Exercise: Operator Approach to Harmonic Oscillator Problem

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1. Determine the expectation values for the kinetic and potential energies in a harmonic oscillator in a state *n*. Make use of the creation and annihilation operators discussed in class. What can you say about $\langle T \rangle$ and $\langle V \rangle$?
2. By methods similar to those we used for the evaluation of $\left\langle x^2 \right\rangle$, evaluate $\left\langle x^4 \right\rangle$ and $\left\langle p^4 \right\rangle$ for the case when $\psi = \psi_n(x)$, where *n* represents arbitrary eigenstate of the microscopic simple harmonic oscillator.
3. Use the expression for the position operator in terms of the creation and annihilation operators that was derived in class to show that the diagonal matrix elements of even powers of *x* in the ground state are:

$$\left\langle 0 \middle| x^{2n} \middle| 0 \right\rangle = \frac{(2n)!}{2^n n!} \left(\frac{\hbar}{2m\omega_0} \right)^n .$$

4. Consider the simple harmonic oscillator Hamiltonian

$$\hat{H} = \hat{H}_0 + \hat{W}, \quad \text{with} \quad \hat{H}_0 = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{C_0}{2}x^2 ,$$

and let $W = C_1 x^2 / 2$ be an oscillator perturbation potential. Calculate the energy eigenvalues of this system.