\ B \end{bmatrix} = M_1 \begin{bmatrix} C \\ D \end{bmatrix} = M_1 M_2 \begin{bmatrix} E \\ F \end{bmatrix} = M \begin{bmatrix} E \\ F \end{bmatrix}$$

- 6.3 Tunneling through a double barrier structure
- 6.4 Tunneling through N barriers - Formation of bandstructure
- 6.5 Analytical and Numerical Solution Strategies

Reference:
piece-wise-constant-potential-barrier tool http://nanohub.org/tools/pcpbt

Single Barrier Transmission Solution

$$\begin{bmatrix} A \\ B \end{bmatrix} = M_1 \begin{bmatrix} C \\ D \end{bmatrix} = M_1 M_2 \begin{bmatrix} E \\ F \end{bmatrix} = M \begin{bmatrix} E \\ F \end{bmatrix}$$



$$T(E) = \left|\frac{E}{A}\right|^2 = \frac{1}{|m_{11}|^2}$$

Case: $E<V_o$ Transmission below barrier

$$T(E) = \left[1 + \left(\frac{\gamma^2 + k^2}{2k\gamma}\right)^2 sh^2(\gamma L)\right]^{-1}$$

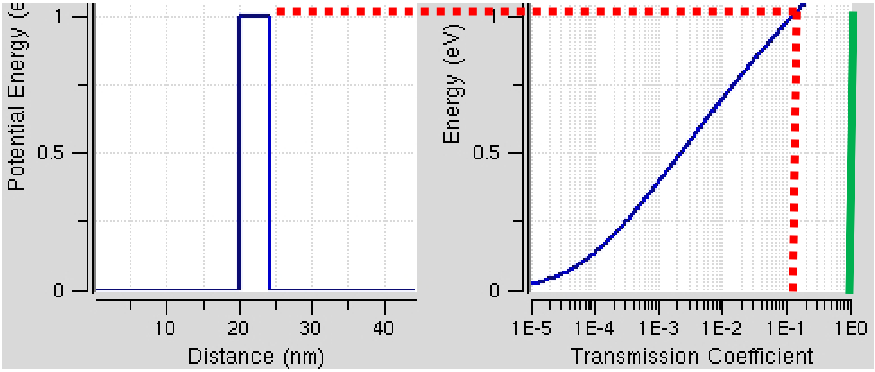
$$k = \sqrt{\frac{2mE}{\hbar^2}}$$ Propagation

$$\gamma = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$$ Attenuation / Decay

Case(γL large): Strong barrier

$$T(E) \approx \left(\frac{4k\gamma}{k^2 + \gamma^2}\right)^2 \exp(-2\gamma L)$$

Exponential decay with
1) barrier width L
2) barrier height V_0



Transmission NOT unity=1
Above Barrier

Single Barrier Transmission Solution

$$\begin{bmatrix}A\\B\end{bmatrix} = M_1\begin{bmatrix}C\\D\end{bmatrix} = M_1 M_2\begin{bmatrix}E\\F\end{bmatrix} = M\begin{bmatrix}E\\F\end{bmatrix}$$

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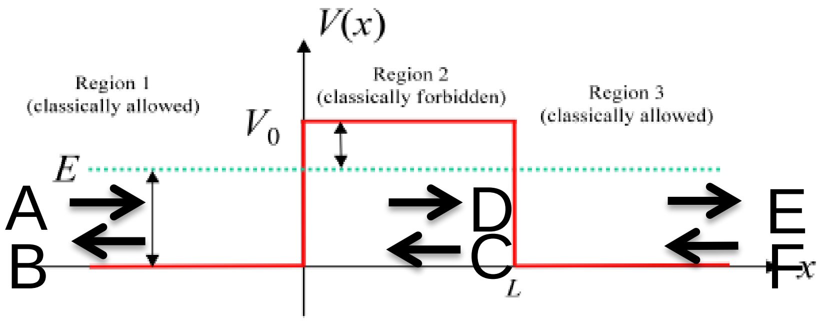

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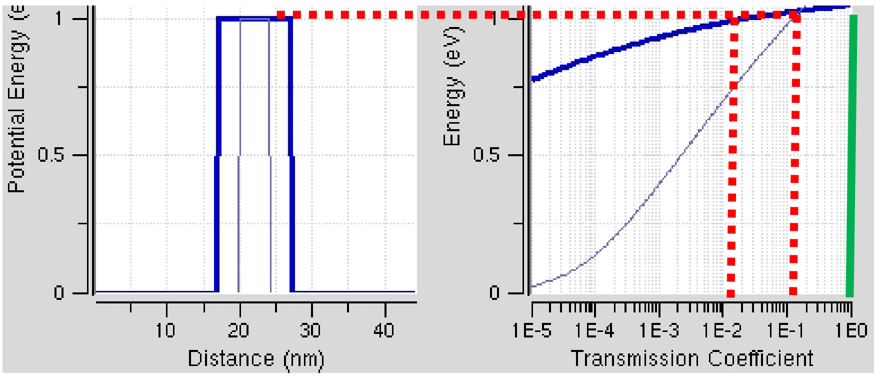

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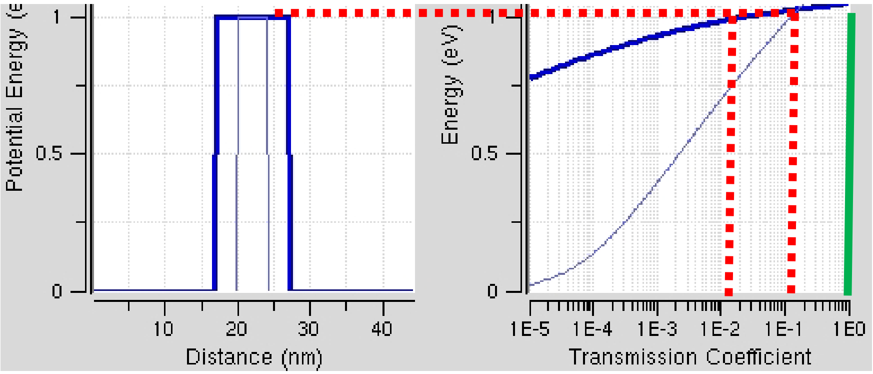

Transmission NOT unity=1
Above Barrier

Case(γL<<1): Weak barrier

$$T(E) \approx \frac{1}{1 + (kL/2)^2}$$

Lorentzian decay with
1) barrier width

Independent of barrier height

Single Barrier Transmission Solution

Case: $E>V_o$ Transmission above barrier

$$T(E) = \left[1 + \left(\frac{k^2 - k_2^2}{2kk_2}\right)^2 \sin^2(k_2 L)\right]^{-1}$$

$k_2 = -i\gamma$ Propagation above barrier

$$T = |t|^2 = \frac{1}{1 + \frac{V_0^2 \sin^2(k_2 L)}{4E(E-V_0)}}$$

Case: $E<V_o$ Transmission below barrier

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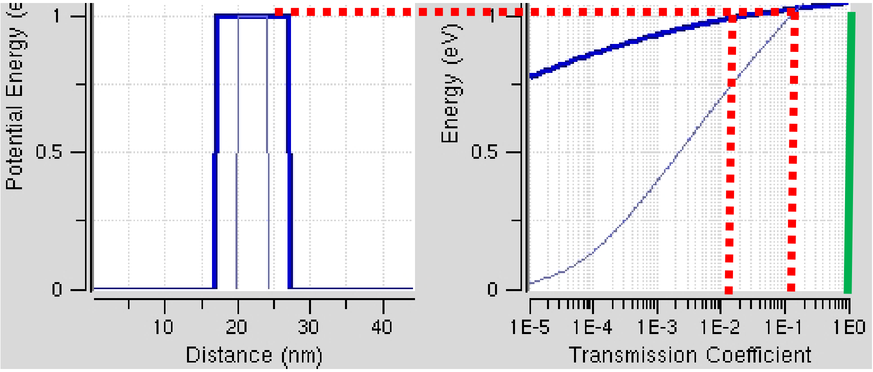

Transmission NOT unity=1 Above Barrier

Transmission oscillates Above Barrier Period is k_2L

Single Barrier Transmission Solution

Case: $E<V_o$ Transmission above barrier

$$T(E) = \left[1 + \left(\frac{k^2 - k_2^2}{2kk_2}\right)^2 \sin^2(k_2 L)\right]^{-1}$$

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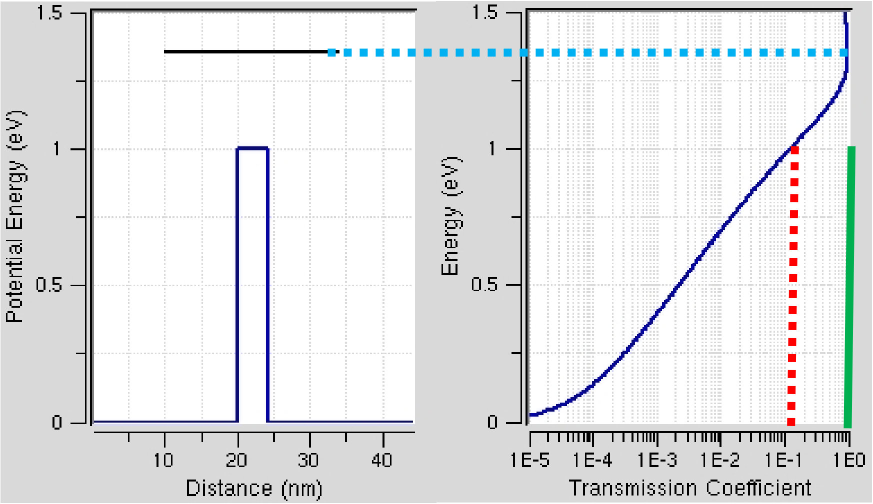

Transmission NOT unity=1
Above Barrier

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Transmission oscillates
Above Barrier
Period is $k_2 L$

Single Barrier Transmission Solution

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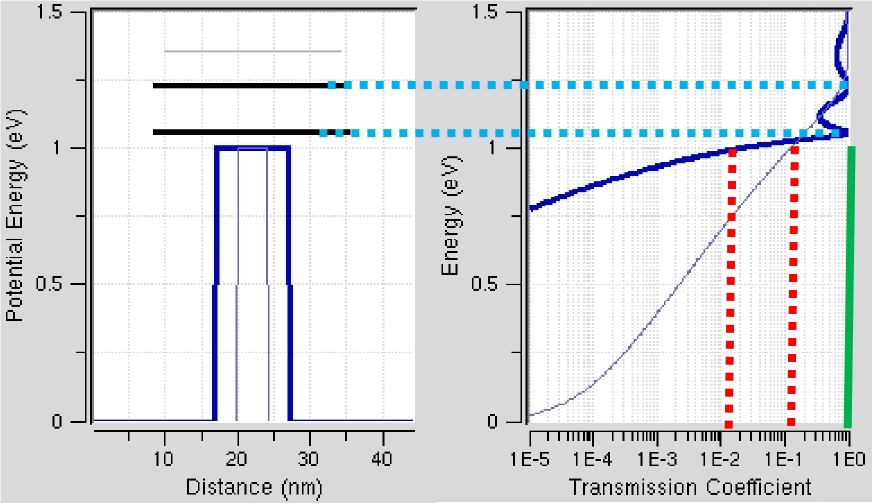
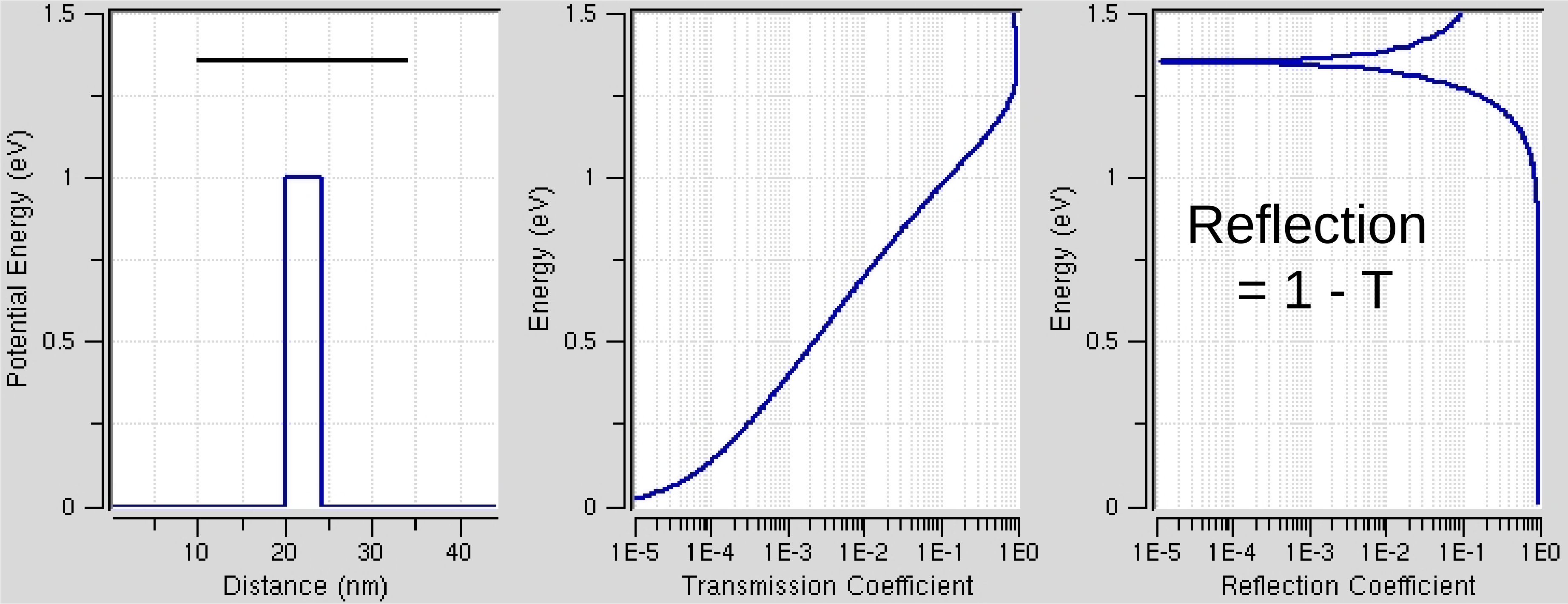


Transmission NOT unity=1
Above Barrier

Transmission oscillates
Above Barrier
Period is k_2L

Single barrier : Concepts



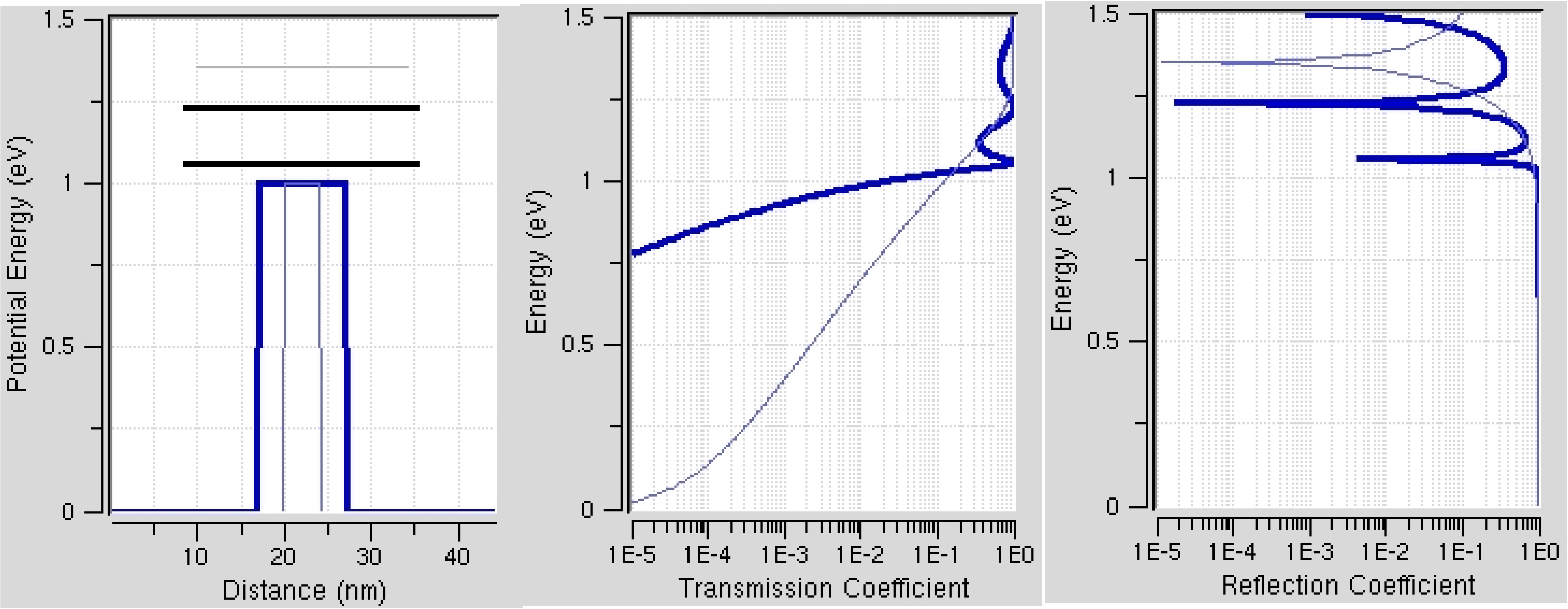
- Transmission is finite under the barrier – tunneling!
- Transmission above the barrier is not perfect unity!
- Quasi-bound state above the barrier. Transmission goes to one.

Case: E>V$_o$

$$T(E) = \left[1 + \left(\frac{k^2 - k_2^2}{2kk_2} \right)^2 \sin^2(k_2 L) \right]^{-1}$$

- Computed with – http://nanohub.org/tools/pcpbt

Effect of barrier thickness below the barrier



- Increased barrier width reduces tunneling probability
- Thicker barrier increase the reflection probability below the barrier height.
- Quasi-bound states occur for the thicker barrier too.

Case: E>V_o

$$T(E) = \left[1 + \left(\frac{k^2 - k_2^2}{2kk_2} \right)^2 \sin^2(k_2 L) \right]^{-1}$$

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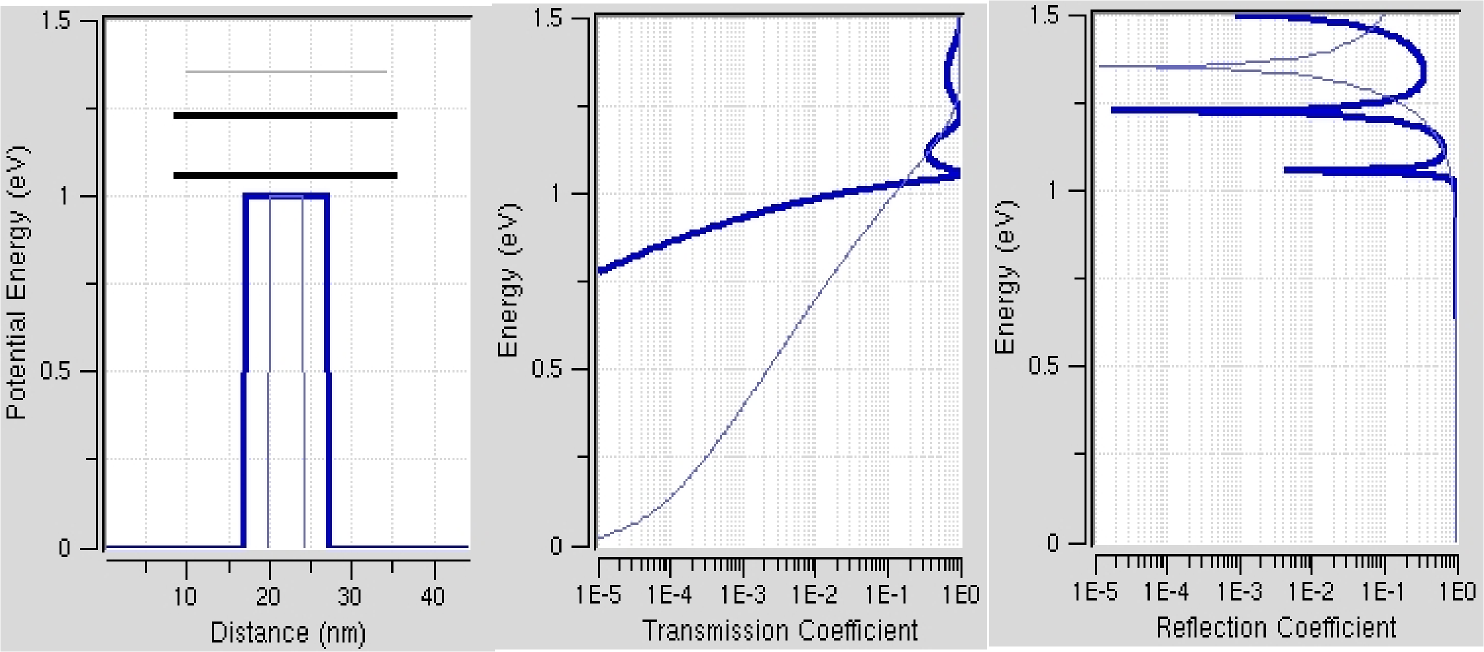
Single Barrier - Key Summary

- Quantum wavefunctions can tunnel through barriers
- Tunneling is reduced with increasing barrier height and width



- Transmission above the barrier is not unity
  - »2 interfaces cause constructive and destructive interference
  - »Quasi bound states are formed that result in unity transmission

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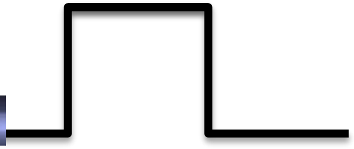

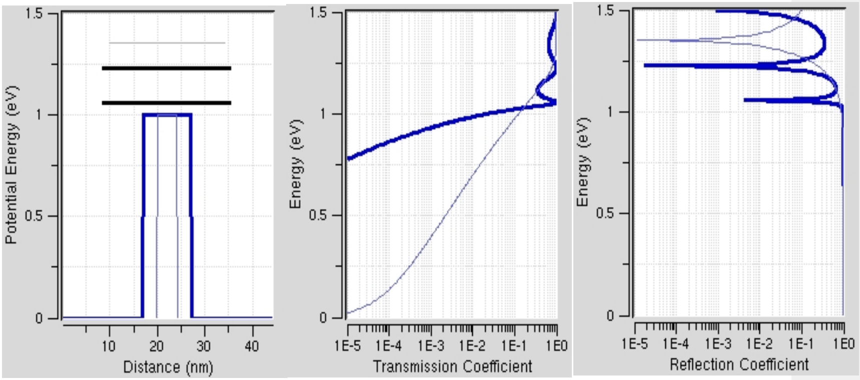

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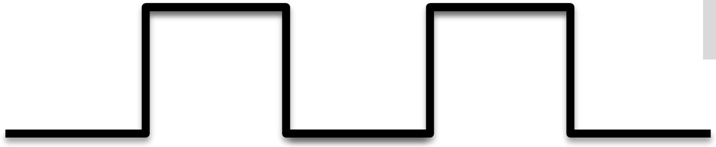

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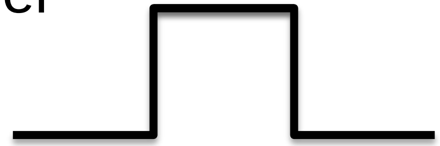

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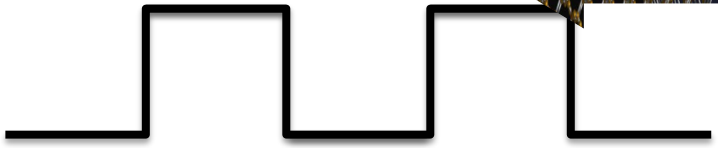

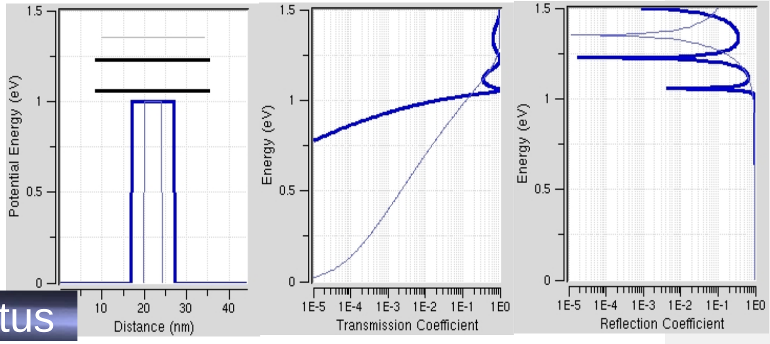

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