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"Electronics from the Bottom Up"

Exercises on Theory of the Ballistic MOSFET Theory: Part 2

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Objective:
The objective of these exercises is to help you gain familiarity with the theory of the ballistic MOSFET as presented in "Physics of Nanoscale Transistors: Lectures 3 and 4: Theory of the Ballistic MOSFET."

1) In Lecture 3, we developed an expression for the ballistic drain current for a planar MOSFET (i.e. one with a 2D channel). We obtained:

$$I_{DS} = I^+ - I^-$$

where

$$I^+ = Wq\left(\frac{N_{2D}}{2}\upsilon_T\right)\mathcal{F}_{1/2}\left(\eta_{F1}\right)$$
$$I^- = Wq\left(\frac{N_{2D}}{2}\upsilon_T\right)\mathcal{F}_{1/2}\left(\eta_{F2}\right)$$

Develop the analogous expressions for a 1D channel MOSFET (i.e. for a nanowire MOSFET).

2) In Lecture 3, we also showed that the states at the top of the barrier in a 2D MOSFET are filled according to:

$$n_S^+(0) = \frac{N_{2D}}{2}\mathcal{F}_0\left(\eta_{F1}\right)\text{ cm}^{-2}$$
$$n_S^-(0) = \frac{N_{2D}}{2}\mathcal{F}_0\left(\eta_{F2}\right)\text{ cm}^{-2}$$

where

$$N_{2D} = \frac{m^* k_B T}{\pi \hbar^2} \text{ \#/cm}^2$$
$$\eta_{F1} \equiv \left(E_{F1} - \varepsilon_1(0)\right) / k_B T$$
$$\eta_{F2} \equiv \eta_{F1} - qV_{DS} / k_B T$$

Develop analogous expressions for the electron density per unit length, $n_L^+(\eta_{F1})$ and $n_L^+(\eta_{F2})$.

3) In Lecture 3, we developed an expression for the I-V characteristic of a ballistic MOSFET as

$$I_{DS} = WC_{ox}\left(V_{GS} - V_T\right)\tilde{\upsilon}_T \left[\frac{1 - \mathcal{F}_{1/2}(\eta_{F2}) / \mathcal{F}_{1/2}(\eta_{F1})}{1 + \mathcal{F}_0(\eta_{F2}) / \mathcal{F}_0(\eta_{F1})}\right]$$

where

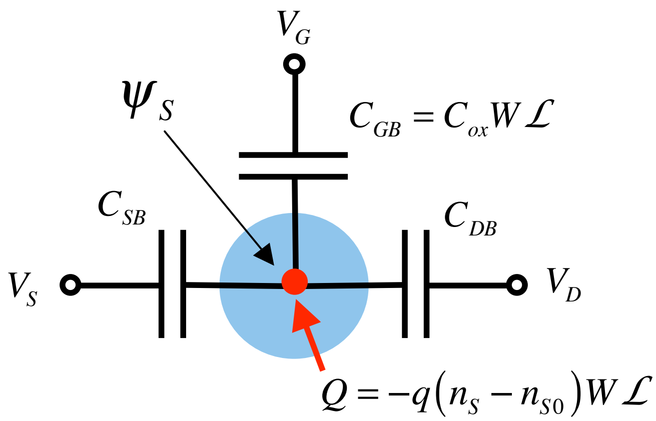
$$\tilde{\upsilon}_T \equiv \sqrt{\frac{2k_B T}{\pi m^*}} \frac{\mathcal{F}_{1/2}(\eta_{F1})}{\mathcal{F}_0(\eta_{F1})} = \upsilon_T \frac{\mathcal{F}_{1/2}(\eta_{F1})}{\mathcal{F}_0(\eta_{F1})}.$$

Use the results of problems 1) and 2) to develop similar expressions for a 1D (nanowire) MOSFET. Note that for the planar MOSFET, $C_{ox} = \varepsilon_0 \kappa_{ox} / t_{ox}$ F/cm^2. For a nanowire MOSFET with a cylindrical gate, the analogous expression would be $C_{ox} = 2\pi\varepsilon_0\kappa_{ox} / \ln\left[\left(2t_{ox} + t_{wire}\right)/t_{wire}\right]$, where t_{wire} is the diameter of the nanowire.

4) For a planar MOSFET, we know the saturated drain current varies with V_{GS} according to $I_D \propto \left(V_{GS} - V_T\right)^{\alpha}$ where $\alpha = 1$ in the non-degenerate case and $\alpha = 1.5$ in the degenerate case. What happens for a nanowire transistor? Use the results of problem 3) to determine the range of α for a ballistic nanowire MOSFET.

5) In Lecture 3, we showed how to derive the density-of-states and conductivity effective masses for the unprimed subbands of (100) Si. Repeat the derivation for the primed subbands. Do this for a transport direction of [100] and for the industry standard direction of [110].

6) Use the three capacitor model below to relate the surface potential, ψ_S, to the applied voltages and charge at the top of the barrier.



Prove that $\psi_S = V_G\left(\frac{C_{GB}}{C_\Sigma}\right) + V_D\left(\frac{C_{DB}}{C_\Sigma}\right) + V_S\left(\frac{C_{SB}}{C_\Sigma}\right) - \frac{q\left[n_S(\psi_S) - n_{S0}\right] W \mathcal{L}}{C_\Sigma}$.

HINT: Use superposition. Begin by assuming that the charge, Q, is zero, and then use superposition again to relate V_G to ψ_S assuming V_S and $V_D = 0$. Repeat for V_S and then for V_D. Next, assume $V_G = V_S = V_D = 0$, and relate ψ_S to the charge, Q. Add the soution for $Q = 0$ to that for Q nonzero.

7) Use the capacitor model,

$$\psi_S = V_G\left(\frac{C_{GB}}{C_\Sigma}\right) + V_D\left(\frac{C_{DB}}{C_\Sigma}\right) + V_S\left(\frac{C_{SB}}{C_\Sigma}\right) - \frac{q\left[n_S(\psi_S) - n_{S0}\right] W \mathcal{L}}{C_\Sigma}$$

and assume that $C_{GB} = C_{ox} W \mathcal{L}$ is known, then answer the following questions.

7a) Show that the subthreshold swing, S, is given by

$$S = 2.3 \frac{k_B T}{q}\left(\frac{C_{GB}}{C_\Sigma}\right)$$

7b) Show that the drain-induced barrier lowering, DIBL, is given by

$$DIBL = \frac{C_{DB}}{C_{GB}}.$$

HINT: Recall that $I_D = Wq\left(N_{2D}\upsilon_T/2\right)\left[\mathcal{F}_{1/2}\left(\eta_{F1}\right) - \mathcal{F}_{1/2}\left(\eta_{F2}\right)\right]$, so for non-degenerate carrier statistics, $I_D \approx e^{q\psi_S/k_BT}$. Because S and DIBL are defined below threshold non-degenerate statistics can safely be used.

7c) Show also that above threshold,

$$R_O = \frac{1}{2.3I_D}\frac{S}{DIBL}$$

Explain why the expression for output resistance may not be as accurate as the expressions for S and DIBL.

8) For T = 0K, the ballistic channel conductance is $G_{CH} = M\left(2e^2/h\right)$ where M is the number of conducting channels. Show that the ballistic channel conductance of a planar MOSFET depends only on the inversion layer density, n_S, and not on the effective mass.