23rd Symposium on Microelectronics Technology and Devices: SB Micro 2008
IEEE / EDS Mini Colloquium
September 1, 2008, Gramado, Brazil

Chip in the Pampa

Hotel Serra Azul - Gramado, Brazil
September 1 to 4, 2008

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Physics of Nanoscale Transistors:

An Introduction to Electronics from the Bottom Up

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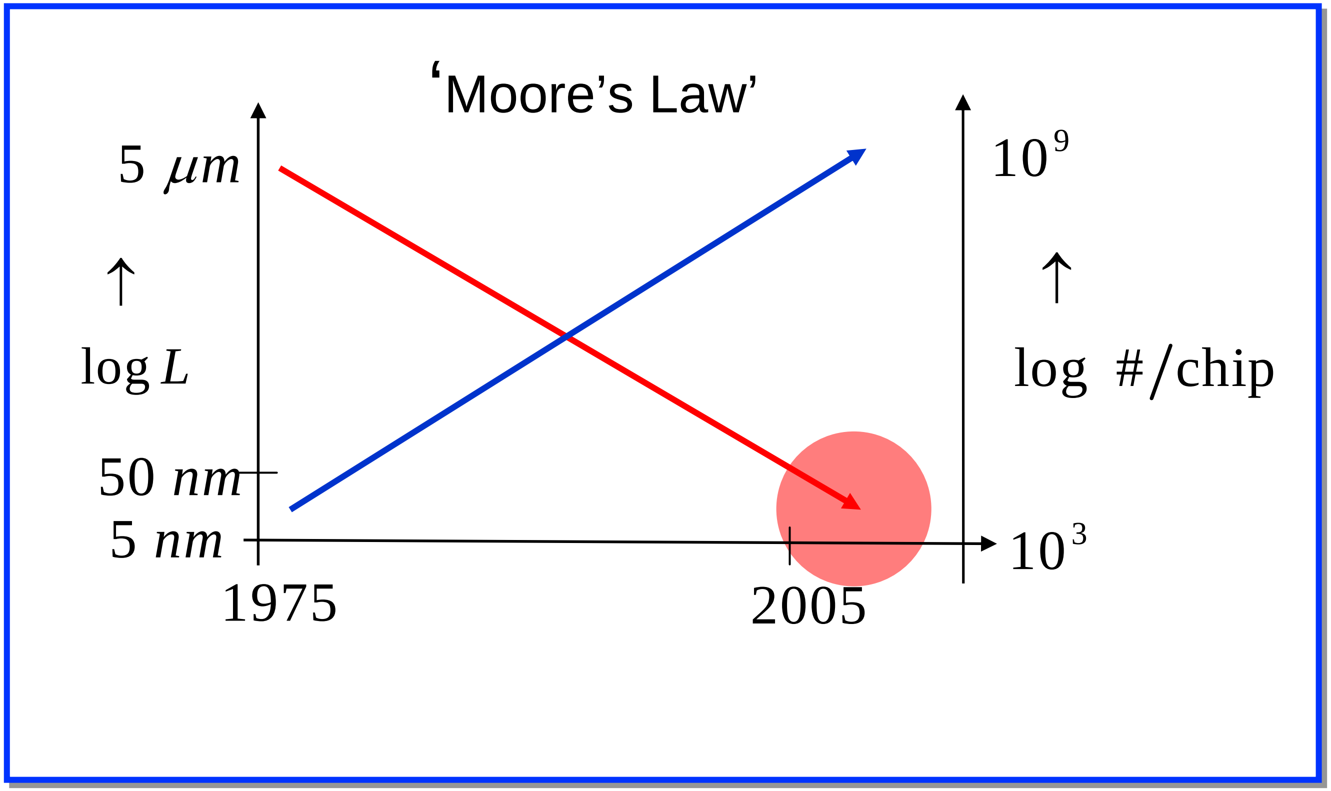
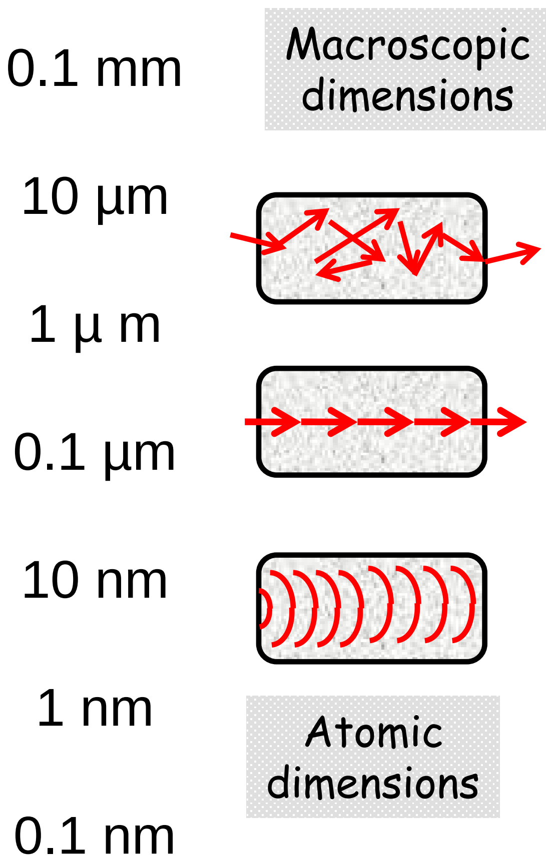
technology trends…..



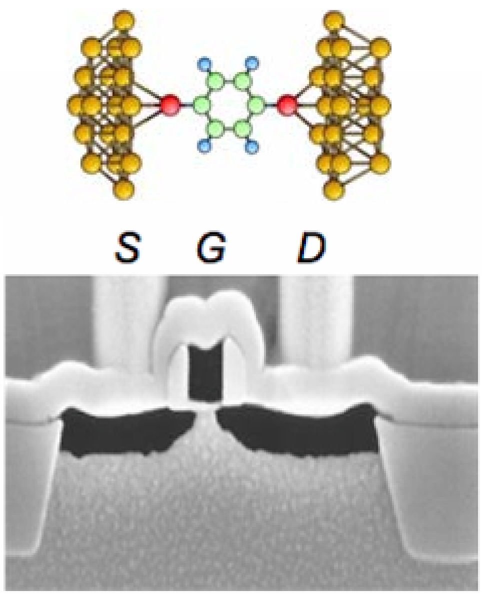
models for devices (conceptual and computational)

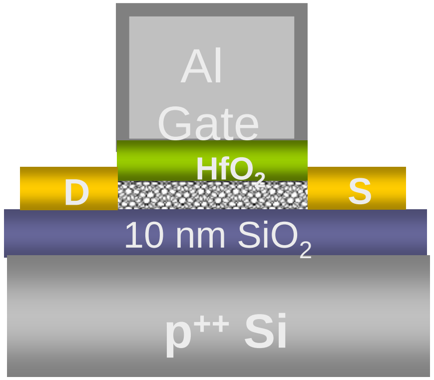


- drift-diffusion
- drift-diffusion + velocity saturation
- Boltzmann for velocity overshoot
- quasi-ballistic
- quantum mechanical

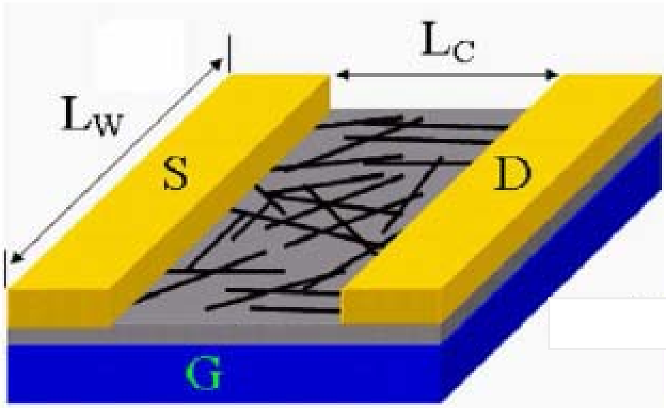
21st Century electronic devices

molecular electronics

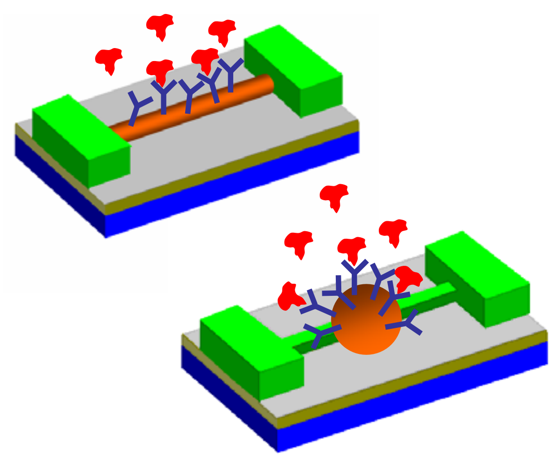


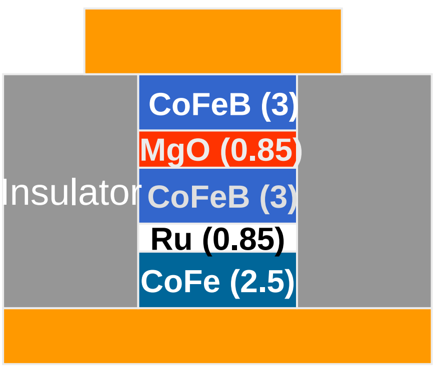


carbon nanotube electronics



nanonets





spin torque devices

nanowire PV

nanowire bio-sensors

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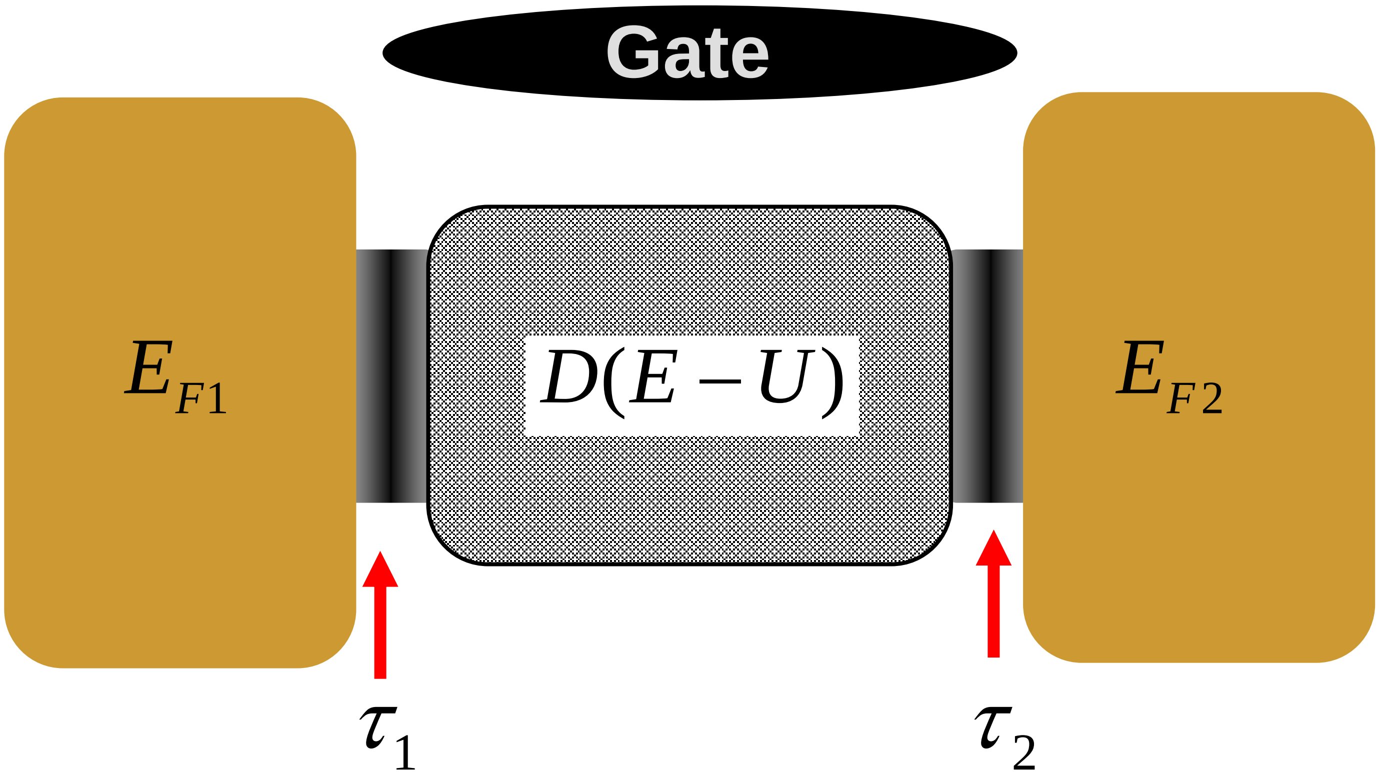
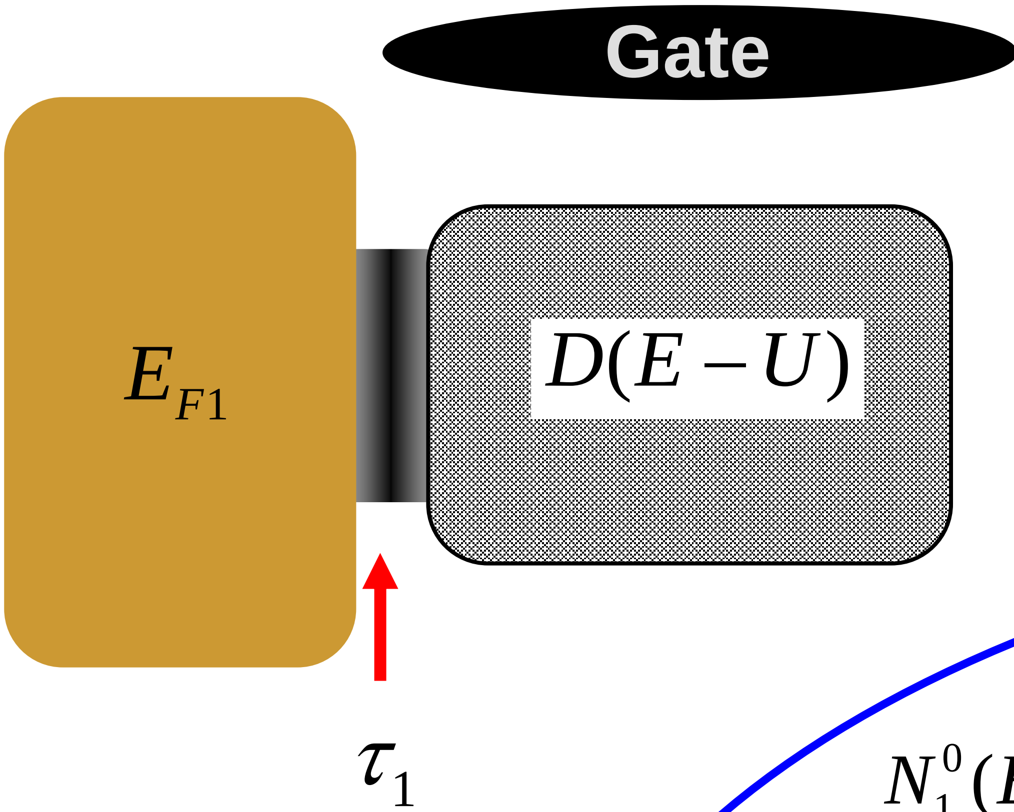
1) Introduction
2) **Generic model of a nanodevice**
3) The ballistic MOSFET
4) Scattering in nano-MOSFETs
5) Discussion
6) Summary

generic model



S. Datta, *Quantum Transport: Atom to Transistor*, Cambridge, 2005
("Concepts of Quantum Transport" **nanohub.org**)

filling states from the left contact



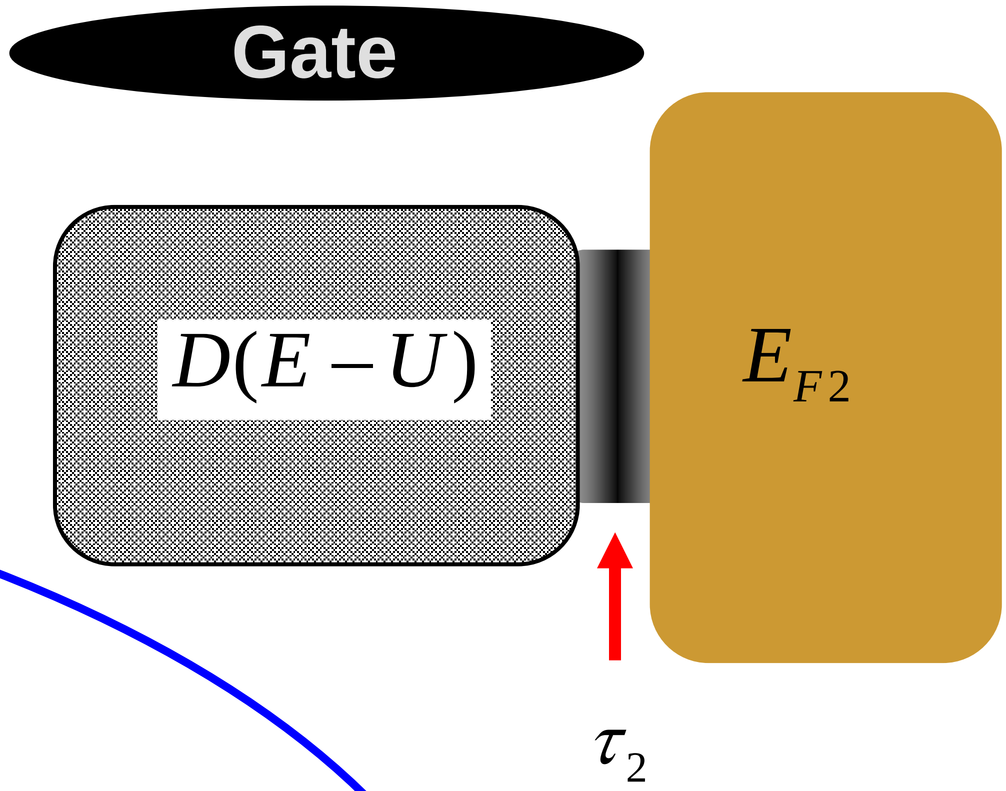
Assumption:
Each energy channel is independent

includes spin

$$N_1^0(E) = D(E-U)\,f_1(E)$$

$$\frac{dN(E)}{dt} = \frac{N_1^0(E) - N}{\tau_1}$$

filling states from the right contact



$$N_2^0(E) = D(E-U)\,f_2(E)$$

$$\frac{dN(E)}{dt} = \frac{N_2^0(E) - N}{\tau_2}$$

steady-state

$$\frac{dN(E)}{dt} = \frac{N_1^0 - N}{\tau_1} + \frac{N_2^0 - N}{\tau_2} = 0$$

$$(1/\tau_1)N_1^0 - (1/\tau_1)N + (1/\tau_2)N_2^0 - (1/\tau_2)N = 0$$

$$\boxed{N(E) = \frac{(1/\tau_1)}{(1/\tau_1)+(1/\tau_2)}N_1^0(E) + \frac{(1/\tau_2)}{(1/\tau_1)+(1/\tau_2)}N_2^0(E)}$$

$$\left\{ \begin{array}{ll} N_1^0(E) \equiv D(E-U)\,f_1(E) & \gamma_1 = h/\tau_1 \\ N_2^0(E) \equiv D(E-U)\,f_2(E) & \gamma_1 = h/\tau_2 \end{array} \right\}$$

steady-state electron number, *N*(*E*)

$$N(E) = \frac{\gamma_1}{\gamma_1 + \gamma_2} D(E-U) f_1(E) + \frac{\gamma_2}{\gamma_1 + \gamma_2} D(E-U) f_2(E)$$

$$N(E) = D_1(E-U) f_1(E) + D_2(E-U) f_2(E)$$

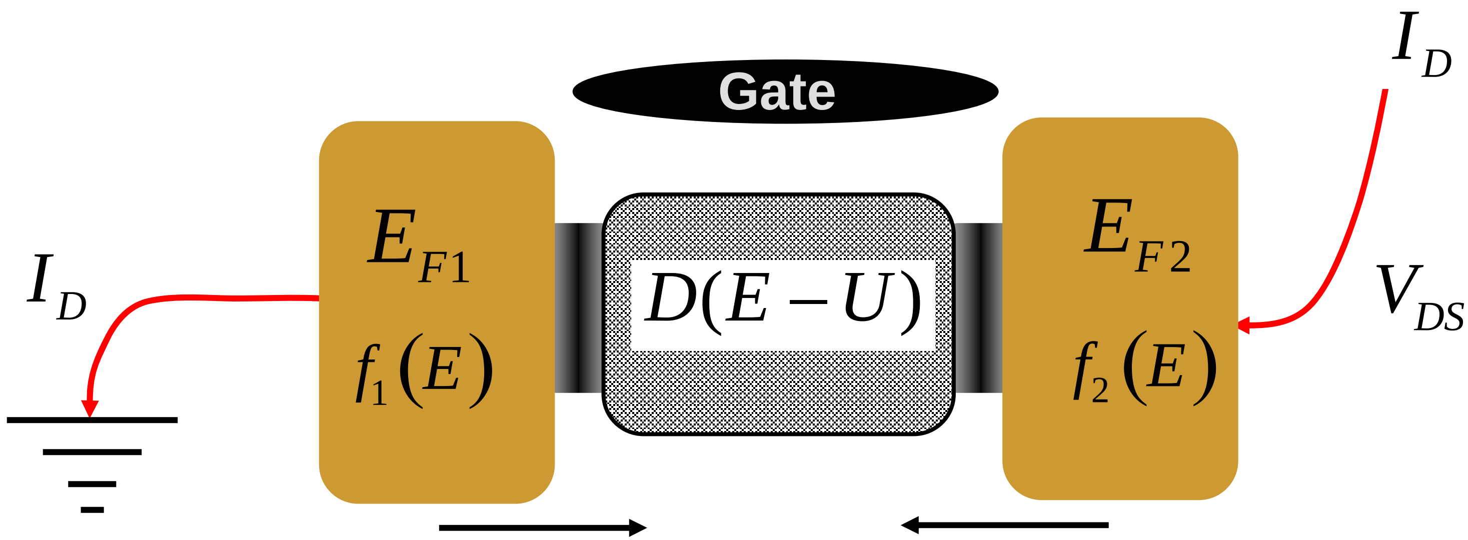
$$D_1(E-U) = \frac{\gamma_1}{\gamma_1 + \gamma_2} D(E-U)$$

DOS that can be filled by contact 1

$$D_2(E-U) = \frac{\gamma_2}{\gamma_1 + \gamma_2} D(E-U)$$

DOS that can be filled by contact 2

steady-state current, *I*



$$\left.\frac{dN(E)}{dt}\right|_1 = \frac{N_1^0(E) - N(E)}{\tau_1} \qquad \left.\frac{dN(E)}{dt}\right|_2 = \frac{N_2^0(E) - N}{\tau_2}$$

$$I_D(E) = +q\left.\frac{dN(E)}{dt}\right|_1 = -q\left.\frac{dN(E)}{dt}\right|_2$$

result

$$I(E) = \frac{q}{\mathrm{h}}\left(\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}\right)D(E-U)\left(f_1 - f_2\right)$$

$$\gamma_1 = \gamma_2 = \gamma$$

$$I_D = \int I(E)dE = \frac{2q}{h}\int\left(\frac{\gamma}{2}\right)\pi D(E-U)\left(f_1 - f_2\right)dE$$

$$N = \int N(E)dE = \int\left[\frac{D(E-U)}{2}\left(f_1(E) + f_2(E)\right)\right]dE$$

final results

$$\gamma_1 = \gamma_2 = \gamma = \hbar/\tau$$

$$D'(E-U) = \frac{D(E-U)}{2}$$

density-of-states per spin

$$I_D = \frac{2q}{h}\int \gamma\,\pi D'(E-U)(f_1 - f_2)\,dE$$

$$N = \int D'(E-U)(f_1 + f_2)\,dE$$

determining τ (γ)

energy channels are independent:

$$I_D(E) = \frac{2q}{h}\gamma\,\pi D'(E-U)\left(f_1 - f_2\right)$$

$$N(E) = D'(E-U)\left[f_1(E) + f_2(E)\right]$$

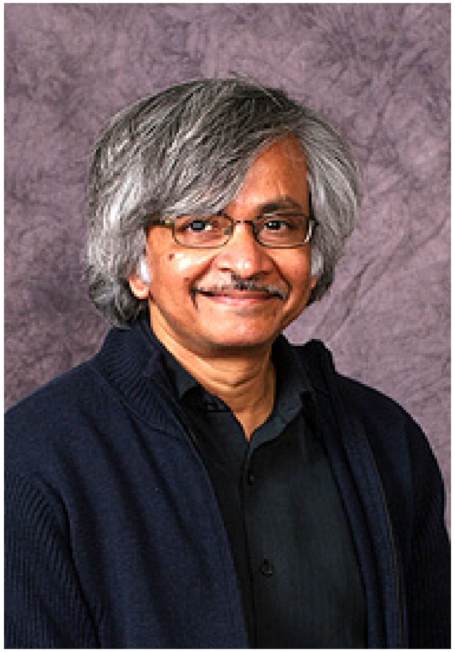
if $f_1 >> f_2$ (source injects, drain collects), then:

$$\frac{qN}{I_D} = \frac{\hbar}{\gamma} = \tau$$

$$I_D = \frac{Q}{\tau} = \frac{\text{stored charge}}{\text{transit time}}$$

"Nanoelectronics and the Meaning of Resistance"

1) What and where is the resistance?
2) Microscopic model for electrical resistance
3) Spins and magnets
4) Energy conversion
5) Beyond the one-electron picture



Supriyo Datta

"Electronics from the Bottom Up" on nanoHUB.org

outline

1) Introduction
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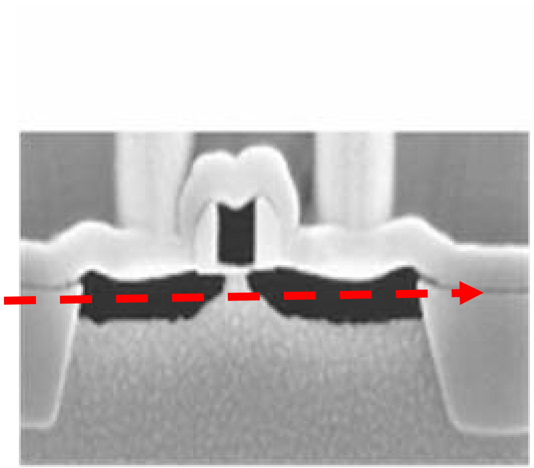
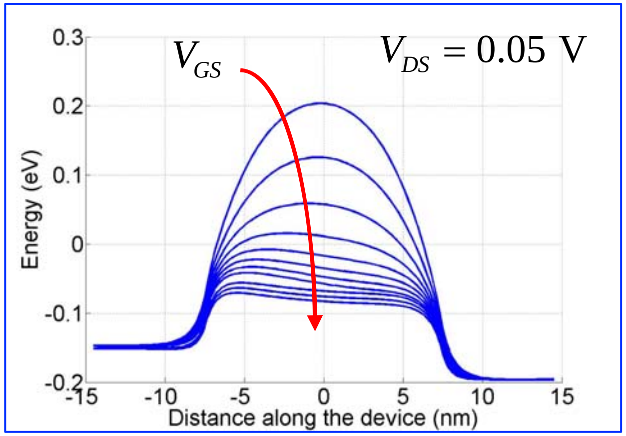
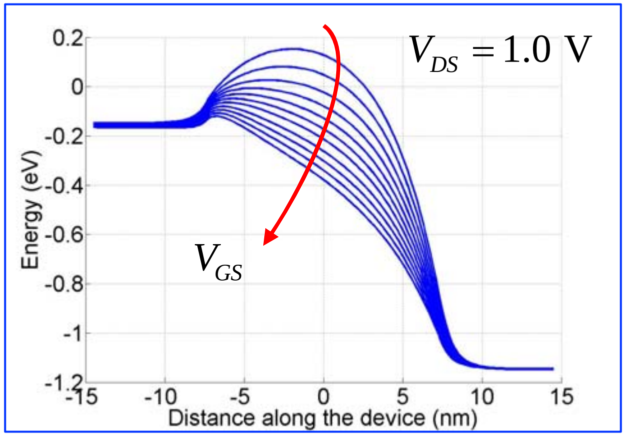
controlling current with energy barriers



$$E = -qV$$

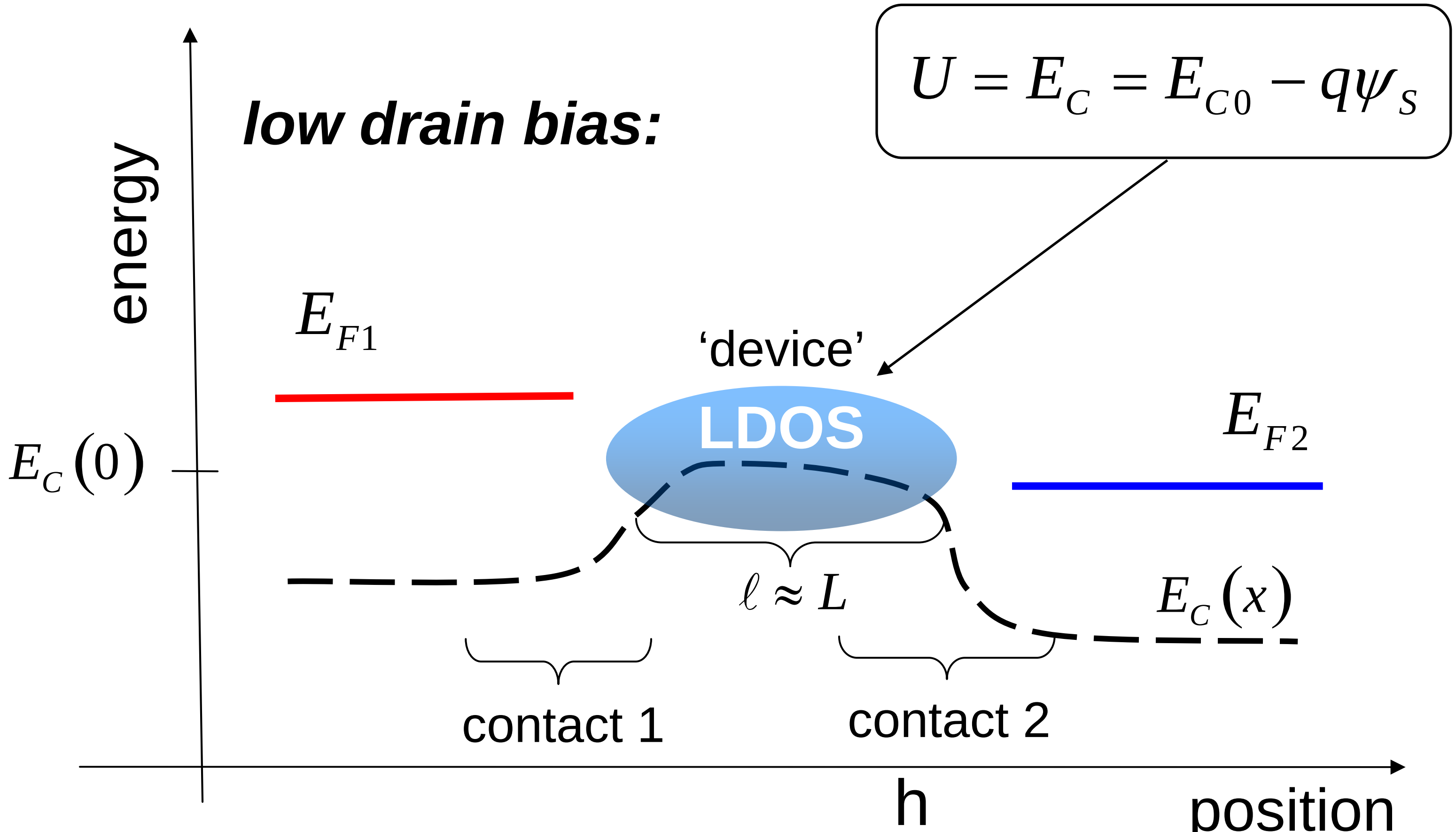
electron energy vs. position





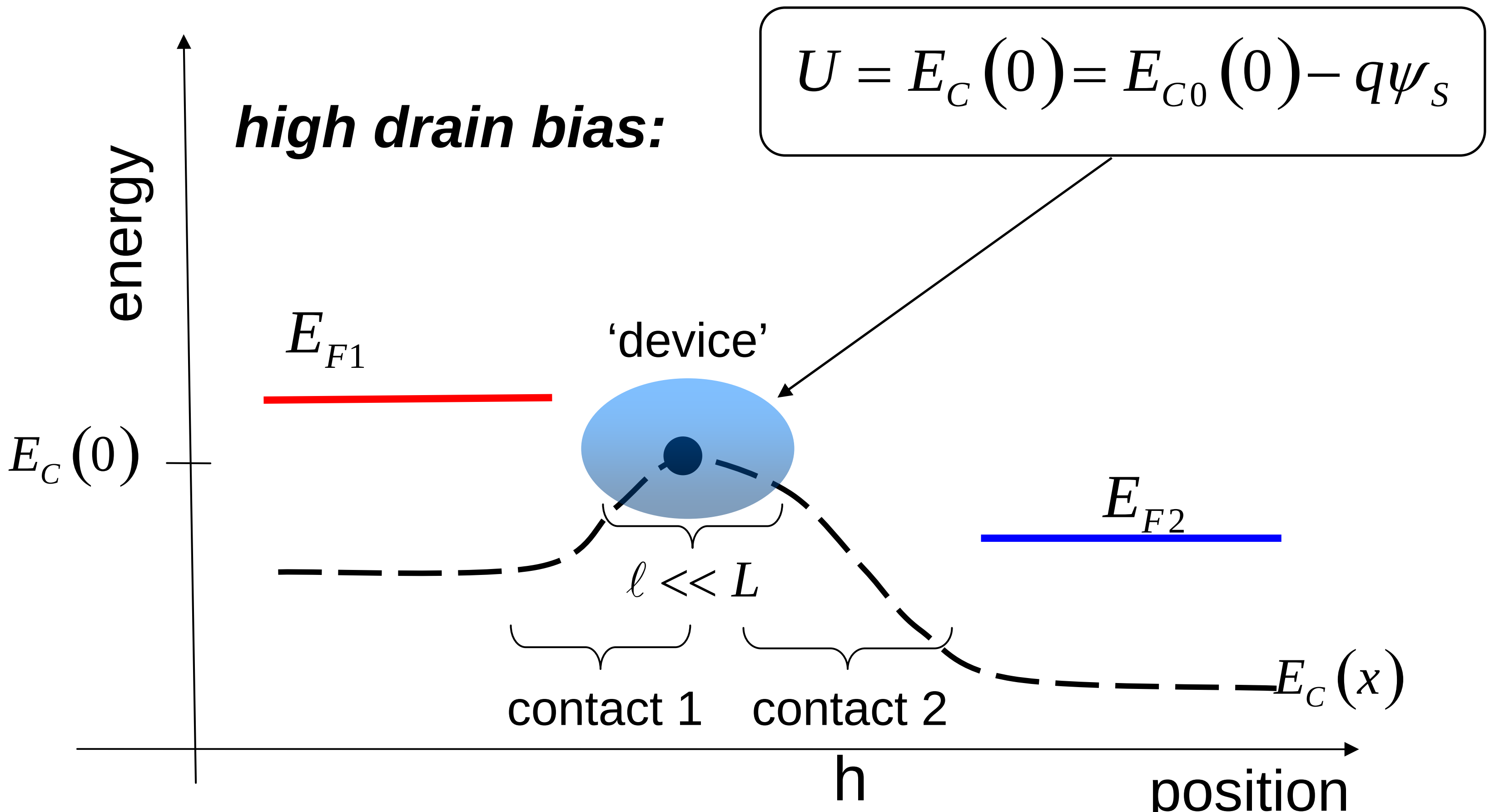
E.O. Johnson, "The Insulated-Gate Field Effect Transistor: A Bipolar Transistor in Disguise," *RCA Review*, **34**, pp. 80-94, 1973.

“top of the barrier” MOSFET model



$$\gamma_1 = \gamma_2 = \frac{\mathrm{h}}{\langle \tau \rangle}$$

“top of the barrier” MOSFET model

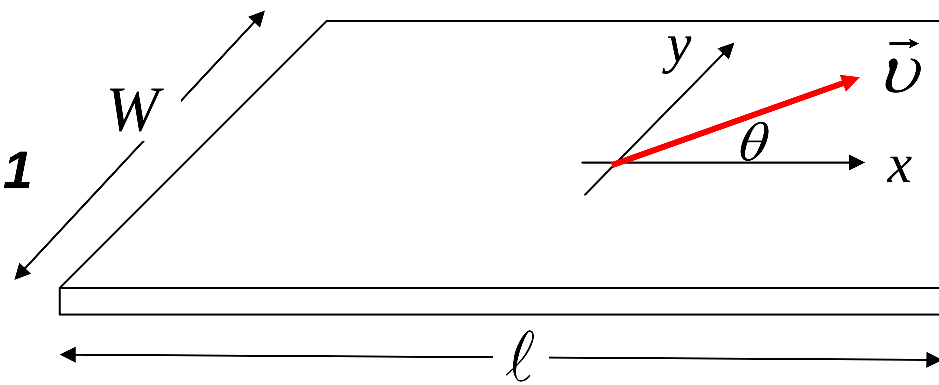


$$\gamma_1 = \gamma_2 = \frac{\hbar}{\langle \tau \rangle}$$

electron density

$$U = E_C(0)$$

contact 1



contact 2

$$N = \int D'(E-U)(f_1 + f_2)dE$$

$$D' = \frac{m^*}{2\pi\hbar^2} W\ell \qquad f_1(E) = \frac{1}{1 + e^{(E_{F1} - E_C(0))/k_B T}} \qquad f_2(E) = \frac{1}{1 + e^{(E_{F1} - qV_{DS} - E_C(0))/k_B T}}$$

electron density

$$N = \int D'(E-U)\left(f_1 + f_2\right)dE$$

$$n_S(0) = \frac{N}{W\mathsf{l}} = \frac{N_{2D}}{2}\left[\mathcal{F}_0(\eta_{F1}) + \mathcal{F}_0(\eta_{F2})\right]$$

$$N_{2D} = m^* k_B T / \pi \hbar^2$$

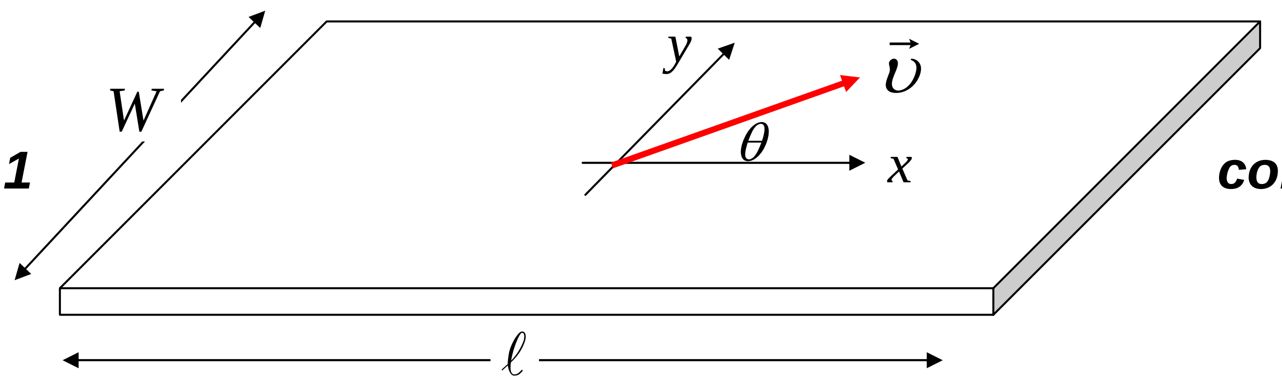
$$\eta_{F1} \equiv \left[E_{F1} - E_C(0)\right] / k_B T$$

$$\eta_{F2} = \eta_{F1} - qV_{DS}/k_B T$$

current

$$U = E_C(0)$$

contact 1



contact 2

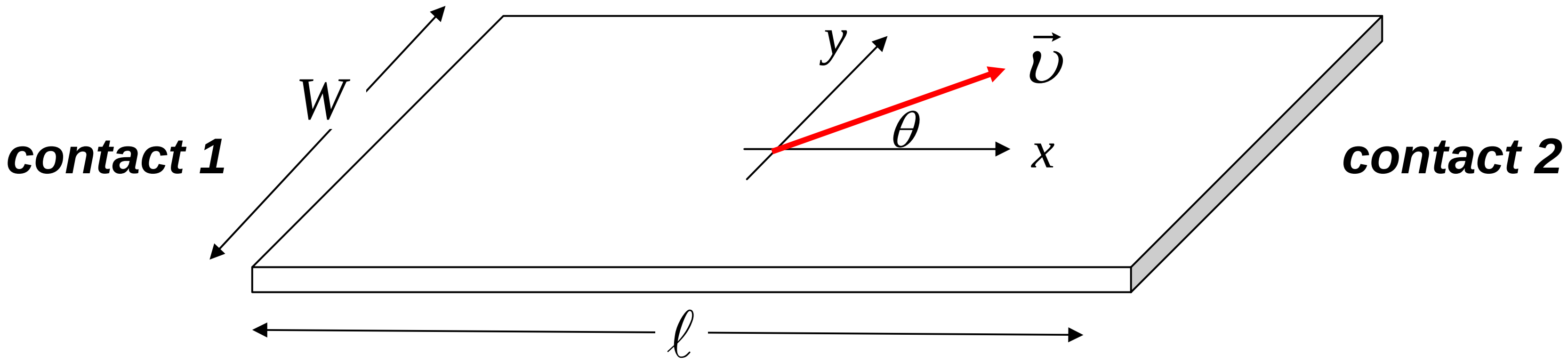
$$I_D = \frac{2q}{h}\int \gamma\, \pi D'\left(f_1 - f_2\right) dE$$

$$\gamma = \frac{\hbar}{\langle \tau \rangle} = \frac{\hbar \langle \upsilon_x \rangle}{\ell}$$

$$D' = \frac{m^*}{2\pi\hbar^2} W \ell$$

current (cont.)

$$U = E_C(0)$$



$$\gamma = \frac{\hbar}{\langle \tau \rangle} = \frac{\hbar \langle \upsilon_x \rangle}{l}$$

$$\upsilon_x = \upsilon \cos\theta = \sqrt{2(E - E_C)/m^*}\cos\theta$$

$$\langle \upsilon_x \rangle = \int_{-\pi/2}^{\pi/2} \upsilon \cos\theta \, d\theta = \upsilon \frac{2}{\pi} = \sqrt{\frac{2(E - E_C)}{m^*}}\frac{2}{\pi}$$

current (cont.)

$$I_D = \frac{2q}{h}\int \gamma\,\pi D'\left(f_1 - f_2\right)dE$$

$$\gamma\,\pi D'\left(E\right) = \frac{\mathrm{h}}{\mathrm{l}}\sqrt{\frac{2\left(E - E_C\right)}{m^*}}\,\frac{2}{\pi}\times\pi\times\frac{m^*}{2\pi\mathrm{h}^2}W\mathrm{l}$$

$$\gamma\,\pi D'\left(E\right) = W\,\frac{\sqrt{2m^*\left(E - E_C\right)}}{\pi\mathrm{h}} = M\left(E\right)$$

$$\left(M\left(E\right) = \frac{W}{\left(\lambda/2\right)}\right)$$

$$I_D = \frac{2q}{h}\int M\left(E\right)\left(f_1 - f_2\right)dE$$

current (final result)

$$I_D = \frac{2q}{h}\int M(E)\left(f_1 - f_2\right)dE$$

$$I_D = Wq\left(\frac{N_{2D}}{2}\upsilon_T\right)\left[\mathcal{F}_{1/2}(\eta_{F1}) - \mathcal{F}_{1/2}(\eta_{F2})\right]$$

$$N_{2D} = m^* k_B T / \pi\hbar^2 \qquad \eta_{F1} \equiv \left[E_{F1} - E_C(0)\right]/k_B T$$

$$\upsilon_T = \sqrt{2k_B T/\pi m^*} \qquad \eta_{F2} = \eta_{F1} - qV_{DS}/k_B T$$

re-cap

$$I_D = Wq\left(\frac{N_{2D}}{2}\upsilon_T\right)\left[\mathcal{F}_{1/2}\left(\eta_{F1}\right) - \mathcal{F}_{1/2}\left(\eta_{F2}\right)\right] \qquad (1)$$

$$n_S\left(0\right) = \frac{N_{2D}}{2}\left[\mathcal{F}_0\left(\eta_{F1}\right) + \mathcal{F}_0\left(\eta_{F2}\right)\right] \qquad (2)$$

Solve (2) for N_{2D}, then insert in (1):

I-V characteristic

$$I_D = Wqn_S(0)\upsilon_T \left[\frac{\mathcal{F}_{1/2}(\eta_{F1}) - \mathcal{F}_{1/2}(\eta_{F2})}{\mathcal{F}_0(\eta_{F1}) + \mathcal{F}_0(\eta_{F2})} \right]$$

$$I_D = WQ_I(0)\left\{ \upsilon_T \frac{\mathcal{F}_{1/2}(\eta_{F1})}{\mathcal{F}_0(\eta_{F1})} \right\} \left[\frac{1 - \mathcal{F}_{1/2}(\eta_{F2}) / \mathcal{F}_{1/2}(\eta_{F1})}{1 + \mathcal{F}_0(\eta_{F2}) / \mathcal{F}_0(\eta_{F1})} \right]$$

$$qn_S(0) \approx C_{ox}(V_{GS} - V_T)$$ (simple, 1D MOS electrostatics $V_{GS} > V_T$)

final result

$$I_D = W C_{ox} \left(V_{GS} - V_T \right) \tilde{\upsilon}_T \left[\frac{1 - \mathcal{F}_{1/2}\left(\eta_{F2}\right) / \mathcal{F}_{1/2}\left(\eta_{F1}\right)}{1 + \mathcal{F}_{0}\left(\eta_{F2}\right) / \mathcal{F}_{10}\left(\eta_{F1}\right)} \right]$$

$$\tilde{\upsilon}_T \equiv \sqrt{\frac{2 k_B T}{\pi m^*}} \frac{\mathcal{F}_{1/2}\left(\eta_{F1}\right)}{\mathcal{F}_{0}\left(\eta_{F1}\right)}$$

Boltzmann limit

$$\mathcal{F}_j(\eta_F) \rightarrow e^{\eta_F}$$

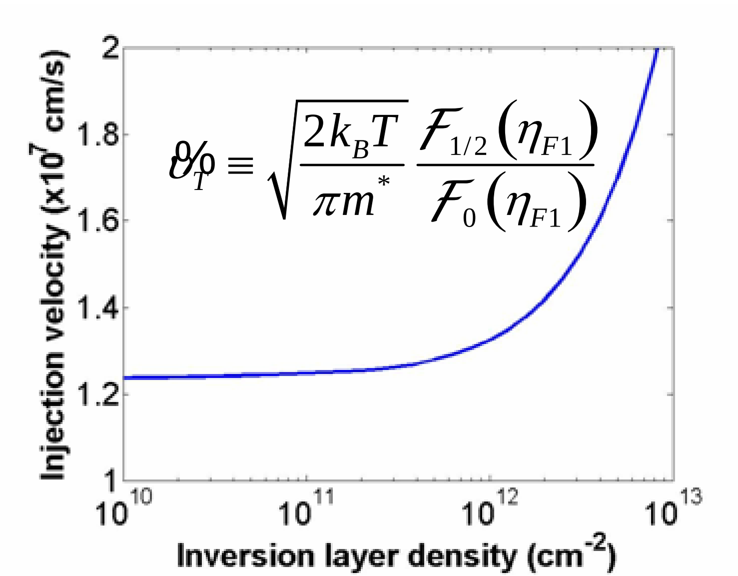
$$I_D = WC_{ox}(V_{GS} - V_T)\upsilon_T \left[\frac{1 - e^{-qV_{DS}/k_BT}}{1 + e^{-qV_{DS}/k_BT}} \right]$$

$$\upsilon_T = \sqrt{\frac{2k_BT}{\pi m^*}}$$

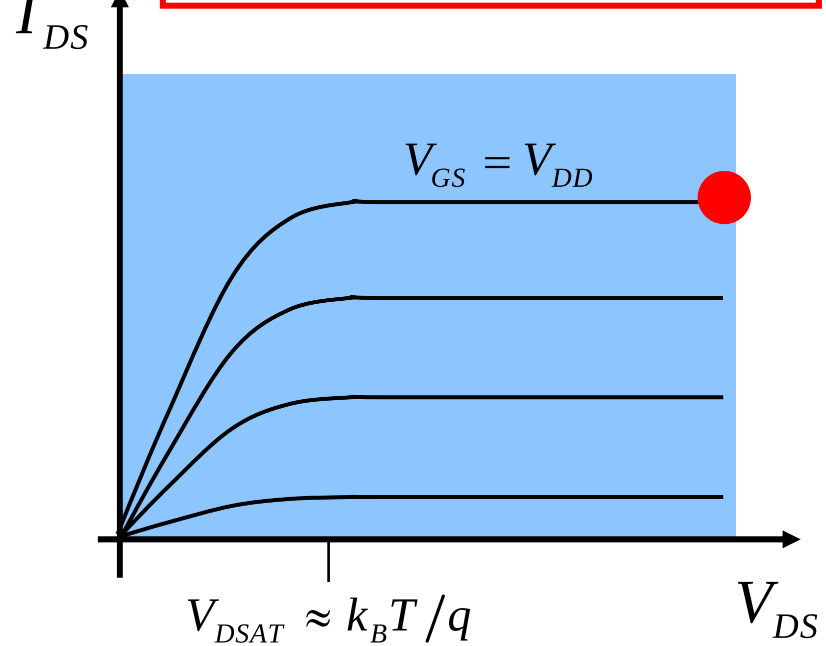
on-current

$$I_D = WC_{ox}\tilde{\upsilon}_T\left(V_{GS} - V_T\right)\left[\frac{1 - \mathcal{F}_{1/2}(\eta_{F2})/\mathcal{F}_{1/2}(\eta_{F1})}{1 + \mathcal{F}_{0}(\eta_{F2})/\mathcal{F}_{0}(\eta_{F1})}\right]$$

$$I_D \rightarrow WC_{ox}\tilde{\upsilon}_T\left(V_{DD} - V_T\right)$$

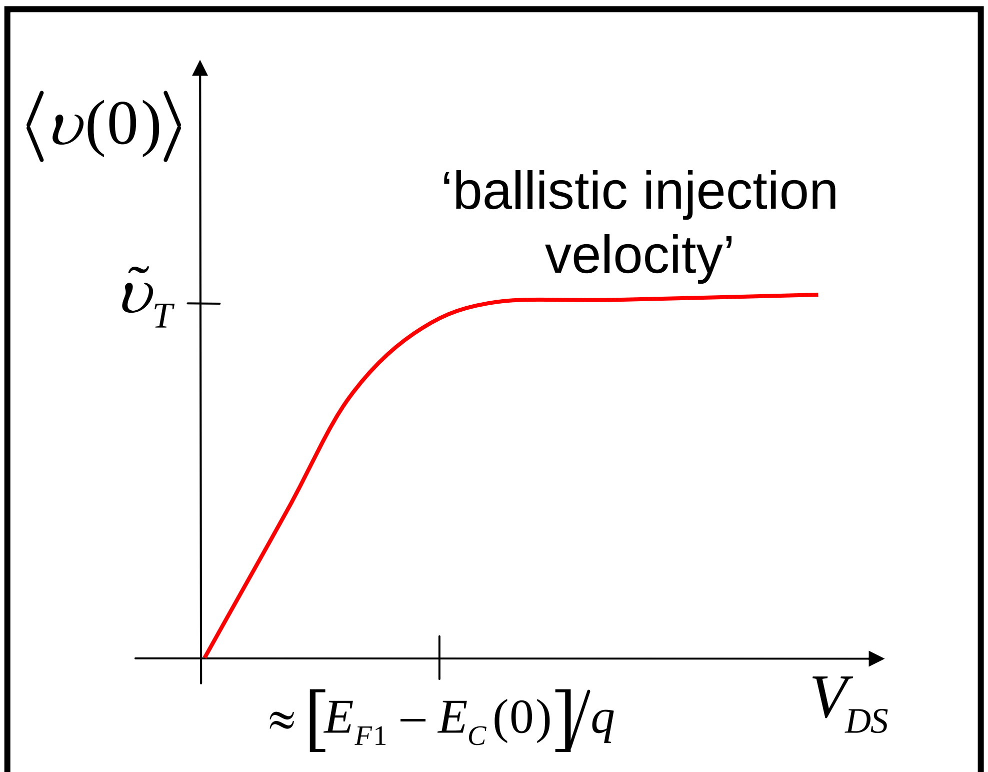


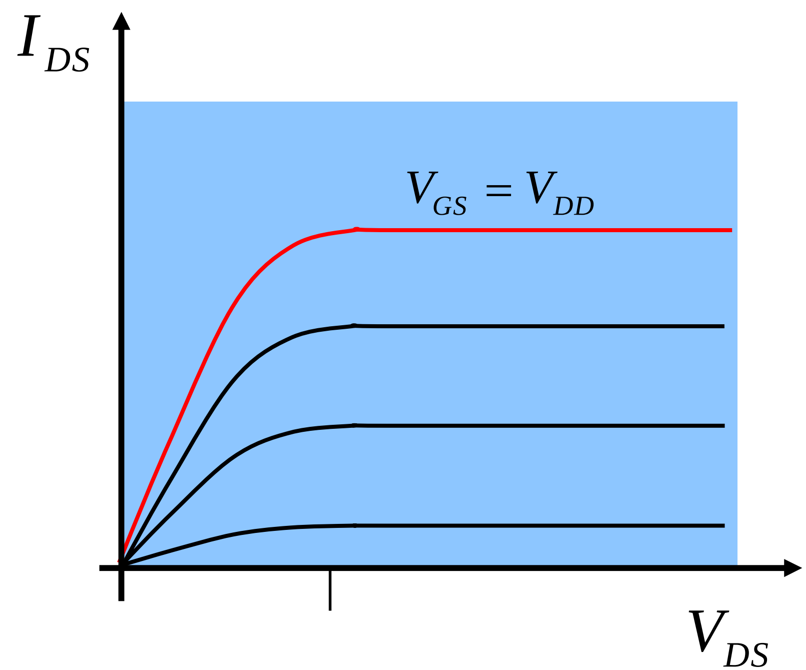
(100) [110] Si (single subband)



velocity saturation in a ballistic MOSFET

$$I_D = WC_{ox}\upsilon_T\left(V_{GS} - V_T\right)\left[\frac{1 - \mathcal{F}_{1/2}\left(\eta_{F2}\right)/\mathcal{F}_{1/2}\left(\eta_{F1}\right)}{1 + \mathcal{F}_0\left(\eta_{F2}\right)/\mathcal{F}_{10}\left(\eta_{F1}\right)}\right] = WQ_I(0)\langle\upsilon(0)\rangle$$

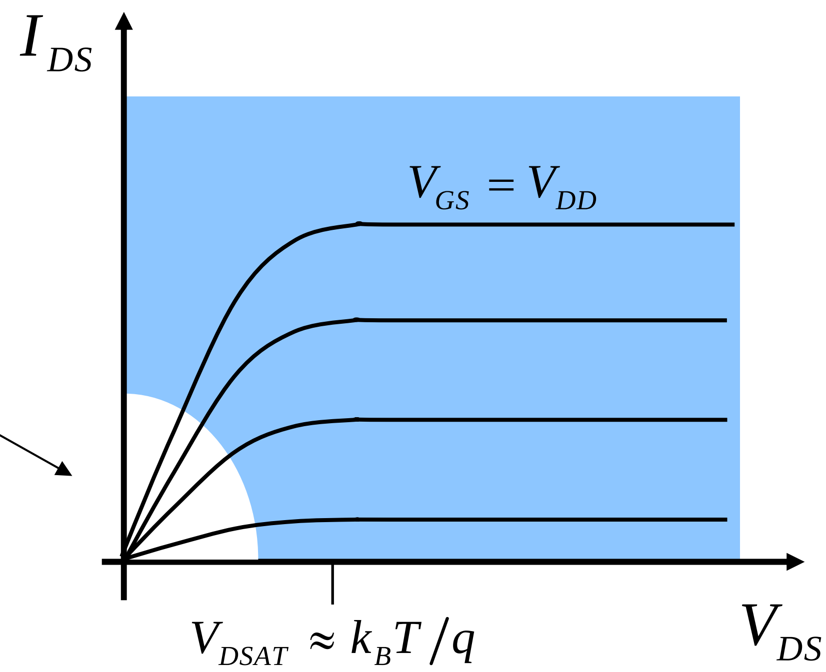




channel resistance of a ballistic MOSFET

$$I_D = WC_{ox}\upsilon_T\left(V_{GS} - V_T\right)\left[\frac{1 - \mathcal{F}_{1/2}\left(\eta_{F2}\right)/\mathcal{F}_{1/2}\left(\eta_{F1}\right)}{1 + \mathcal{F}_{0}\left(\eta_{F2}\right)/\mathcal{F}_{10}\left(\eta_{F1}\right)}\right]$$

finite channel resistance



as $T \rightarrow 0$

$$G_{CH} = \frac{1}{R_{CH}} \rightarrow M\left(E_{F1}\right)\frac{2q^2}{h}$$

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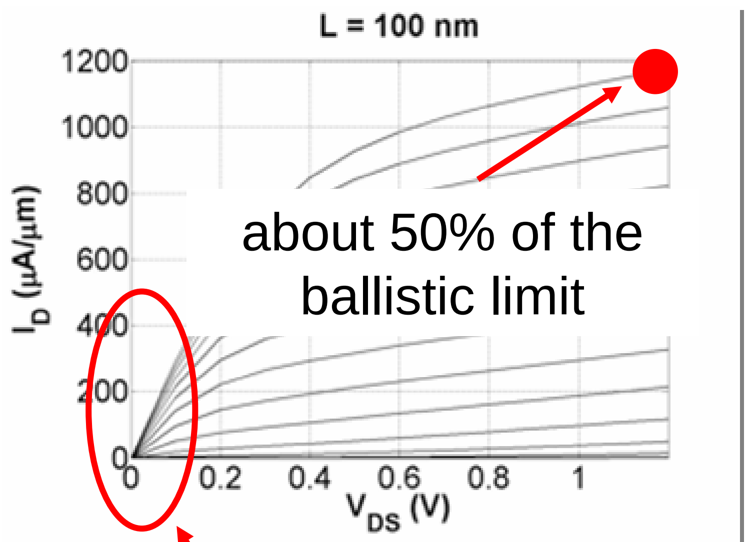
is a nanoscale MOSFET really ballistic?

Typical N-channel MOSFET:

$$I_{ON} \approx 1\ \text{mA}/\mu\text{m}$$



(Courtesy, Shuji Ikeda, ATDF, Dec. 2007)

$$I_{ON}(\text{ballistic}) = -WQ_I(0)\upsilon_T$$

$$-Q_I(0)/q = C_{inv}(V_{DD} - V_T)$$

$$\approx 0.8 \times 10^{13}\ \text{cm}^{-2}$$

$$\upsilon_T \approx 1.8 \times 10^{7}\ \text{cm/s}$$

$$I_{ON}/W(\text{ballistic}) \approx 2\ \text{mA}/\mu\text{m}$$

about 10% of the ballistic limit.

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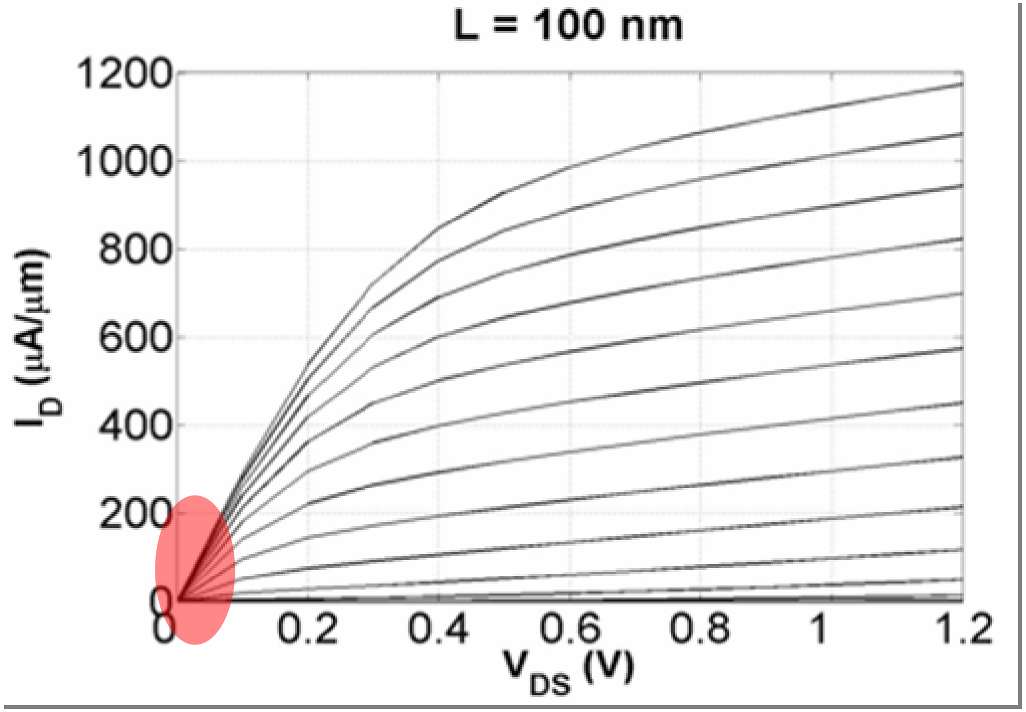
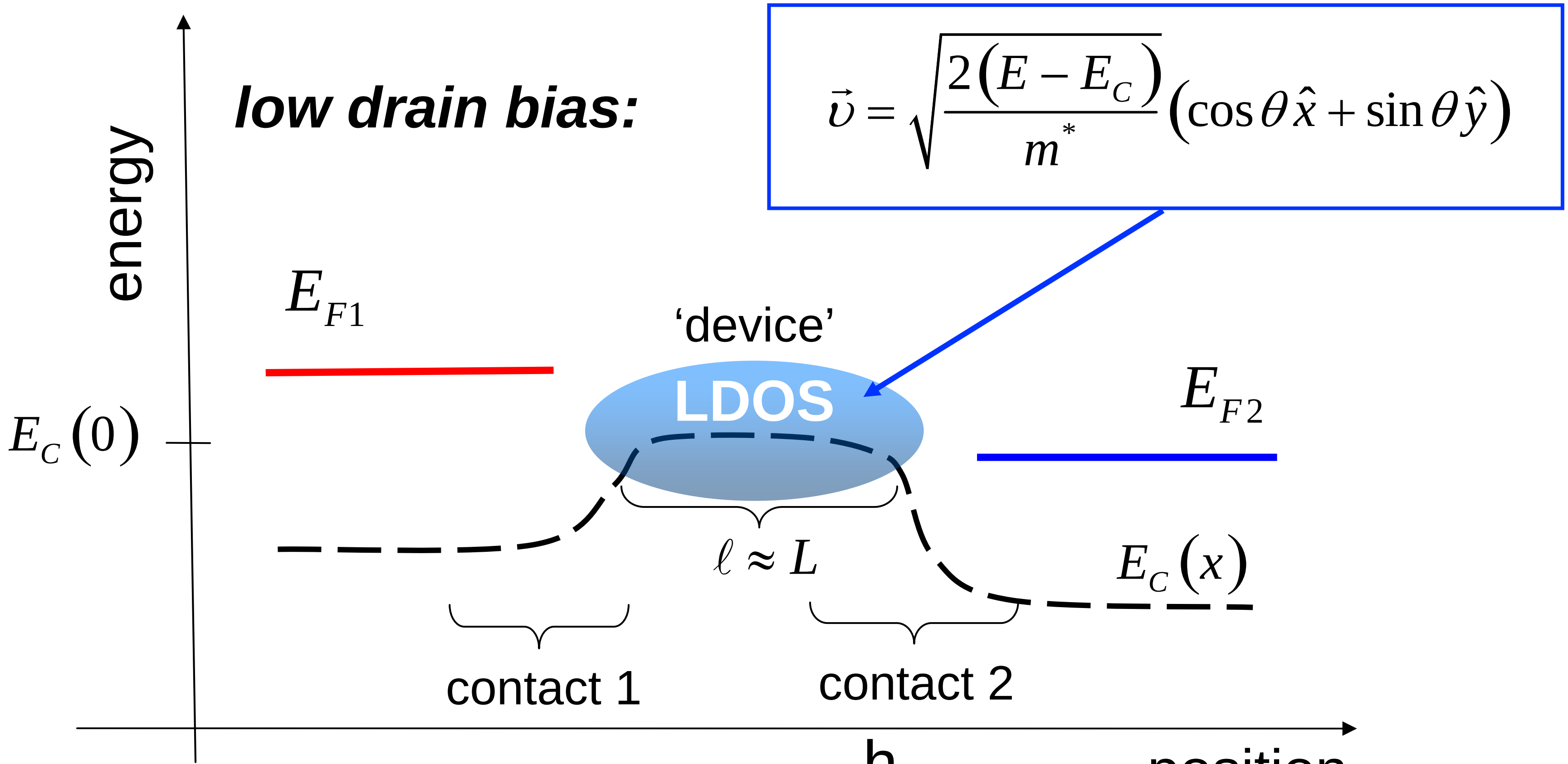
scattering in Si n-channel nano-MOSFETs



$$I_D = \frac{W}{L}\mu_n C_{ox}\left(V_{GS} - V_T\right)V_{DS} \rightarrow ?$$

“top of the barrier” MOSFET model

(ballistic)



$$\gamma_1 = \gamma_2 = \frac{\hbar}{\langle\tau\rangle}$$

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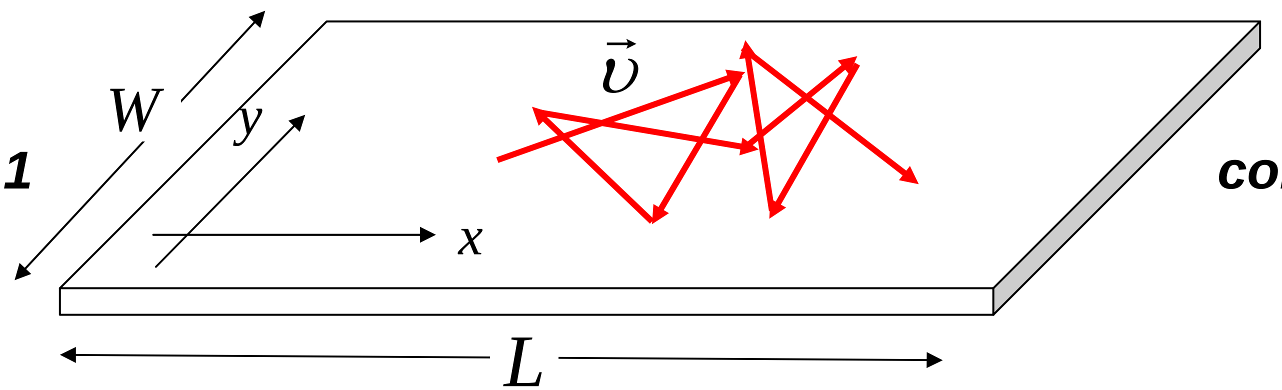
diffusive current

$$U = E_C(0)$$

contact 1



contact 2

$$I_D = \frac{2q}{h}\int \gamma\, \pi D'\left(f_1 - f_2\right) dE$$

$$\gamma = \frac{\hbar}{\langle\tau\rangle} = \frac{\hbar\langle\upsilon_x\rangle}{L} \quad \text{ballistic}$$

$$\gamma = \frac{\hbar}{\langle\tau\rangle} = ? \quad \text{diffusive}$$

$$\langle\tau\rangle = \frac{L^2}{2D_n}$$

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between ballistic and diffusive

$$\gamma = \frac{h}{\tau_B + \tau_D} = \left(\frac{1}{1 + \tau_D / \tau_B}\right) \frac{h}{\tau_B}$$

$$D_n(E) = \frac{\langle \upsilon_x \rangle}{2} \lambda_0 = \frac{\upsilon}{2} \left(\frac{2}{\pi}\right) \lambda_0$$

$$T = \frac{\lambda_0}{\lambda_0 + L}$$

$$\gamma \, \pi D' = \frac{\lambda_0}{\lambda_0 + L} \times \gamma_B \, \pi D'$$

$$I_D = W C_{ox} \left(V_{GS} - V_T\right) \left(\frac{\lambda_0}{\lambda_0 + L}\right) \upsilon_T \left[\frac{1 - \mathcal{F}_{1/2}(\eta_{F2}) / \mathcal{F}_{1/2}(\eta_{F1})}{1 + \mathcal{F}_0(\eta_{F2}) / \mathcal{F}_{10}(\eta_{F1})}\right]$$

linear-region current

Boltzmann statistics:

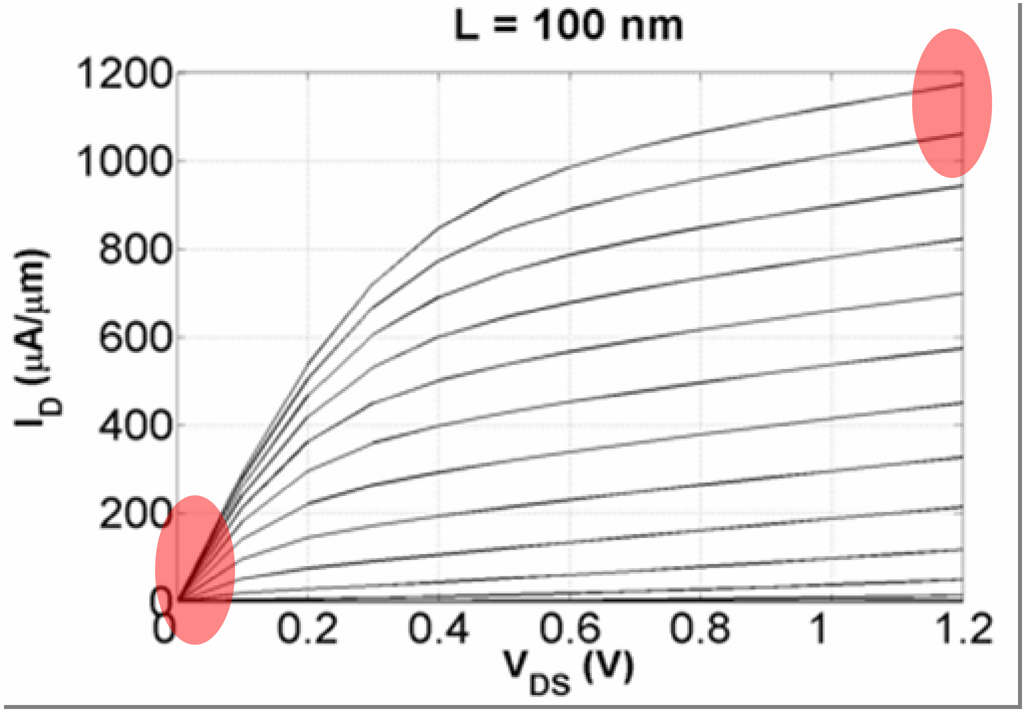
$$I_D = WC_{ox}\left(V_{GS} - V_T\right)\left(\frac{\lambda_0}{\lambda_0 + L}\right)\upsilon_T\left[\frac{1 - e^{-qV_{DS}/k_BT}}{1 + e^{qV_{DS}/k_BT}}\right]$$

Low V_{DS}:

$$I_D = WC_{ox}\left(V_{GS} - V_T\right)\left(\frac{\lambda_0}{\lambda_0 + L}\right)\frac{\upsilon_T}{2\left(k_BT/q\right)}V_{DS}$$

$$I_D = \frac{W}{L + \lambda_0}\mu_n C_{ox}\left(V_{GS} - V_T\right)V_{DS}$$

scattering in Si n-channel nano-MOSFETs

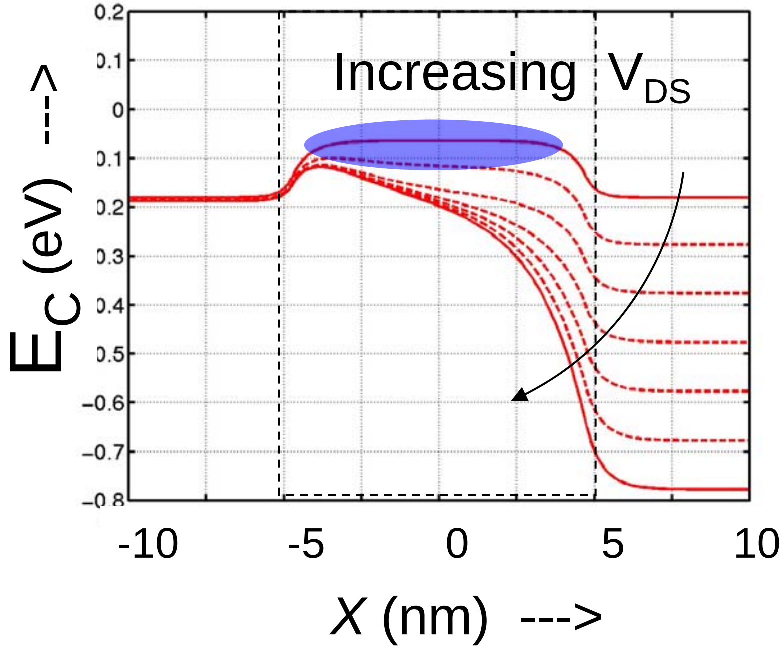


on-current

$$I_D = \frac{W}{L}\mu_n C_{ox}\left(V_{GS} - V_T\right)V_{DS} \rightarrow I_D = \frac{W}{L + \lambda_0}\mu_n C_{ox}\left(V_{GS} - V_T\right)V_{DS}$$

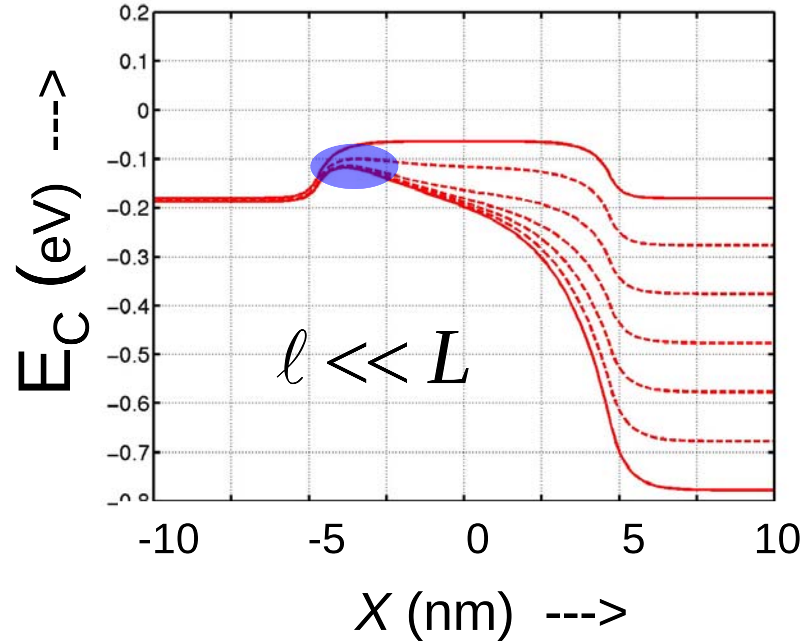
where scattering matters (the most)

E_C vs. x for V_{GS} = 0.5V



$$T = \frac{\lambda_0}{\lambda_0 + L}$$

E_C vs. x for V_{GS} = 0.5V



$$T = \frac{\lambda_0}{\lambda_0 + \ell}$$

outline

1) Introduction
2) Generic model of a nanodevice
3) The ballistic MOSFET
4) Scattering in nano-MOSFETs
5) **Discussion**
6) Summary

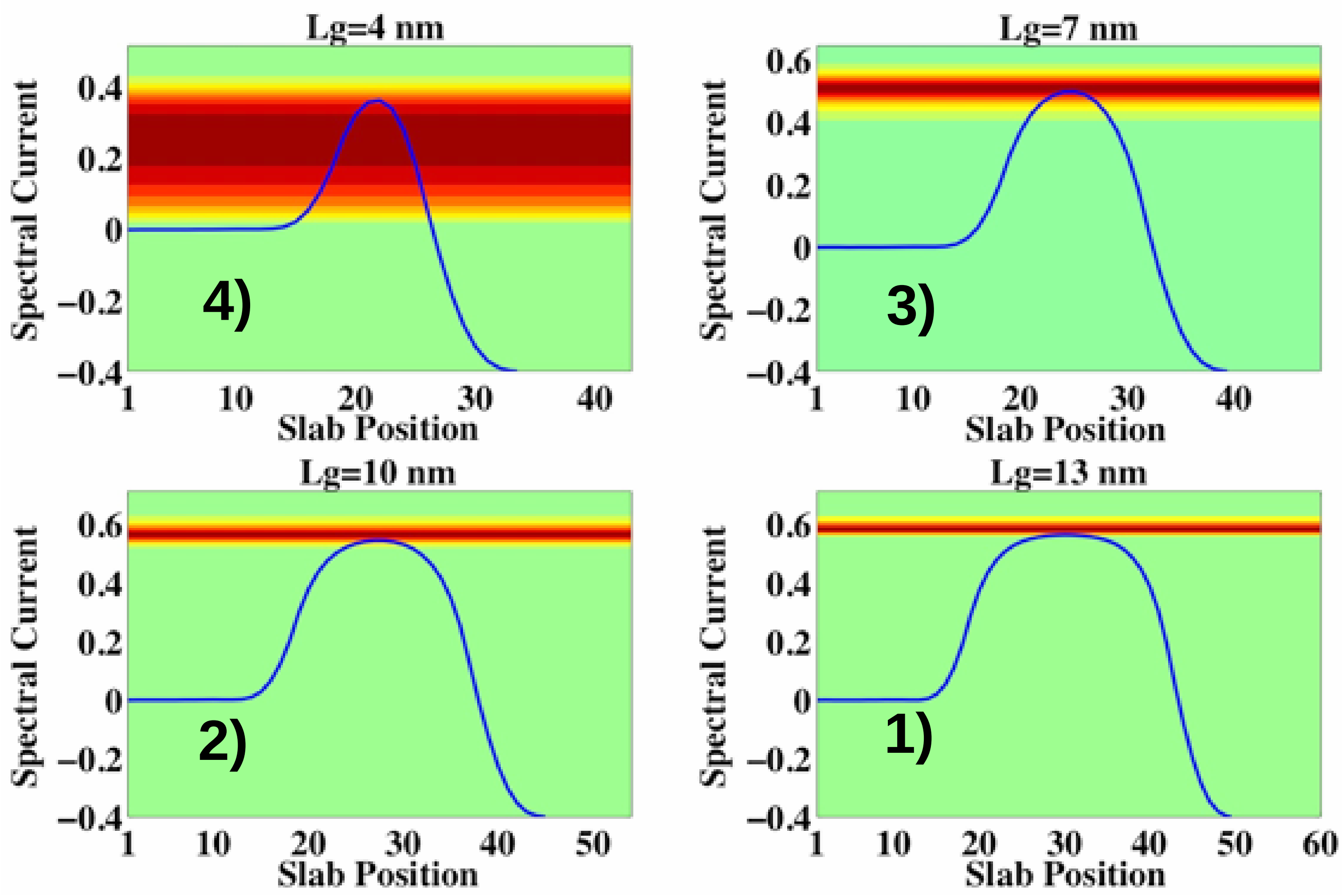
“Physics of Nanoscale Transistors”

1) Review of MOSFET Fundamentals
2) Elementary Theory of the Nanoscale MOSFET
3) Theory of the Ballistic MOSFET
4) Scattering in Nanoscale MOSFETs
5) Application to State-of-the-Art MOSFETs
6) Quantum Transport in Nanoscale MOSFETs
7) Connection to the Bottom Up Approach

Mark Lundstrom

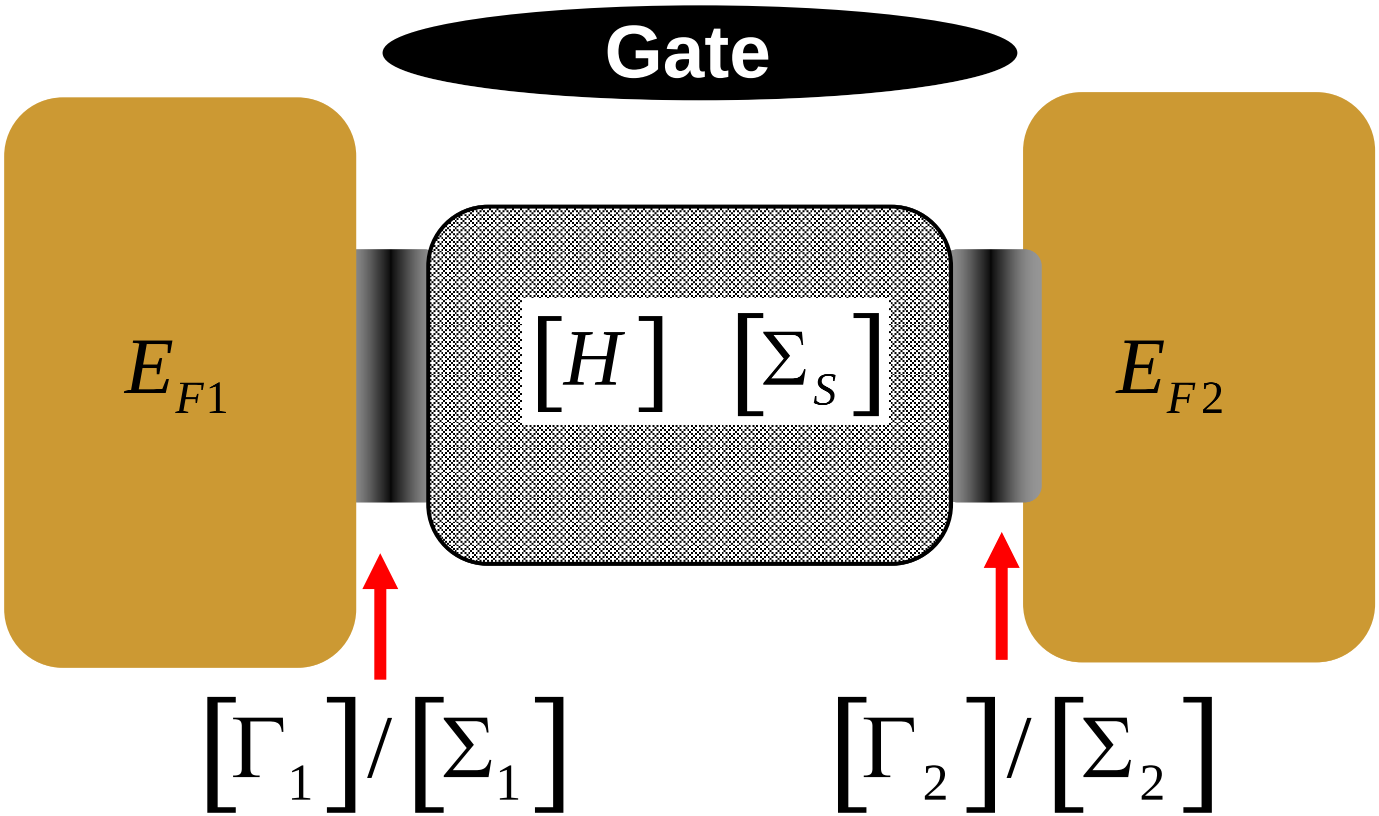
“Electronics from the Bottom Up” on nanoHUB.org

S/D quantum mechanical tunneling



from M. Luisier, ETH Zurich

generic model to NEGF

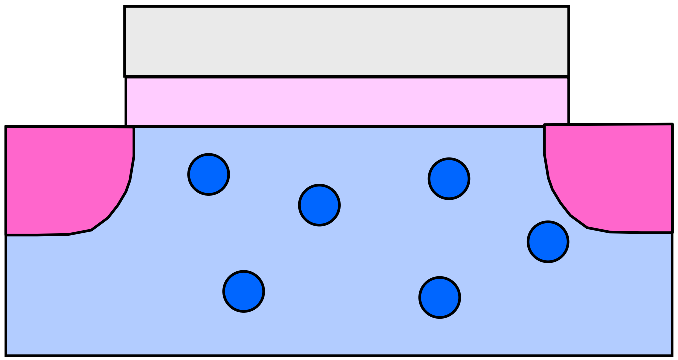


S. Datta, *Quantum Transport: Atom to Transistor*, Cambridge, 2005
("Concepts of Quantum Transport" **nanohub.org**)

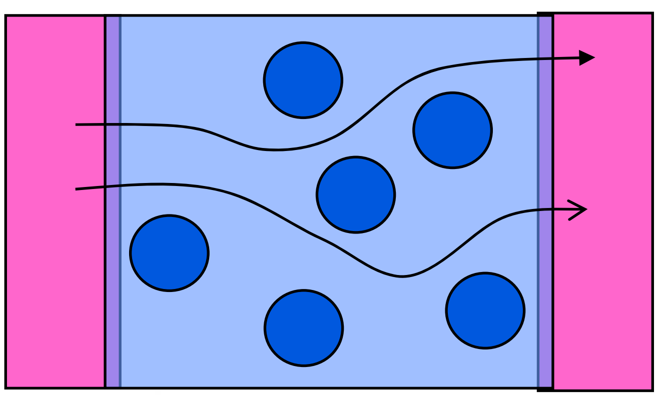
randomness is the rule - not the exception!

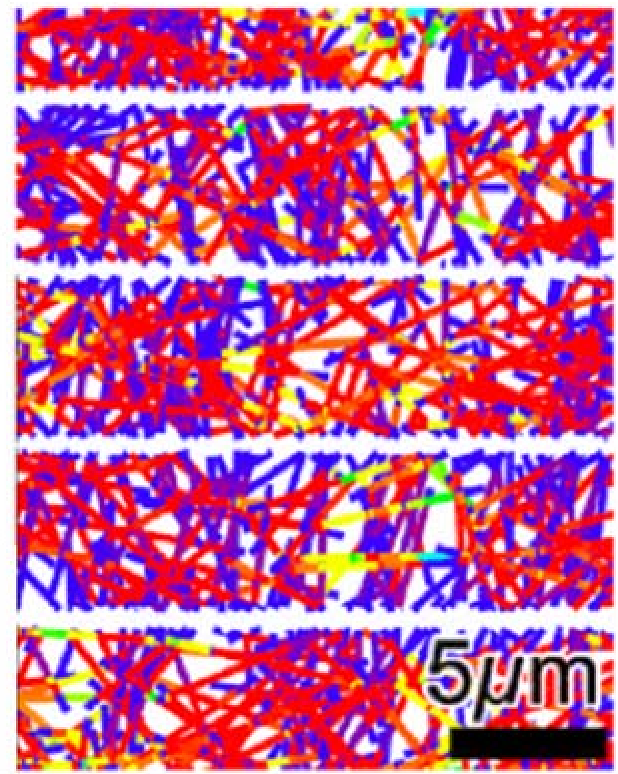
Random dopant fluctuations

side view



top view





nanonets

“Percolation in Electronic Devices”

1) Percolation in Electronic Devices

2) Thresholds, Islands, and Fractals

3) Nonlinear Electrical Conduction in Percolative Systems

4) Stick Percolation and Nanonet Electronics

5) 2D Nets in 3D World: Sensors, Solar Cells, and Antennas

M. Ashraf Alam

“Electronics from the Bottom Up” on nanoHUB.org

outline

1) Introduction
2) Generic model of a nanodevice
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6) **Summary**

summary

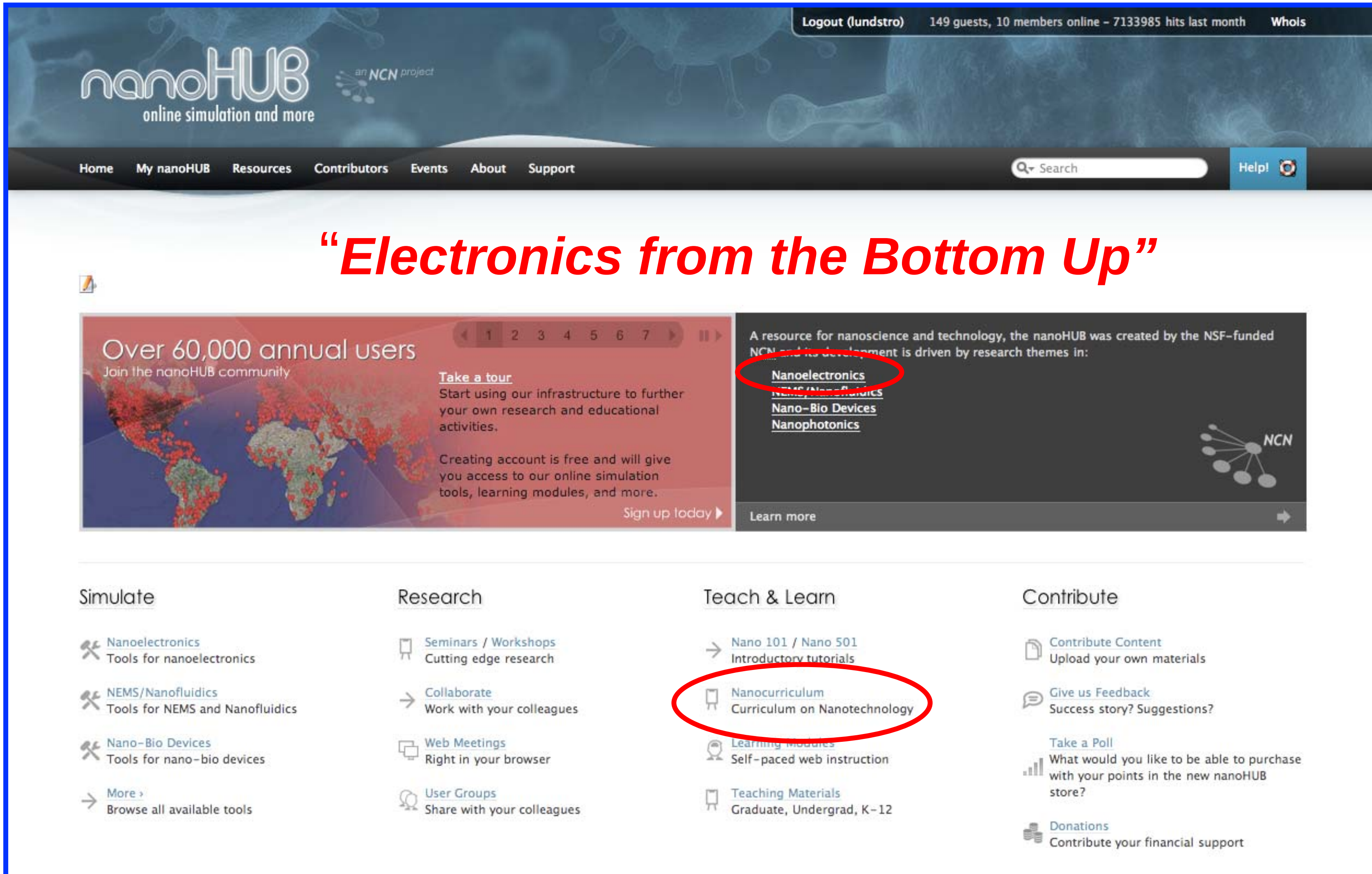
1) The bottom-up view provides a simple, but rigorous approach to nanoelectronics.

2) It's useful for familiar devices, like MOSFETs.

3) It's also a good starting point for new devices.

4) You can learn more on nanoHUB.org or by attending the annual "Electronics from the Bottom Up" summer schools at Purdue University.

nanoHUB.org



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