Section 4 - Elements of Quantum Mechanics

4.2 Strange Experimental Results
=> The Advent of Quantum Mechanics

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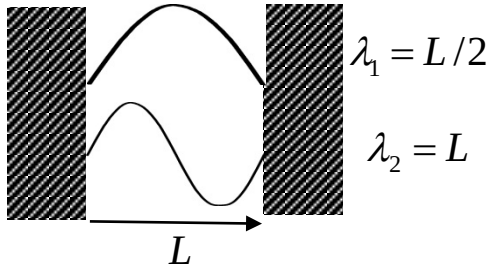
Section 4
Elements of Quantum Mechanics

One Video Segment

- 4.1 Classical Systems
  - » Particles
  - » Propagating Waves
  - » Standing Waves
  - » Chromatography



One Video Segment

- 4.2 Strange Experimental Results => The Advent of Quantum Mechanics
  - » Black Body Radiation — status ion is quantized
  - » Discrete Optical Spectra => light emission/absorption quantized – Bohr Atom
  - » Photoelectric Effect => light is described by particles
  - » Particle-Wave Duality

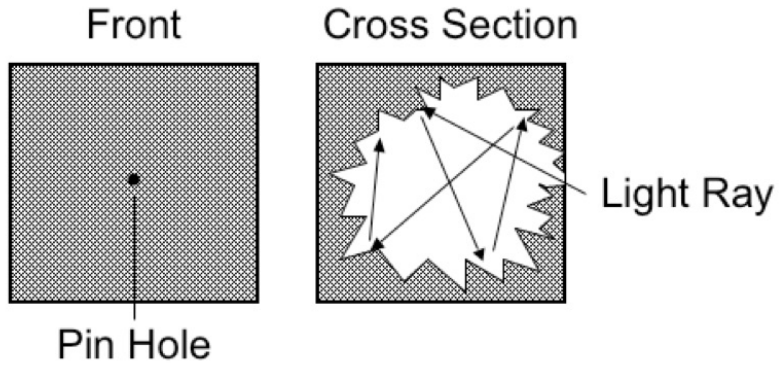
One Video Segment

- 4.3 Why do we need quantum mechanics?

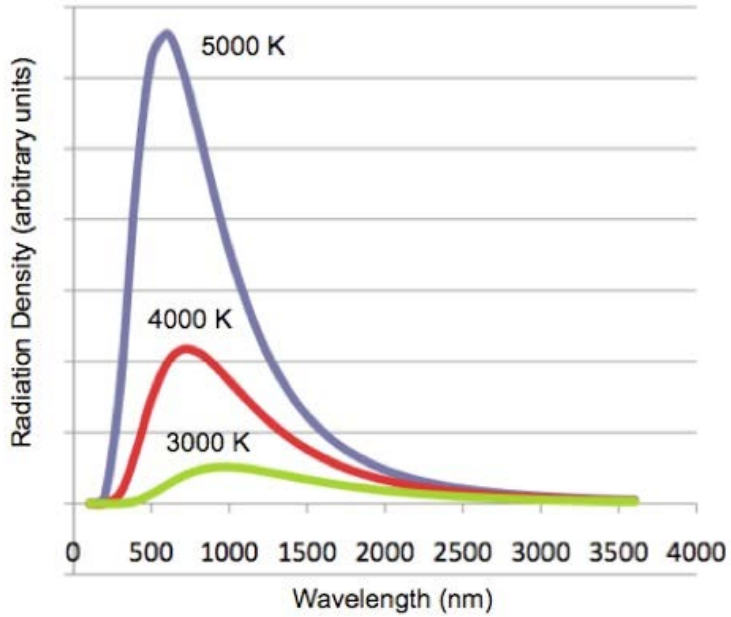
One Video Segment

- 4.4 Formulation of Schrödinger's Eq.

Black-body Radiation



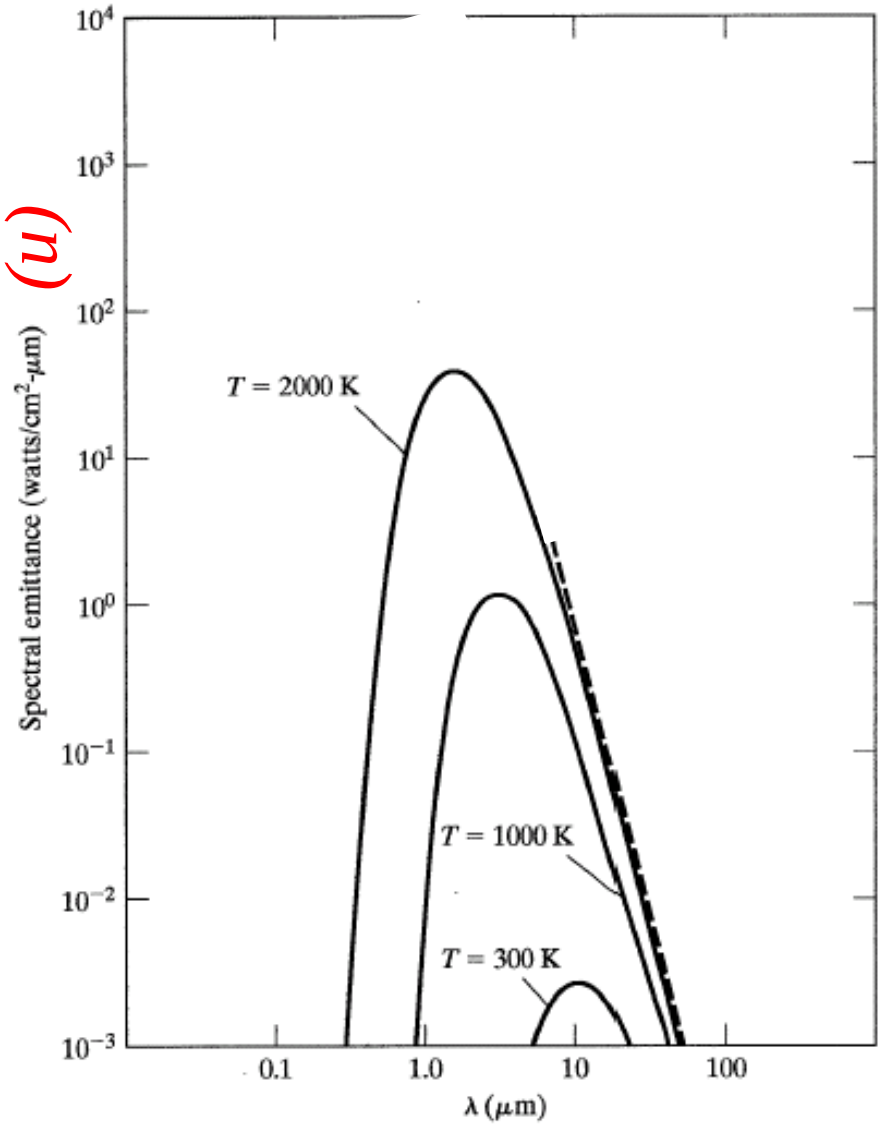
Experiment:
Measure wavelength and intensity of radiation

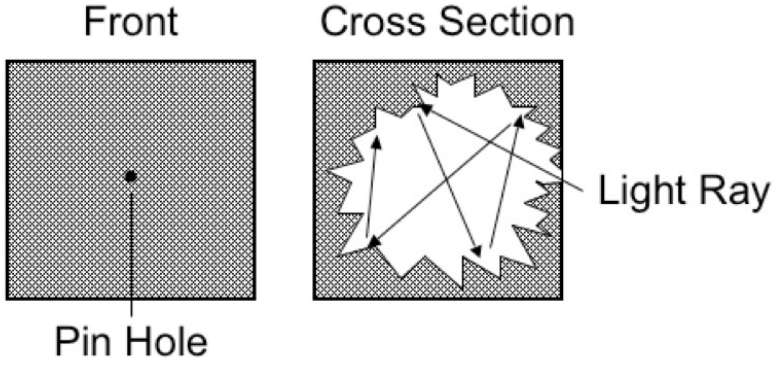


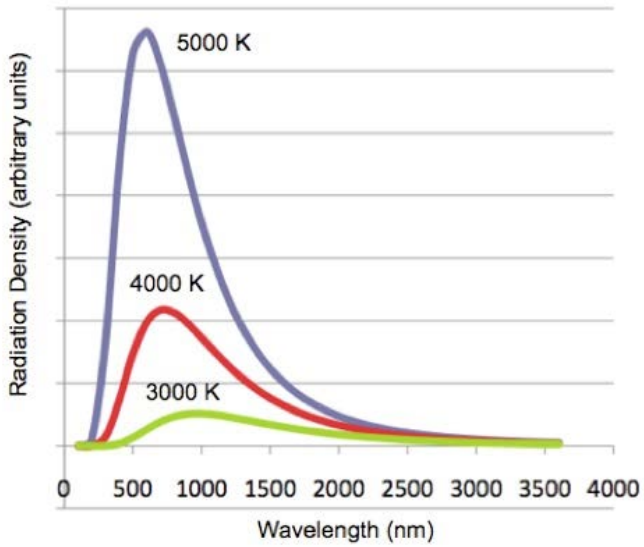
Observations:
Radiation intensity is wavelength and temperature dependent

https://en.wikipedia.org/wiki/Black-body_radiation

Black-body Radiation

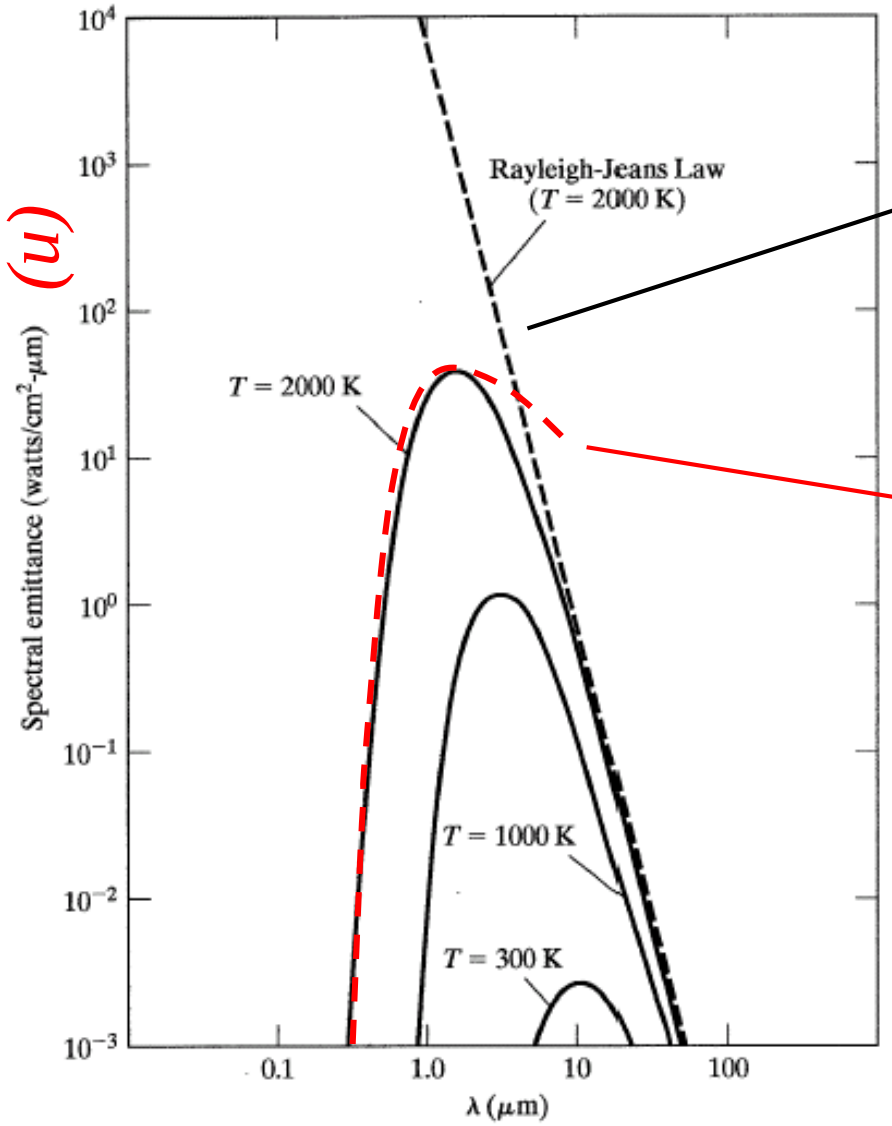






Observations:
Intensity peak moves significantly

Black-body Radiation



Rayleigh-Jeans Formula

$$u(\lambda,T) \propto k_B T / \lambda^4$$

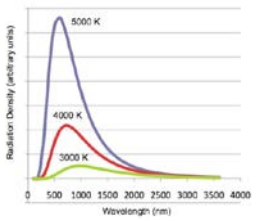
$$\log(u) = -4\log(\lambda) + \log(T)$$

Wein's Formula

$$u \propto \frac{e^{-\beta/\lambda T}}{\lambda^5}$$

Plank's fitting formula

$$u(\lambda,T) \propto \frac{1}{\lambda^5}\left[\frac{1}{e^{\beta/\lambda T} - 1}\right]$$

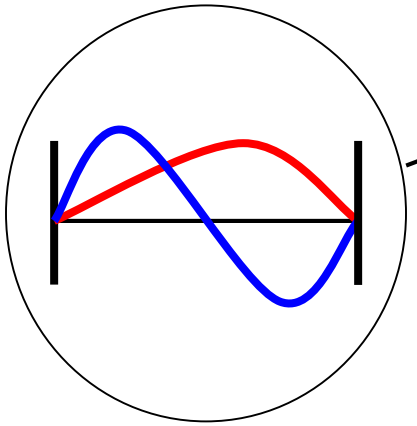


Interpretation of Plank's Formula

$$u(\lambda,T) \propto \frac{1}{\lambda^5}\left[\frac{1}{e^{\beta/\lambda T}-1}\right]$$

$$u(f,T) = u(\lambda,T)\frac{d\lambda}{df} \sim \frac{1}{\lambda^5}\left[\frac{1}{e^{\beta/\lambda T}-1}\right]\frac{d\lambda}{df} \qquad \lambda = \frac{c}{f}$$

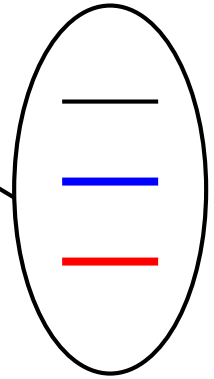
$$\sim f^2 \times hf \times \left(\frac{1}{e^{hf/kT}-1}\right)$$



nos. of modes

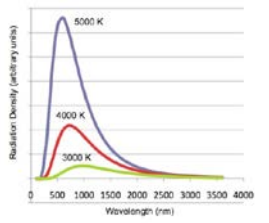
Energy of mode

Occupation Probability

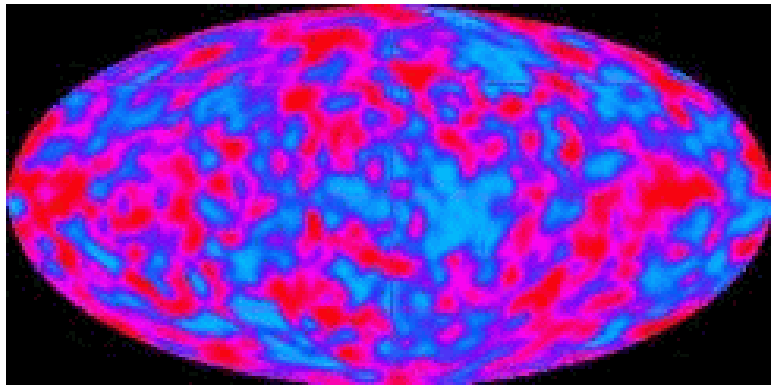


EM emission occurs in discrete quanta of

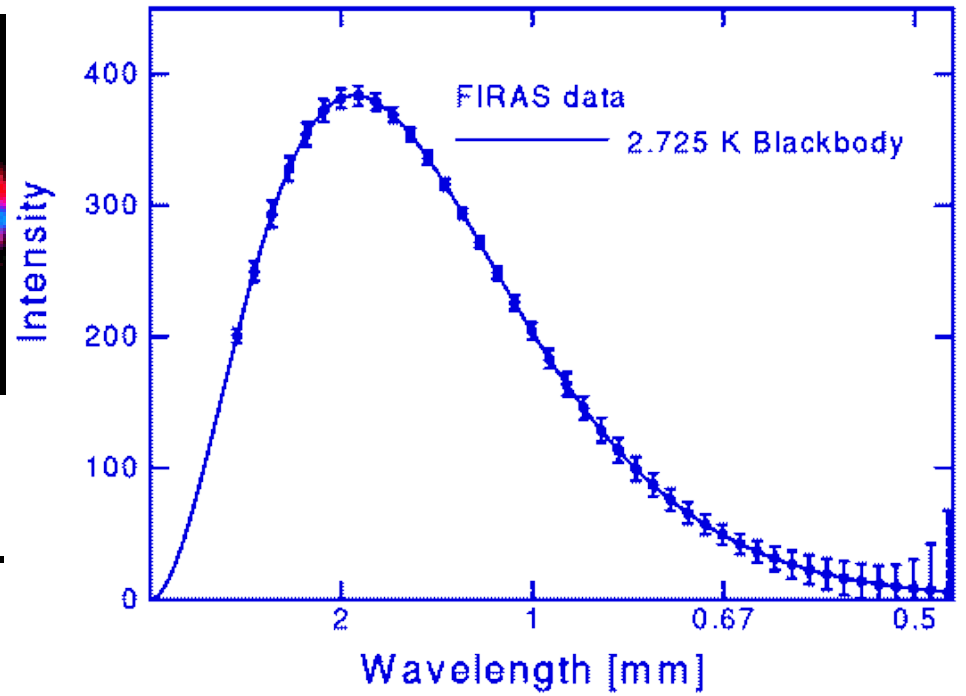
$$E = hf \qquad \text{n=1,2, ... N}$$

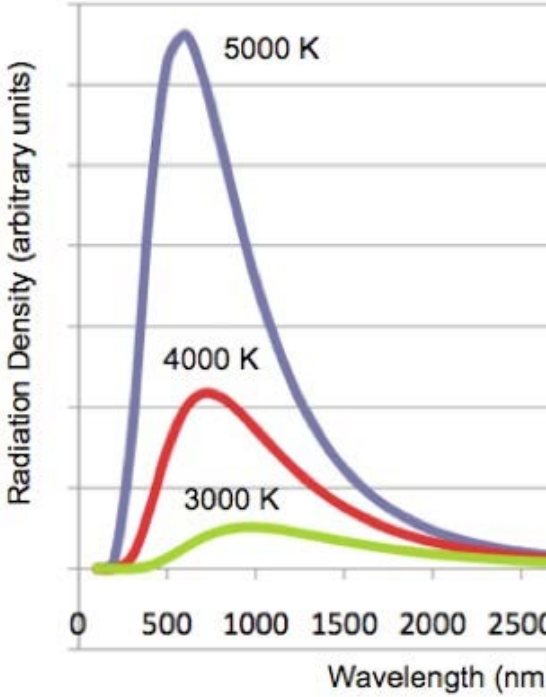


COBE Satellite Data
Measuring Black Body Radiation



J.C. Mather, Astrophysics J., 1990.





Data points are shown superimposed on the theoretical blackbody curve. The fit of the Planck radiation formula is so precise that it provides a powerful confirmation of the idea that it is a remnant of big bang expansion.
Shows that the cosmic background temperature is ~3K.

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One Video Segment

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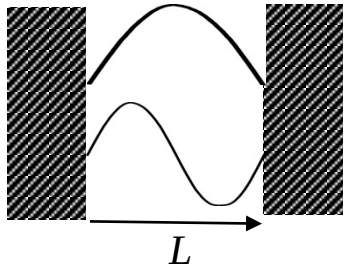

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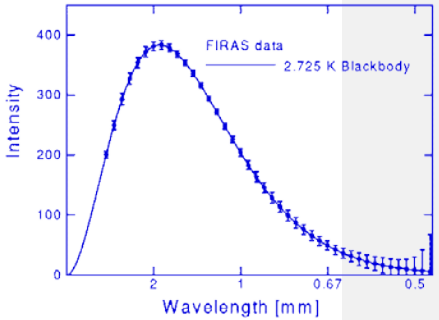
status



- 4.3 Why do we need quantum mechanics?

- 4.4 Formulation of Schrödinger's Eq.

Strange Experimental Observations
The Advent of Quantum Mechanics

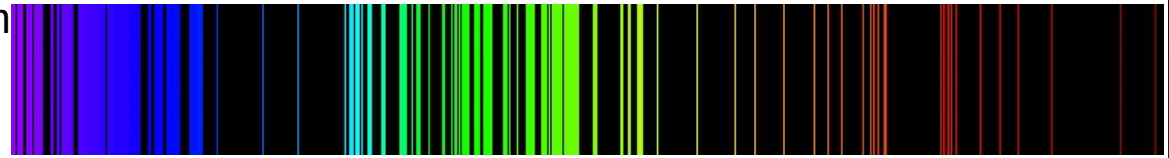
Discrete light spectrum:

- Light emitted from hot elemental materials has a discrete spectrum
- The spectrum is characteristic for the material (fingerprint)

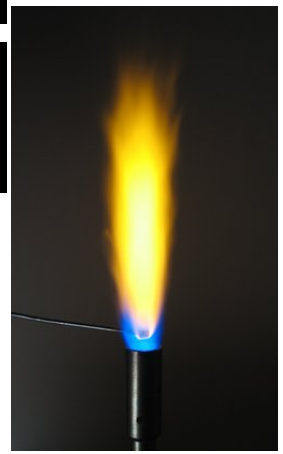
Images from: http://en.wikipedia.org

- E.g.: H spectrum

- E.g.: Iron spectrum



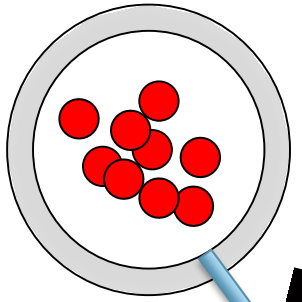
- E.g. application - bright yellow Na lamps
  => lot of excitation energy converted into single frequency



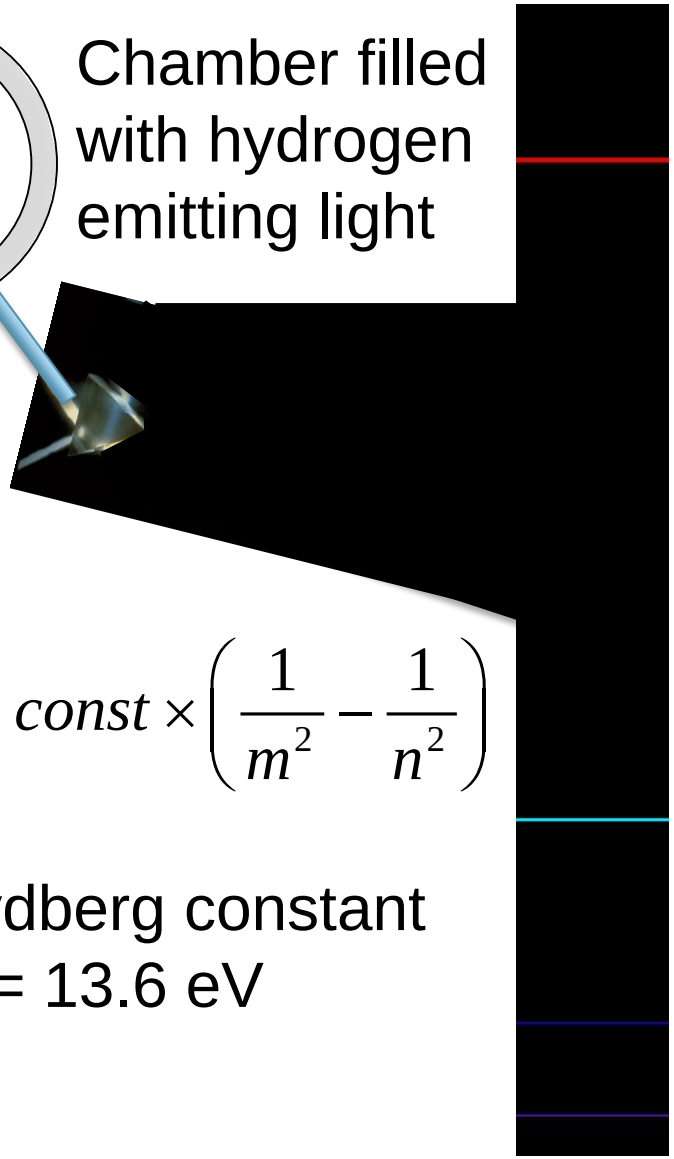
Development of atomic models

- Bohr atom model - electrons in looping orbits

Mapping Observations to a Model
Hydrogen Emission Spectra

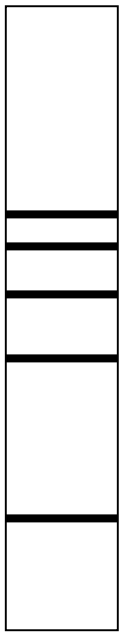


Chamber filled with hydrogen emitting light

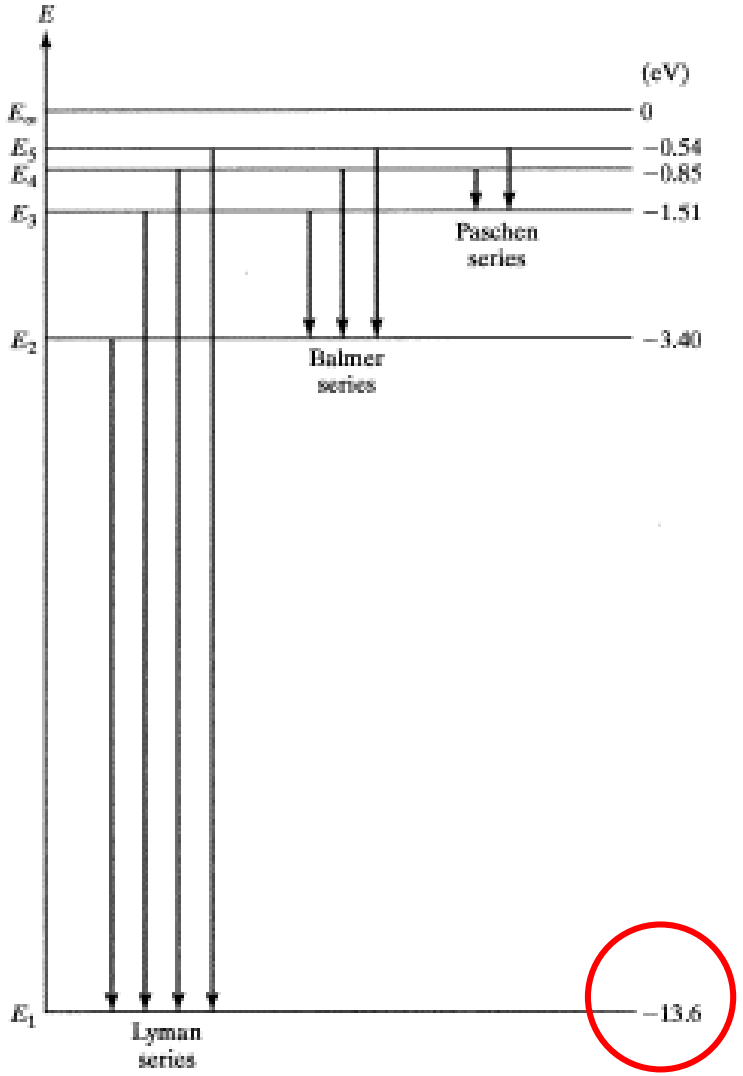


$$E_{m,n} = const \times \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$$

Rydberg constant
R = 13.6 eV



frequency



Bohr Atom Model
Charge Orbiting another Charge

Assume that angular momentum is quantized:

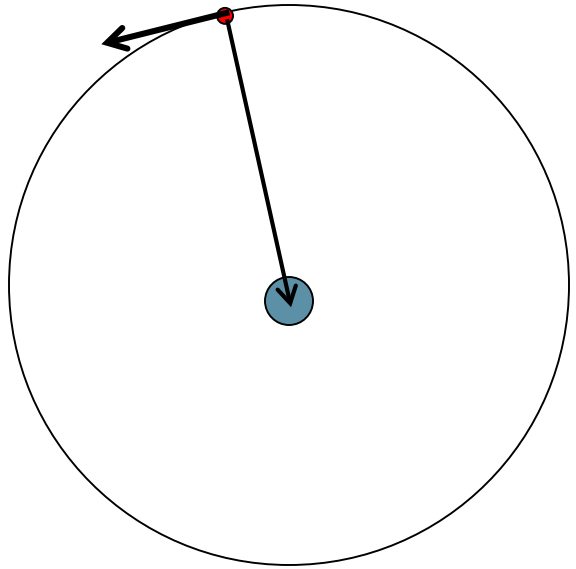
$$L_n = m_0 v r_n = n\hbar$$

$$\upsilon = n\hbar / m_0 r_n$$

$$n = 1, 2, 3, \ldots$$

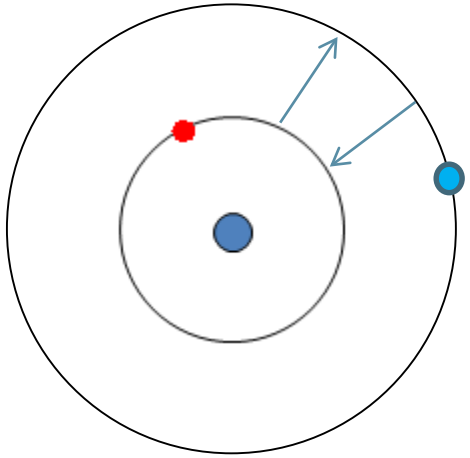
$$\frac{m_0 v^2}{r_n} = \frac{q^2}{4\pi\varepsilon_0 r_n^2}$$

$$r_n = \frac{4\pi\varepsilon_0 (n\hbar)^2}{m_0 q^2}$$



The Bohr Atom Model

$$r_{\mathbf{n}} = \frac{4\pi\varepsilon_0(\mathbf{n}\hbar)^2}{m_0 q^2}$$



$$\text{K.E.} = \frac{1}{2}m_0 v^2 = \frac{1}{2}\left(q^2/4\pi\varepsilon_0 r_{\mathbf{n}}\right)$$

$$\text{P.E.} = -q^2/4\pi\varepsilon_0 r_{\mathbf{n}} \qquad (\text{P.E. set} = 0 \text{ at } r = \infty)$$

$$E_{\mathbf{n}} = \text{K.E.} + \text{P.E.} = -\frac{1}{2}\left(q^2/4\pi\varepsilon_0 r_{\mathbf{n}}\right)$$

$$\boxed{E_{\mathbf{n}} = -\frac{m_0 q^4}{2(4\pi\varepsilon_0 \mathbf{n}\hbar)^2} = -\frac{13.6}{\mathbf{n}^2}\,\text{eV}}$$

$$E_{m,n} = const \times \left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

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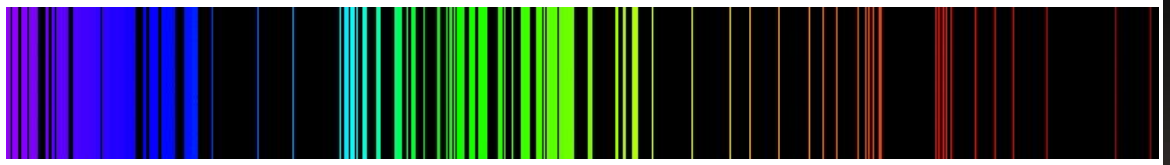

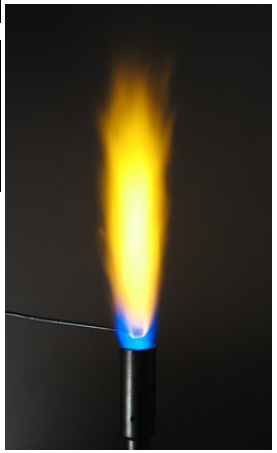

- E.g. application - bright yellow Na lamps
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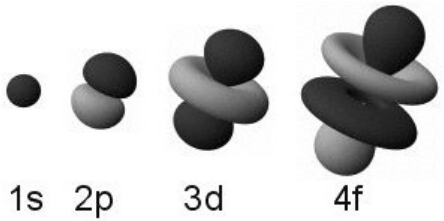
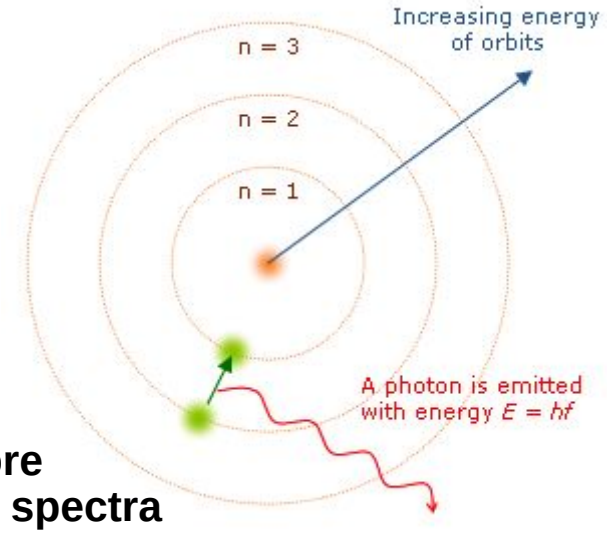
Development of atomic models

- Bohr atom model - electrons in looping orbits
- Quantum mechanical model





=> electrons are standing waves bound to a core
=> discrete transition energies lead to discrete spectra

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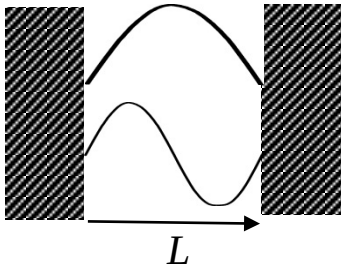

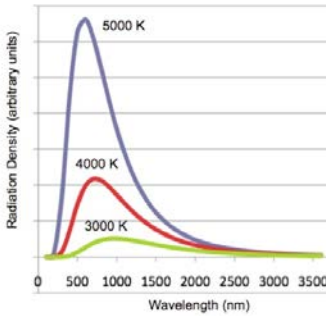

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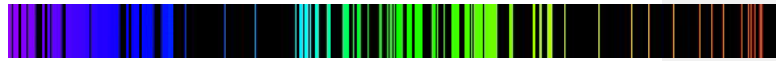

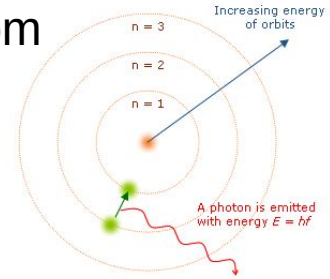

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- 4.4 Formulation of Schrödinger's Eq.

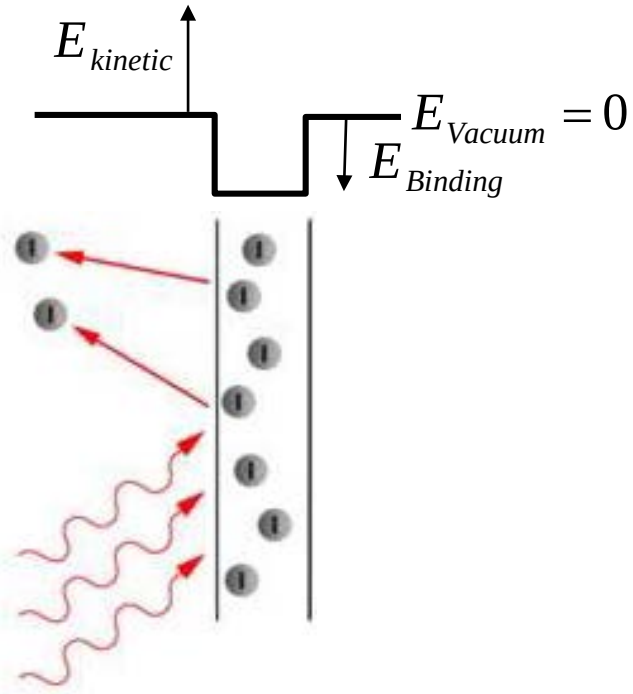
Strange Experimental Observations
The Advent of Quantum Mechanics

Photoelectric Effect:

- Light can eject electrons from a clean metal
- Observed by many researchers but not explained for 55 years:
  1839, 1873, 1887, 1899, 1901
  see details: http://en.wikipedia.org/wiki/Photoelectric_effect

Unexplained problems:

- Electrons emitted immediately, no time lag
- Increasing light intensity increases number of electrons but not their energy
- Red light will not cause emission, no matter what intensity
- Weak violet light will eject few electrons with high energy
  => Light had to have a minimum frequency / color to excite electrons
- => Emitted electrons have light dependent energy f_m



http://en.wikipedia.org/wiki/Photoelectric_effect

Light consists of particles

Photons

The solution in 1905 (Nobel prize for Einstein in 1921)

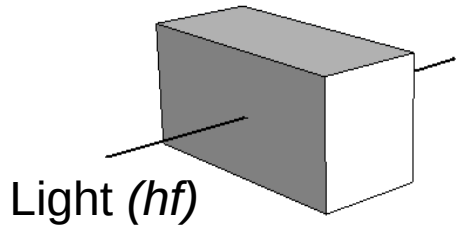
$$\Delta E \propto (f - f_m)$$

$$E = hf$$

- Light can be described by discrete particles of discrete energy
- Planck's constant - h
- Light energy is not divisible
- Have to have minimum energy to kick out an electron from the bound state

$$E_{Binding} = hf_m \qquad E_{kinetic} = E_{light} - E_{Binding} = h(f_{light} - f_m) \geq 0$$

Photoelectric Effect

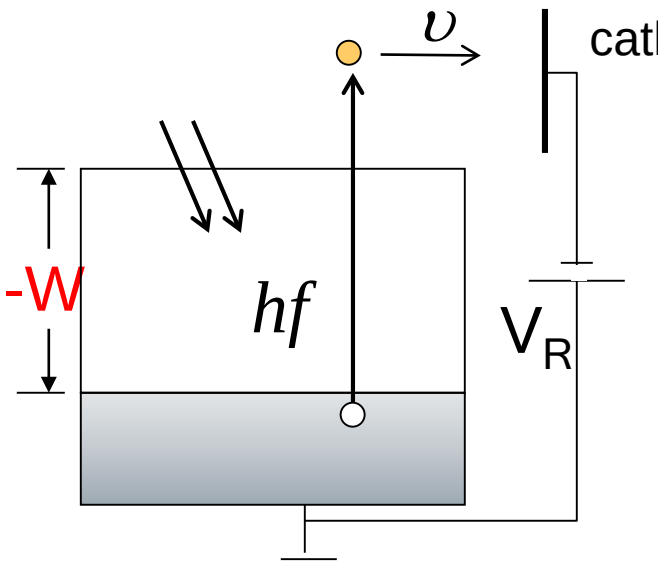


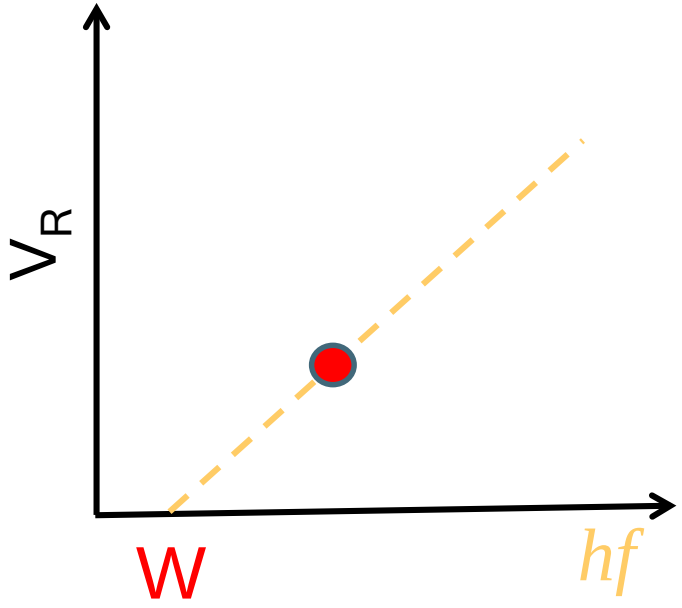
E = (hf-W)

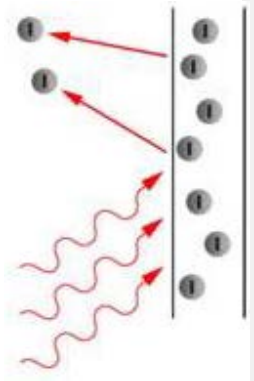
Electrons

Light *(hf)*

$$qV_R \approx \left(\tfrac{1}{2}\right) m_0 \upsilon^2 = hf - W$$







Absorption occurs in quanta as well, consistent with photons having $E=hf$

Wave - Particle Duality

All particles have a wave property

- Can interfere
- Can diffract
- Can form standing waves

All waves have particle properties

- Have momentum
- Have an energy
- Can be created and destroyed

Typical descriptions:

- Energy E, frequency f, Momentum k
- A set of discrete quantum numbers

- Choose wave/particle description according to problem

Wave - Particle Duality

Photons act both as wave and particle – simple math formulation

$$E = \sqrt{m_0^2 c^4 + p^2 c^2}$$

$$hf = pc \qquad m_0 = 0 \text{ (photon rest mass)}$$

$$\begin{aligned} p &= hf / c \\ &= h / \lambda \quad (\text{because } c = \lambda f) \\ &= \hbar k \quad (\text{because } k = 2\pi / \lambda) \end{aligned}$$

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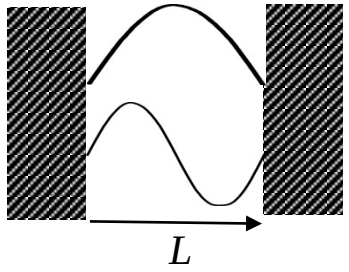

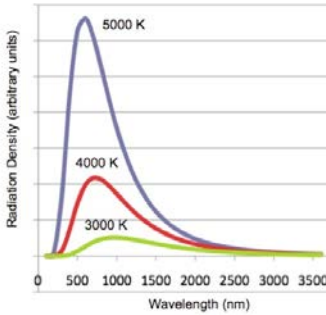

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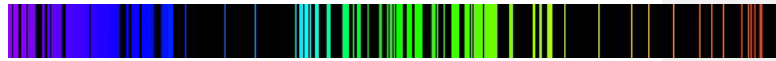
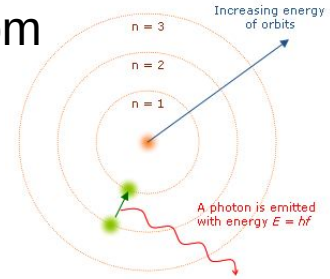
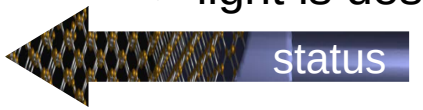
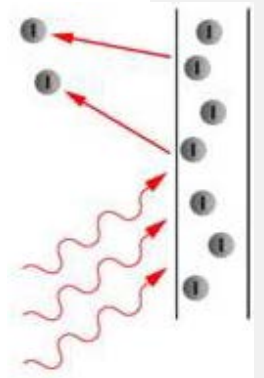






$$p = hf/c$$
$$= h/\lambda \quad (\text{because } c = \lambda f)$$
$$= \hbar k \quad (\text{because } k = 2\pi/\lambda)$$



- 4.3 Why do we need quantum mechanics?
- 4.4 Formulation of Schrödinger's Eq.

Section 4
Elements of Quantum Mechanics

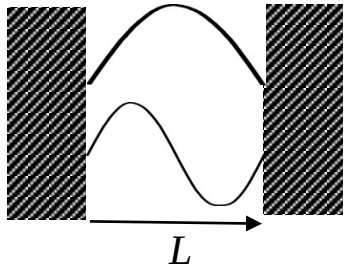
One Video Segment

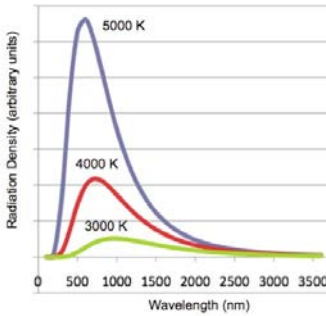
One Video Segment

One Video Segment

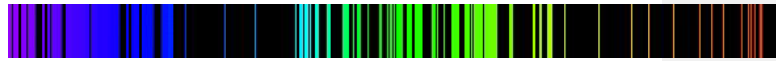
One Video Segment

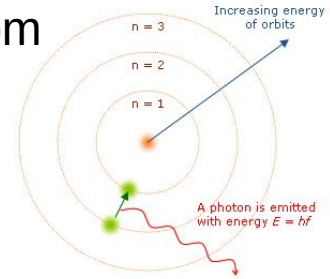
- 4.1 Classical Systems
  - » Particles
  - » Propagating Waves
  - » Standing Waves
  - » Chromatography





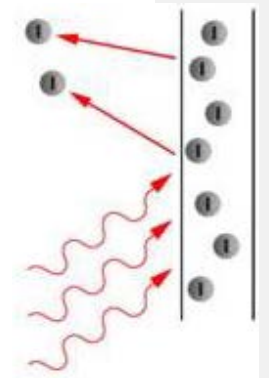
- 4.2 Strange Experimental Results => The Advent of Quantum Mechanics
  - » Black Body Radiation => light emission is quantized
  - » Discrete Optical Spectra => light emission/absorption quantized – Bohr Atom
  - » Photoelectric Effect => light is described by particles
  - » Particle-Wave Duality





$$p = hf/c$$
$$= h/\lambda \quad (\text{because } c = \lambda f)$$
$$= \hbar k \quad (\text{because } k = 2\pi/\lambda)$$

- 4.3 Why do we need quantum mechanics?



- 4.4 Formulation of Schrödinger's Eq.