Computational Electronics: Finite Difference Discretization

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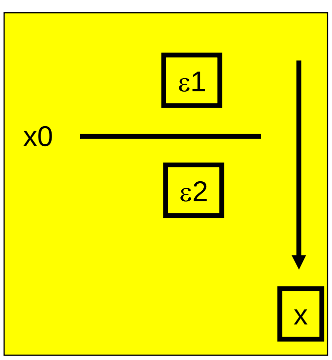
1. Derive general finite difference scheme for the 1D Poisson equation for non-uniform mesh.

2. Consider a 1D sample, such that for $x < x_b$ the semiconductor has a dielectric constant ε_1, and for $x > x_b$ has dielectric constant ε_2. At the interface between the two semiconductor materials ($x = x_b$), there are no interface charges. Starting from the condition

$$\varepsilon_1 \left.\frac{\partial \psi}{\partial x}\right|_{x=x_b} = \varepsilon_2 \left.\frac{\partial \psi}{\partial x}\right|_{x=x_b}$$

and using Taylor series expansion for ψ around $x = x_b$ (for $x < x_b$ and $x > x_b$), calculate the finite difference approximation of the Poisson equation at $x = x_b$. To arrive at the proper solution follow the outlined procedure below:



Start with the general finite-difference discretization

$$f(x0+\Delta x) = f(x0+) + \left.\frac{\partial f}{\partial x}\right|_{x=x0+} \Delta x + \left.\frac{\partial^2 f}{\partial x^2}\right|_{x=x0+} \frac{(\Delta x)^2}{2} + \ldots$$

$$f(x0-\Delta x) = f(x0-) - \left.\frac{\partial f}{\partial x}\right|_{x=x0-} \Delta x + \left.\frac{\partial^2 f}{\partial x^2}\right|_{x=x0+} \frac{(\Delta x)^2}{2} + \ldots$$

$$f(x0+) = f(x0-)$$

$$\left.\frac{\partial^2 f}{\partial x^2}\right|_{x=x0+} = -\frac{\rho(x0+)}{\varepsilon_2}; \quad \left.\frac{\partial^2 f}{\partial x^2}\right|_{x=x0-} = -\frac{\rho(x0-)}{\varepsilon_1}$$

Express the first derivatives and apply the condition

Find the corresponding coefficients in front of f(x0), f(x0+Δx) and f(x0- Δx). Those are your discretization coefficients.

In general net charges at x0+ and x0- might not be zero