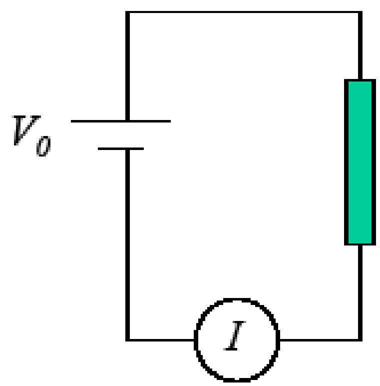
Diffusive vs. Ballistic Transport

Dragica Vasileska

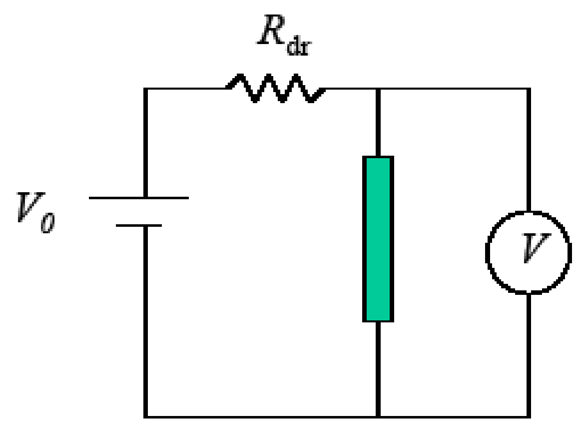
Low frequency measurements: 2-terminal

- what most people think of when discussing electrical conduction measurements.



conductance

$$G = \frac{I}{V_0}$$

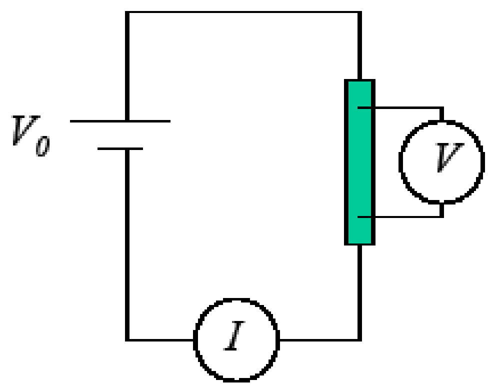


resistance

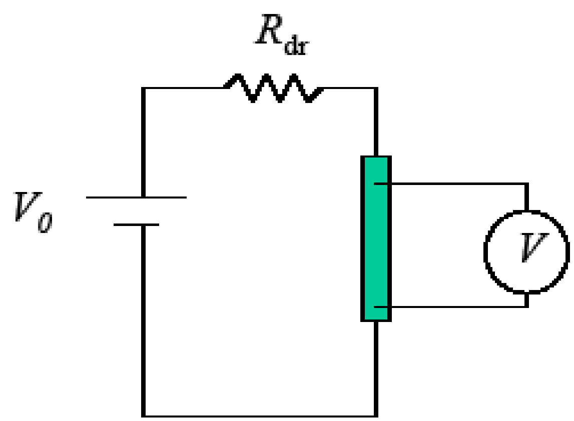
$$R \approx \frac{V}{V_0} R_{dr} \quad \text{for } R << R_{dr}$$

- contact effects contribute directly.

Low frequency measurements: 4-terminal



$$G = \frac{I}{V}$$



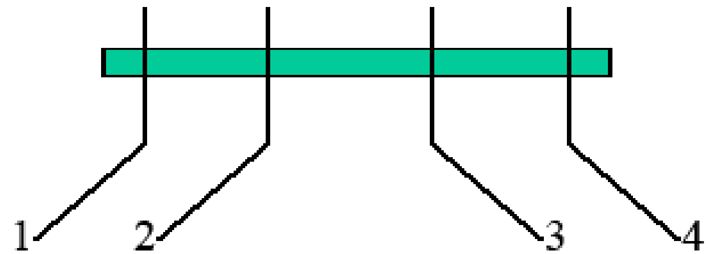
$$R \approx \frac{V}{V_0} R_{dr} \quad \text{for } R << R_{dr}$$

- Mitigates lead effects b/c no current flows through ideal voltage probes.

- Truly ideal voltage probe: capacitive coupling allows electrostatic potentials to equalize w/o any charge exchange.

Transport terminology

Can define multiterminal resistances and conductances:

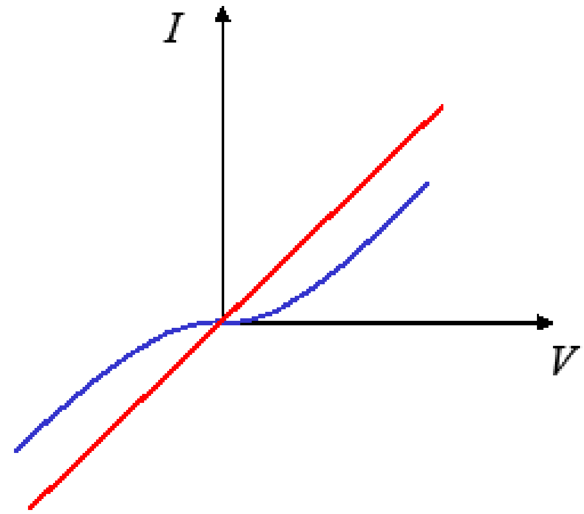


Usual 4-terminal resistance: $R_{14,23} = \frac{V_{23}}{I_{14}}$

"Non-local" resistances: $R_{23,14} = \frac{V_{14}}{I_{23}}$ → Seems like should = $R_{14,23}$

$R_{12,34} = \frac{V_{34}}{I_{12}}$ → Seems like should = 0

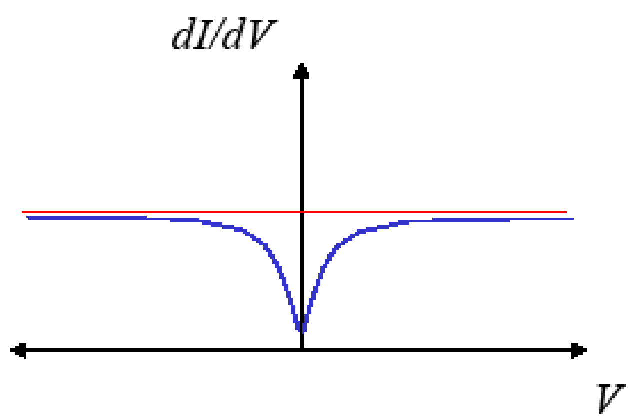
I-V characteristics



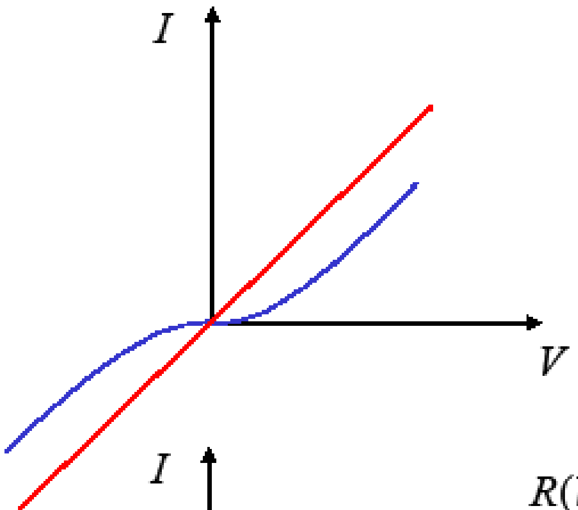
Red curve is Ohmic: $I \sim V$.
Blue curve is nonlinear & subOhmic.

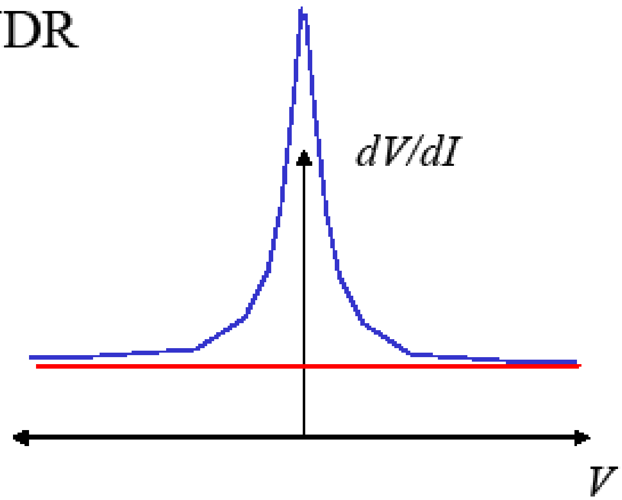
Differential conductance:

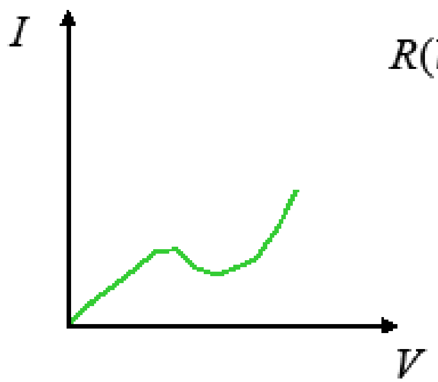
$$G(V) \equiv \left.\frac{dI}{dV}\right|_V$$



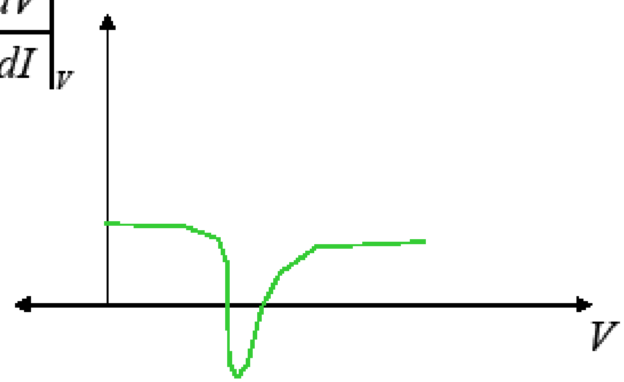
Differential resistance and NDR







$$R(V) \equiv \left.\frac{dV}{dI}\right|_V$$



- dV/dI can be < 0 !
- Useful for "active" circuit elements – gain.

Transport coefficients

Intensive quantities:

J = current density
σ= conductivity
ρ = resistivity

$$\mathbf{J} = \sigma \cdot \mathbf{E} \qquad \begin{pmatrix} J_x \\ J_y \\ J_z \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix} \cdot \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

$$\mathbf{E} = \rho \cdot \mathbf{J} \qquad \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \begin{pmatrix} \rho_{xx} & \rho_{xy} & \rho_{xz} \\ \rho_{yx} & \rho_{yy} & \rho_{yz} \\ \rho_{zx} & \rho_{zy} & \rho_{zz} \end{pmatrix} \cdot \begin{pmatrix} J_x \\ J_y \\ J_z \end{pmatrix}$$

Onsager relation: $\sigma_{xy}(\mathbf{B}) = \sigma_{yx}(-\mathbf{B})$

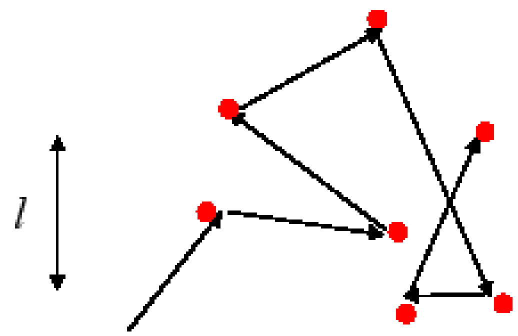
Comes from basic symmetries of equations of motion

Reciprocity relation: $R_{ab,cd}(\mathbf{B}) = R_{cd,ab}(-\mathbf{B})$

Kinetic concepts

Treat electrons like classical gas of noninteracting particles (Drude), with density n_{3d} and charge q.

Elastic mean free path = l: typical distance traveled before scattering event randomizes direction of momentum, while conserving energy.



Scattering is off static defects and disorder.

Assume typical speed = v_F.

Collision time (momentum relaxation time): $\tau = l / v_F$

On scales short compared to l, motion is ballistic.
On scales long compared to l, motion is diffusive.

Drude conductivity and mobility

Consider what happens when such a gas is put into an electric field:

Particles accelerate in response to ***E***, but experience collisions in time τ that randomize direction of momentum.

Result: a drift velocity. $\mathbf{v}_d = \frac{q\mathbf{E}}{m}\tau$

Resulting current density: $\mathbf{J} = n_{3d} q \mathbf{v}_d = \frac{n_{3d} q^2 \tau}{m}\mathbf{E}$

Drude conductivity: $\sigma = \frac{n_{3d} q^2 \tau}{m}$

Mobility: $\mu \equiv \frac{v_d}{E} = \frac{q\tau}{m} \longrightarrow \sigma = n_{3d} q \mu$

More about mobility

$$\mu \equiv \frac{v_d}{E} = \frac{q\tau}{m}$$ Units: cm^2/Vs

- May be extracted from low temperature conductivity measurements.
- Effective mobility limited at high temperatures by inelastic processes (*e.g.* electron-phonon scattering).
- Directly related to elastic scattering time.
- Serves as a measure of material cleanliness.

Typical mobility in single-crystal Si at 300 K: ~ 1000 cm^2/Vs.

→ $\tau \sim 2 \times 10^{-13}$ s.

For $v_F \sim 10^5$ m/s, → $l \sim 20$ nm.

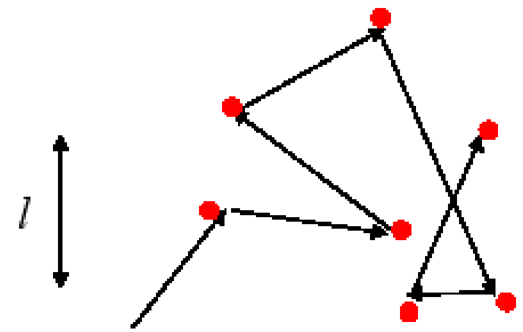
In GaAs 2deg at low temperatures, $\mu \sim 10^7$ cm^2/Vs.

→$\tau \sim 4 \times 10^{-10}$ s.

For $v_F \sim 10^5$ m/s, → $l \sim 40$ μm (!)

Diffusive regime

At scales longer than l transport looks diffusive – random walk of step size l.



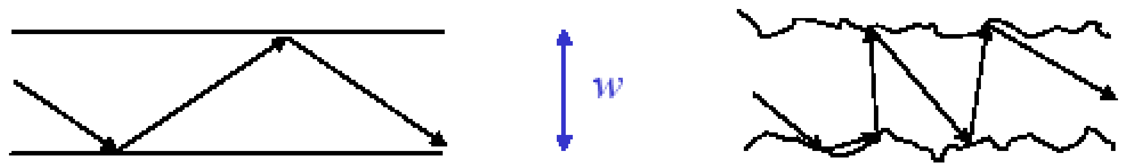
Diffusion constant: $$D = \frac{1}{d} v_F l = \frac{1}{d} v_F^2 \tau$$

Dimensionality d is that for diffusive process. That is, for sample of length L, width w, thickness t,

3d	→	$L, w, t >> l$
2d	→	$L, w >> l >> t$
1d	→	$L >> l >> w, t$

Boundary scattering

An example of a purely classical finite size effect.



Specular scattering.

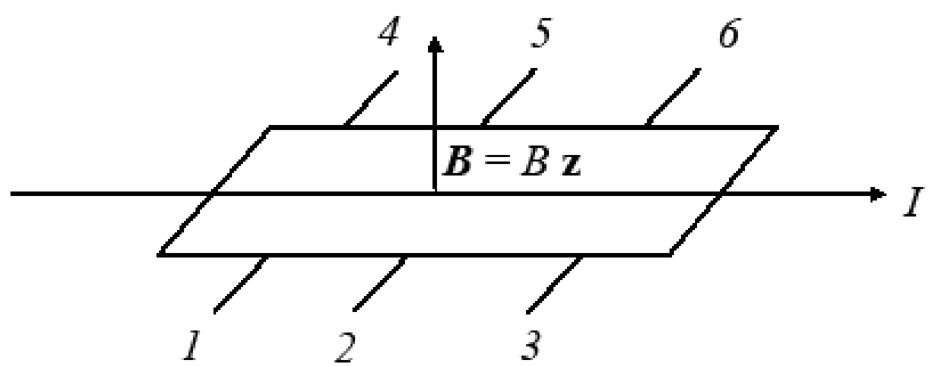
- smooth surfaces
- does not randomize momentum
- does not affect l.

Diffuse scattering.

- rough surfaces
- randomizes momentum
- reduces l.

$$\frac{1}{l_{eff}} \approx \frac{1}{l} + \frac{1}{w} \longrightarrow \sigma_{eff} \approx \sigma \frac{w}{l}$$

Effects of a magnetic field: Hall effect



V_{12} / I = longitudinal resistance

V_{52} / I = Hall resistance

Hall voltage, V_H

Transverse voltage develops across sample.

In equilibrium, average transverse electric force + average Lorenz force + scattering must all balance.

Hall effect – 2d case

$$\frac{m\mathbf{v}_d}{\tau} = e[\mathbf{E} + \mathbf{v}_d \times \mathbf{B}]$$

Writing out components, and remembering definition of ***J***,

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} m / e^2 n_{2d}\tau & -m\mu B / e^2 n_{2d}\tau \\ m\mu B / e^2 n_{2d}\tau & m / e^2 n_{2d}\tau \end{pmatrix} \begin{pmatrix} J_x \\ J_y \end{pmatrix}$$

So,

$$\rho_{xx} = \sigma^{-1},$$

$$\rho_{yx} = -\rho_{xy} = B / |e| n_{2d}$$

Remember,

$$E_x = V_x / L,$$

$$E_y = V_H / w,$$

$$J_x = I / w,$$

$$J_y = 0.$$

Summary of classical transport ideas:

- Transport coefficients relate currents to fields.
- Can do multiterminal measurements, + define differential transport coefficients.
- Treat electrons like independent pointlike particles; motion is ballistic or diffusive, depending on sample size compared to elastic mean free path.
- Can define mobility, and find it using Hall effect.
- Classical size effects: boundary scattering, cyclotron motion & magnetoresistance.

What does *semiclassical* mean

Remember that real single-particle states are Bloch waves, and that we fill those states from the bottom up.

Microscopic equations of motion now relate to evolution of quantum number $\boldsymbol{k}$ rather than to some classical momentum $\boldsymbol{p}$.

Why classical at all?

- Neglect quantum interference effects!
- Act like position and momentum can both be known.

Relevant picture here: with no lattice, Sommerfeld; with lattice + band structure, Bloch.

Semiclassical equations of motion

$$\dot{\mathbf{r}} = \mathbf{v}_n(\mathbf{k}) = \frac{1}{\hbar}\frac{\partial E_n(\mathbf{k})}{\partial \mathbf{k}}$$

$$\hbar\dot{\mathbf{k}} = -e\left[\mathbf{E}(\mathbf{r},t) + \mathbf{v}_n(\mathbf{k}) \times \mathbf{B}(\mathbf{r},t)\right]$$

Assume band index n is constant of the motion.

Electrons not near extrema of $E_{\mathrm{n}}(\boldsymbol{k})$ are always in motion!

No scattering or forces from ion cores here - all that is taken into account by our definition of Bloch waves.

Quantum coherence

A system is said to be quantum coherent if, to calculate probabilities of processes, one must include interference terms.

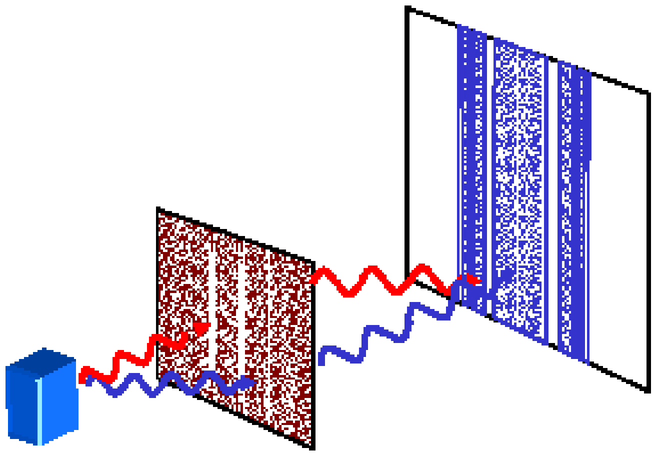
When worrying about an electron in a hydrogen atom, one must use quantum mechanics all the time.

However, one doesn't need to use quantum probabillity rules when discussing electrons in macroscopic objects at room temperature.

Consider dropping an electron into a solid. Quantum coherent corrections to the electron's motion are only needed over a finite length scale….

Quantum interference

Prototypical quantum coherent system: two-slit interference experiment.



Amplitude for path A = ϕ_A

Amplitude for path B = ϕ_B

Probability = $| \phi_A + \phi_B |^2$

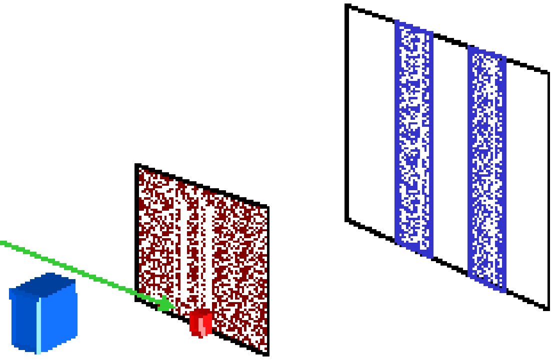
$= \underbrace{| \phi_A |^2 + | \phi_B |^2}_{\text{Classical result}} + 2 \,\mathrm{Re}\, \phi_A^* \phi_B$

Classical result

- Electron beam traveling in isolation, in vacuum.
- Elastic scattering (diffraction) of electrons off slits.
- Coherence seen over large length scale….

Decoherence

Detector wavefunction: $\chi(\eta, t)$



Initial wavefunction:

$$[\phi_A(x, t = 0) + \phi_B(x, t = 0)]\ \chi(\eta, t=0)$$

The wavefunction at time τ (after electron goes through slit) :

$$\phi_A(x,\tau)\ \chi_A(\eta,\tau) + \phi_B(x,\tau)\ \chi_B(\eta,\tau)$$

So, the interference term is now:

$$2\ \mathrm{Re}\ [\ \phi^*_A(x,\tau)\ \phi_B(x,\tau)\ \langle\chi_A(\eta,\tau)\ |\ \chi_B(\eta,\tau)\rangle]$$

The state of the detector multiplies the interference term!

- We see apparent decoherence because inelastic interaction *entangles* electrons with detector, and we trace over detector final states.

Coherence time / length

Equivalent formulation: interaction with detector perturbs relative phases of electron paths in effectively random way; decoherence sometimes known as *dephasing*.

Consider dropping an electron into a solid, where at some rate it undergoes inelastic interactions with the environment (other electrons, phonons, magnetic impurities),

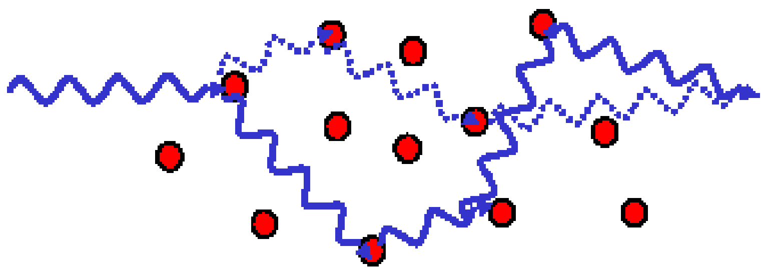
On some time scale τ_ϕ, the phase of the electron becomes essentially uncorrelated with its initial phase. Result: washing out of interference effects.

Distance the electron moves in this time in a diffusive system: $L_\phi = (D\ \tau_\phi)^{1/2}$ = coherence length.

Note: *elastic scattering* off disorder or edges does ***not*** cause decoherence – no change in state of environment.

Why this matters: conductance as transmission

If we have to worry about interference effects, every conductance measurement really becomes an interference experiment!



Should sum amplitudes for all possible trajectories through disordered system, and then square to find probabilities. Many cross terms!

Since L usually $> L_\phi$, we can treat often treat these corrections perturbatively….

Quantum coherence: Aharonov-Bohm effect

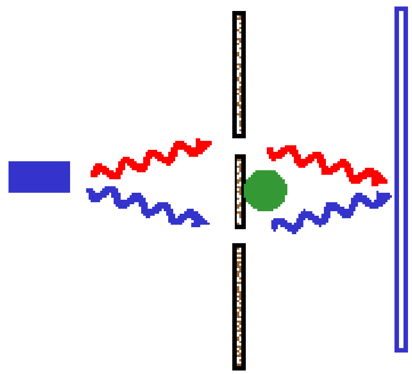
In magnetic field, $\hbar\mathbf{k} \rightarrow \hbar\mathbf{k} + q\mathbf{A}$

the vector potential $\nabla \times \mathbf{A} = \mathbf{B}$,

$$\oint \mathbf{A} \cdot d\mathbf{r} = \Phi$$

Presence of A leads to particles moving along a trajectory picking up an additional quantum mechanical phase:

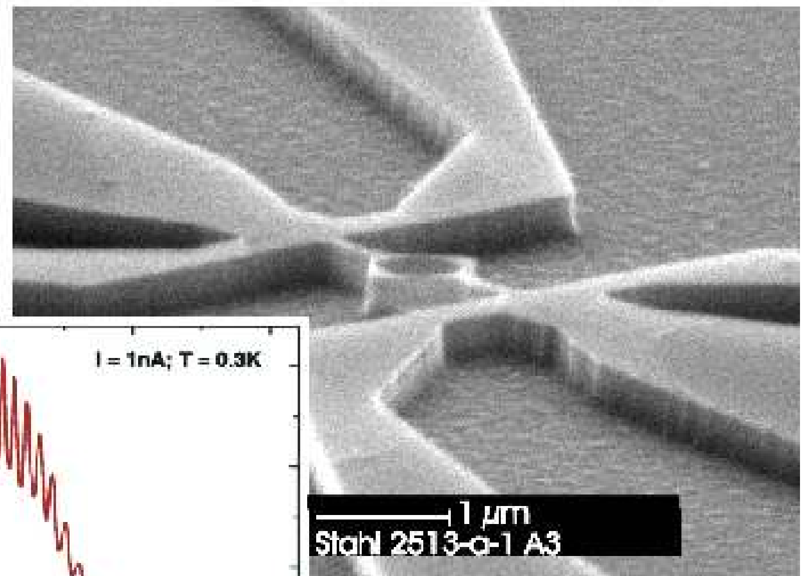
$$\psi \rightarrow \psi \exp\left(i \frac{\oint e\mathbf{A} \cdot d\mathbf{r}}{\hbar} \right)$$

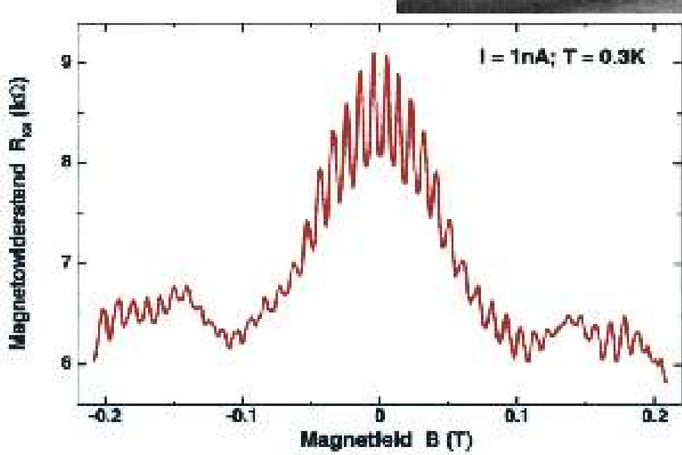


- Interference fringes shift as current through solenoid is varied.
- Complete shift of 2π when flux enclosed = 1 flux quantum, h/e.

Quantum coherence: Aharonov-Bohm effect

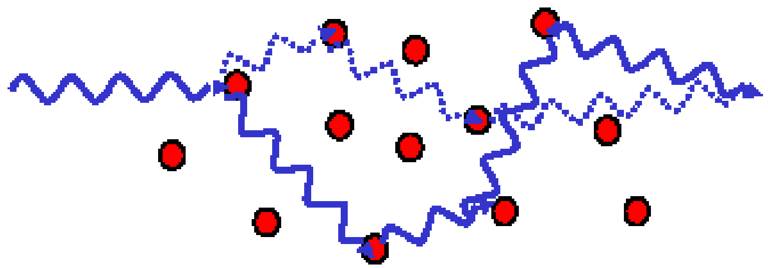
- can see Aharonov-Bohm effect in solids if ring circumference is <~ coherence length!





http://www.physik.rwth-aachen.de/group/physik2b/meso/interference/interf.html

Quantum coherence: conductance fluctuations

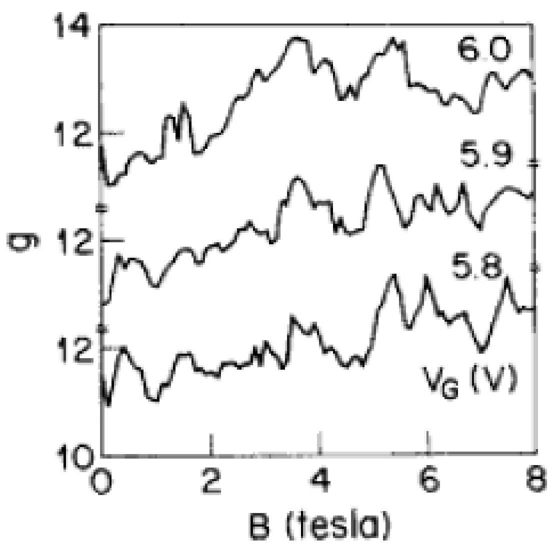


- In fully coherent region, conductance involves adding amplitudes from all possible trajectories, where phases are distributed *randomly*.
- Analogous to diffraction off array of randomly placed slits.
- Interference pattern on a screen would be random bright and dark regions - “speckles”.
- Shifting the relative phases of the waves would move the speckles randomly yet deterministically.

Quantum coherence: conductance fluctuations

One way of shifting relative trajectory phases: Aharonov-Bohm.

Result: applying a magnetic field leads to fluctuations in sample conductance $\delta G(B)$ that depend on exact configuration of scatterers in that sample.

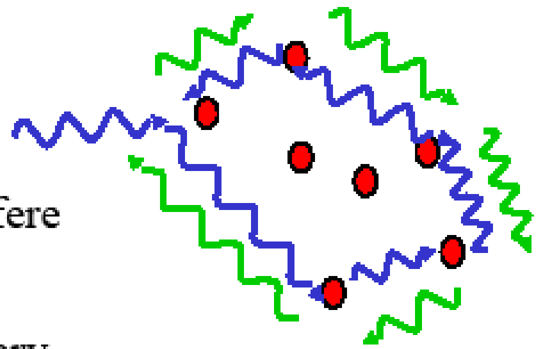


Skocpol *et al.*, PRL **56**, 2865 (1986).

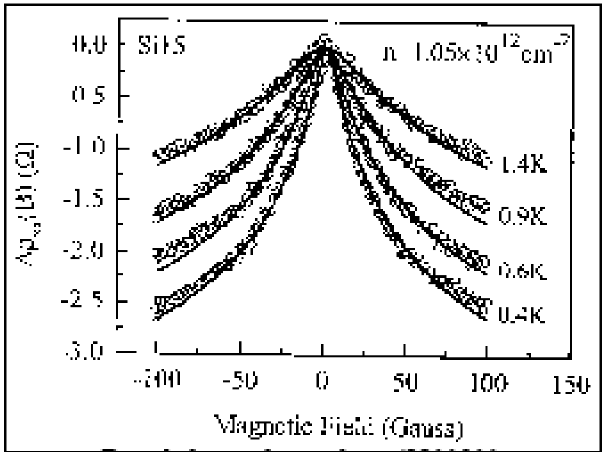
- In coherent volume, rms $\delta G \sim e^2/h$
- Field scale $B_c \sim 1$ flux quantum through a typical coherent area, L_ϕ^2.

Quantum coherence: weak localization

A correction not suppressed by ensemble averaging!

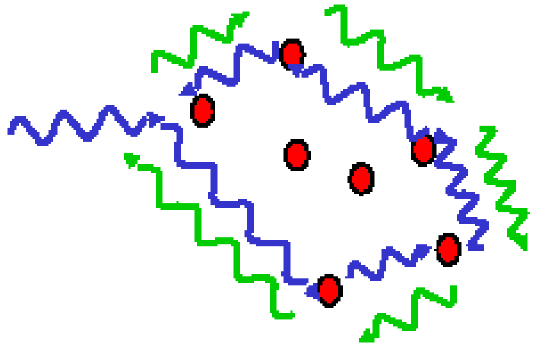


- Time-reversed paths shorter than L_ϕ interfere constructively, affecting the conductance.
- One flux quantum through typical trajectory suppresses this effect due to Aharonov-Bohm phase.
- Result: a magnetoresistance with a size and field scale set by τ_ϕ and conductor properties (dimensionality, D, etc.).



Brunthaler et al., cond-mat/9911011

Quantum coherence: weak localization



Not surpressed by ensemble averaging because each coherent volume's contribution adds in series!

Field scale of magnetoresistance allows one to infer the coherence length as a function of temperature.

In 1d, must thread *h/e* worth of flux through a typical loop trajectory, w x L_ϕ ….

Do we understand quantum coherence in the solid state?

As inelastic processes freeze out, τ_ϕ is expected to **diverge**, with power law exponent set by dimensionality and inelastic mechanism:

electron-3d-phonon dominates: $\tau_\phi \sim T^{-3}$

electron-electron dominates: $\tau_\phi \sim T^{2/(d-4)}$

Still some experimental concerns. In many experiments at very low temperatures, these power laws don't seem to hold. Subtle experimental problems to debug….

Summary:

- Semiclassical picture with Bloch waves does reasonable job reproducing observed electronic transport properties of solids.
- This picture explains why the free-electron Drude model works as well as it does.
- Quantum coherence corrections (interference effects) include: Aharonov-Bohm oscillations, conductance fluctuations, and weak localization.
- Quantum coherence is suppressed on some time (and length) scale by inelastic interactions between the system and dynamic environmental degrees of freedom.
- Understanding coherence is an active area of research, likely to be of technological relevance as devices get very small and / or as spin becomes an important degree of freedom.