EE-612:
Lecture 2
1D MOS Electrostatics: II

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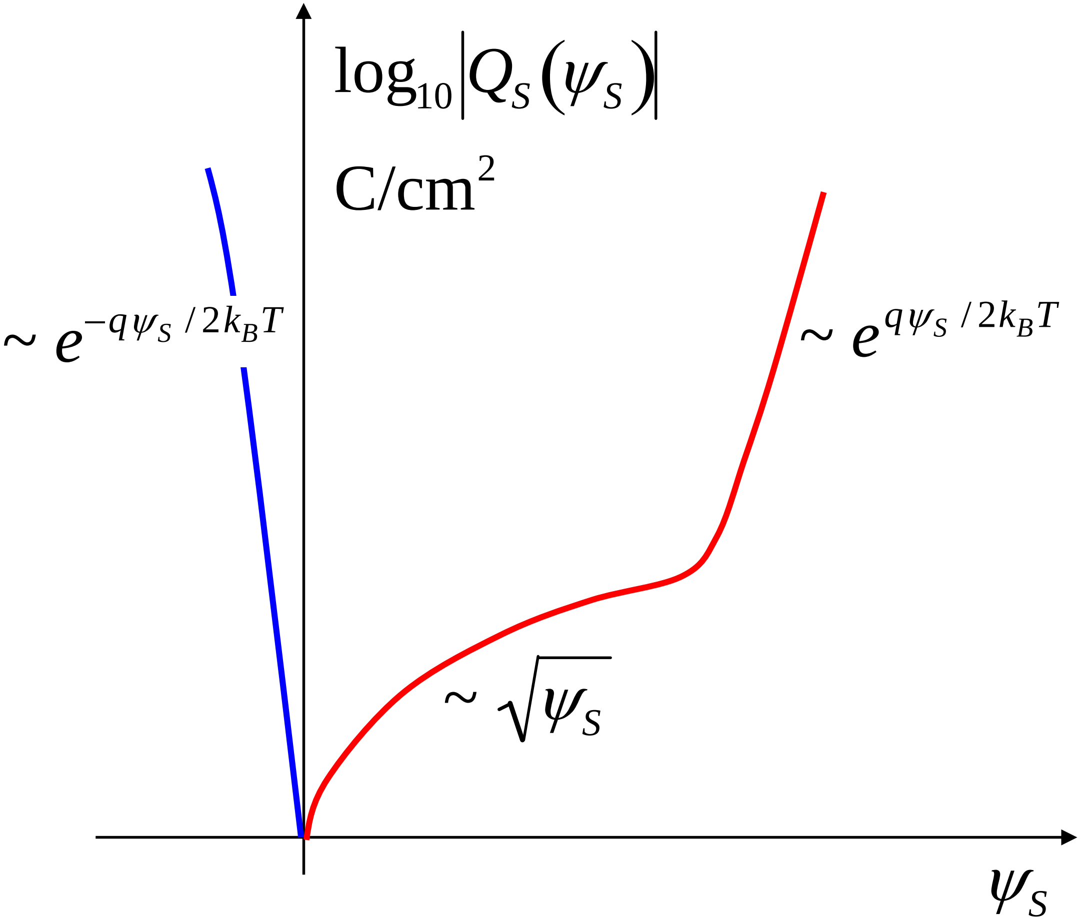
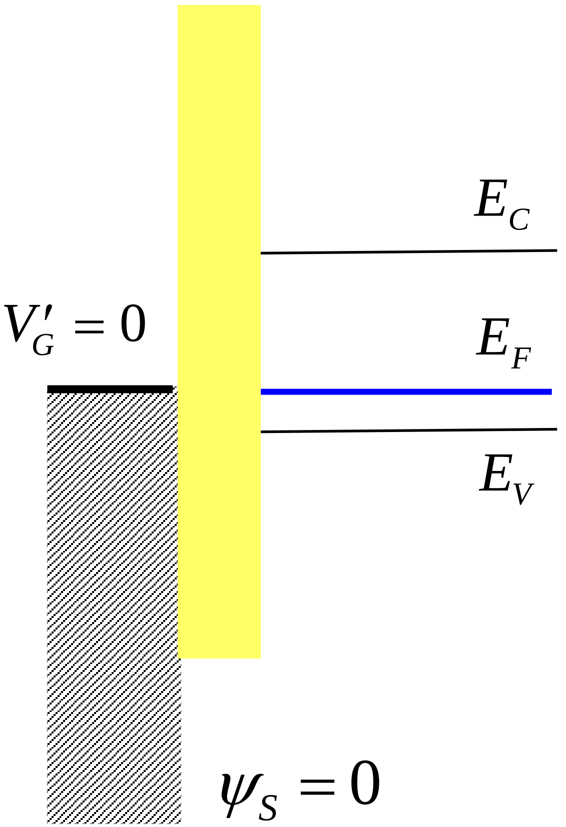
outline

1) **Review**
2) 'Exact' solution (bulk)
3) Approximate solution (bulk)
4) Approximate solution (ultra-thin body)
5) Summary

MOS electrostatics





space charge and sheet charge density

$$\rho(\vec{r}) = q\left[p(\vec{r}) - n(\vec{r}) + N_D^+(\vec{r}) - N_A^-(\vec{r})\right] \quad \text{C/cm}^3$$

$$\nabla \bullet \vec{D} = \rho(\vec{r})$$

$$Q_T = \iiint \rho(\vec{r})\,dx\,dy\,dz \ \text{ C}$$

$$Q_T = \iiint \rho(x)\,dx\,dy\,dz \ \text{ C}$$ (uniform in *y*-*z* plane)

$\rho(\vec{r})$	C/cm^3
Q_S	C/cm^2

$$Q_T = \int \rho(x)\,dx\, A \quad \text{C}$$

$$Q_S = \frac{Q_T}{A} = \int \rho(x)\,dx \quad \text{C/cm}^2$$

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‘exact’ solution of $Q_S(\psi_S)$

$$\nabla \bullet \vec{D} = \rho$$

$$\nabla \bullet \left(\vec{J}_n / -q \right) = (G - R)$$

$$\nabla \bullet \left(\vec{J}_p / \; q \right) = (G - R)$$

equilibrium

$$\frac{d^2\psi}{dx^2} = \frac{-q}{\varepsilon_{Si}} \left[p_0 \{\psi(x)\} - n_0 \{\psi(x)\} + N_D^+ - N_A^- \right]$$

Poisson-Boltzmann equation

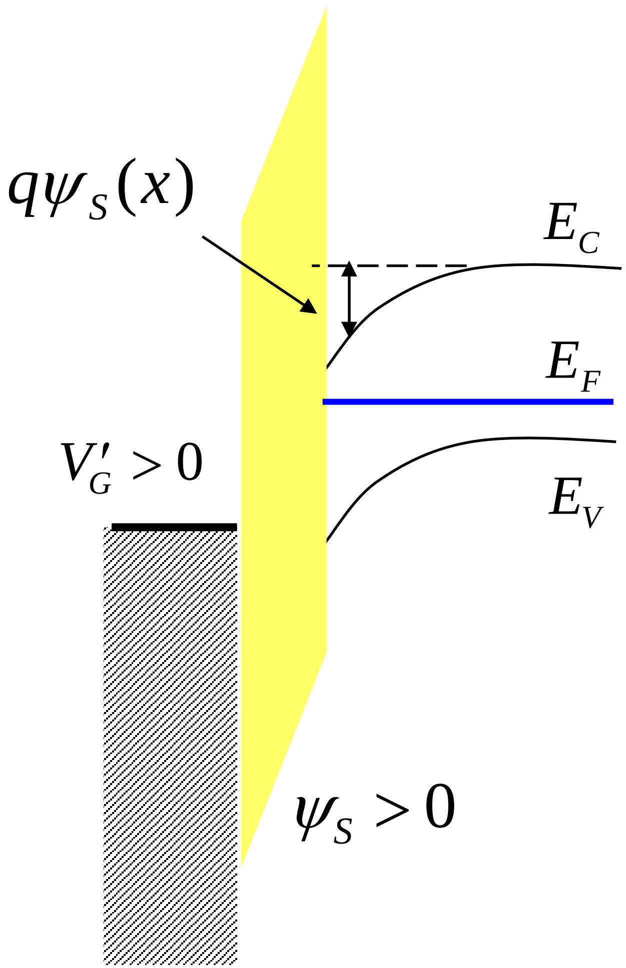
$$n_0(x) = N_C e^{[E_F - E_C(x)]/k_B T}$$

$$p_0(x) = N_V e^{[E_V(x) - E_F]/k_B T}$$

$$E_C(x) = E_C(\infty) - q\psi(x)$$

$$n_0(x) = n_B e^{q\psi(x)/k_B T}$$

$$p_0(x) = p_B e^{-q\psi(x)/k_B T}$$



Poisson equation

$$\frac{d^2\psi}{dx^2} = \frac{-\rho}{\varepsilon}$$

$$\frac{d^2\psi}{dx^2} = \frac{-q}{\varepsilon}\left[p_0(x) - n_0(x) + N_D^+ - N_A^- \right]$$

$$p_0(x) = p_B e^{-q\psi(x)/k_BT} \qquad n_0(x) = n_B e^{q\psi(x)/k_BT}$$

$$\frac{d^2\psi}{dx^2} = \frac{-q}{\varepsilon}\left[p_B e^{-q\psi(x)/k_BT} - n_B e^{q\psi(x)/k_BT} + N_D^+ - N_A^- \right]$$

Poisson-Boltzmann equation

$$\left.\frac{d^2\psi}{dx^2}\right|_{x\to\infty} = 0 = \frac{-q}{\varepsilon}\left[p_B - n_B + N_D^+ - N_A^-\right]$$

$$\left(N_D^+ - N_A^-\right) = -p_B + n_B$$

$$\frac{d^2\psi}{dx^2} = \frac{-q}{\varepsilon}\left[p_B e^{-q\psi(x)/k_BT} - n_B e^{q\psi(x)/k_BT} + N_D^+ - N_A^-\right]$$

$$\frac{d^2\psi}{dx^2} = \frac{-q}{\varepsilon}\left[p_B e^{-q\psi(x)/k_BT} - n_B e^{q\psi(x)/k_BT} - p_B + n_B\right]$$

$$p_B \approx N_A \qquad n_B \approx n_i^2 / N_A$$

Poisson-Boltzmann equation

$$\frac{d^2\psi}{dx^2} = \frac{-q}{\varepsilon}\left[N_A\left(e^{-q\psi/k_BT} - 1\right) - \frac{n_i^2}{N_A}\left(e^{q\psi/k_BT} - 1\right) \right]$$

see:

Taur and Ning, pp. 63-65

and

Lundstrom's notes on the Poisson-Boltzmann equation.

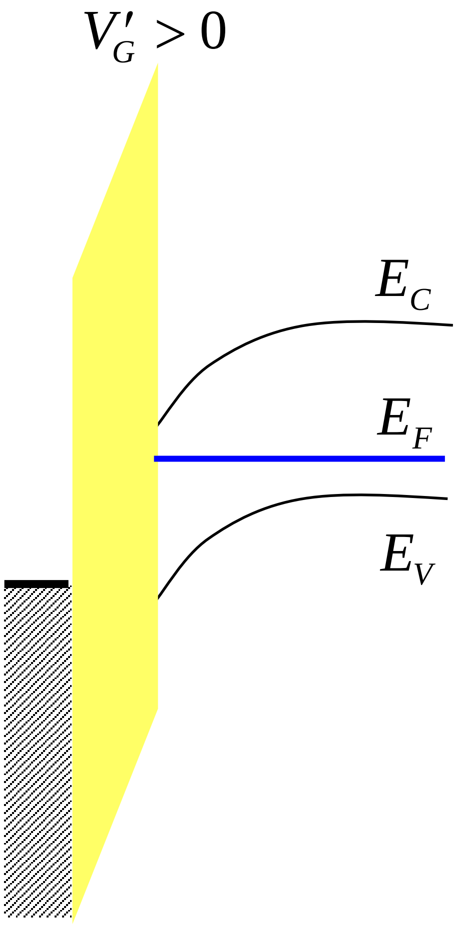
solution of the PB equation

$$\frac{d^2\psi}{dx^2} = \frac{-q}{\varepsilon}\left[N_A\left(e^{-q\psi/k_BT} - 1\right) - \frac{n_i^2}{N_A}\left(e^{q\psi/k_BT} - 1\right)\right]$$

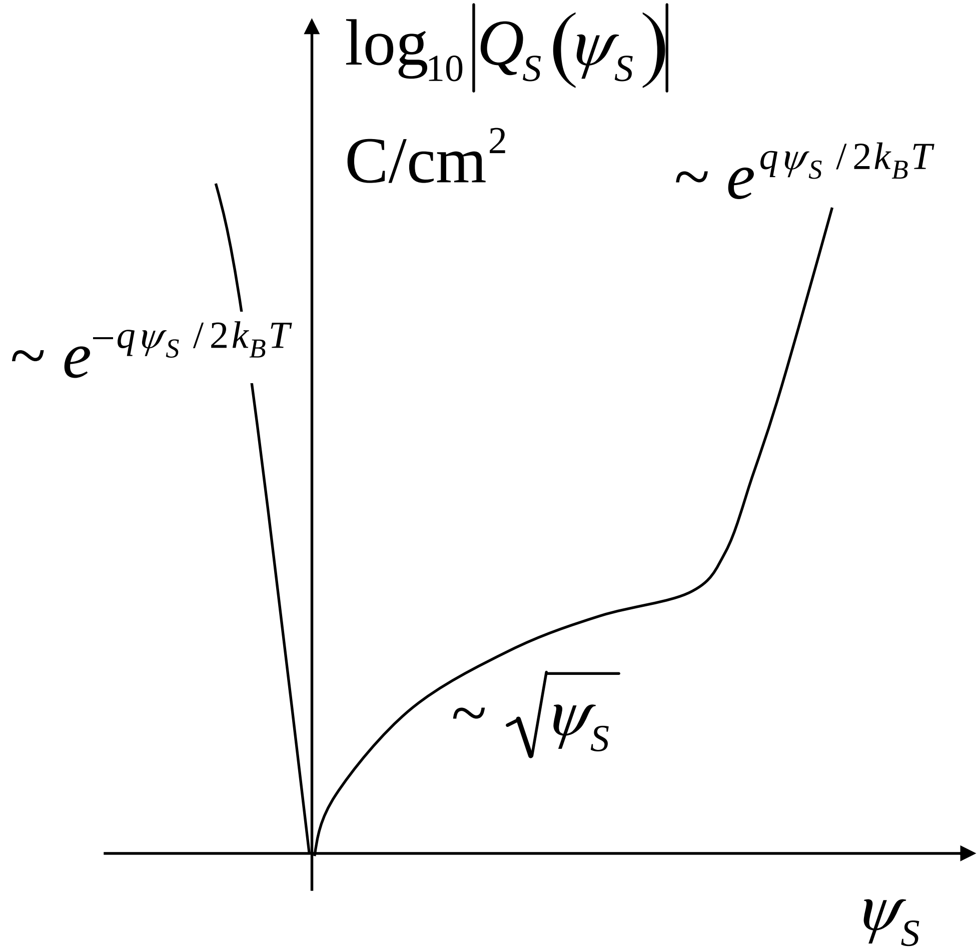
$$E_s = \pm\sqrt{\frac{2k_BT\,N_A}{\varepsilon_{Si}}}\,F(\psi_s)$$

$$F(\psi) = \sqrt{\left(e^{-q\psi/k_BT} + \frac{q\psi}{k_BT} - 1\right) + \frac{n_i^{\,2}}{N_A^{\,2}}\left(e^{+q\psi/k_BT} - \frac{q\psi}{k_BT} - 1\right)}$$

$$Q_S = -\varepsilon_S E_S = \mp\sqrt{\frac{2k_BT\,N_A}{\varepsilon_{Si}}}\,F(\psi_s)$$



solution of the PB equation



strong accumulation:

$$Q_S \approx +\sqrt{2\varepsilon_{Si}k_BT\,N_A}\;e^{-q\psi_s/2k_BT}$$

depletion:

$$Q_S \approx \sqrt{2q\varepsilon_{Si}N_A(\psi_s - k_BT/q)}$$

strong inversion:

$$Q_S \approx +\sqrt{\frac{2\varepsilon_{Si}k_BT\,n_i^2}{N_A}}\;e^{+q\psi_s/2k_BT}$$

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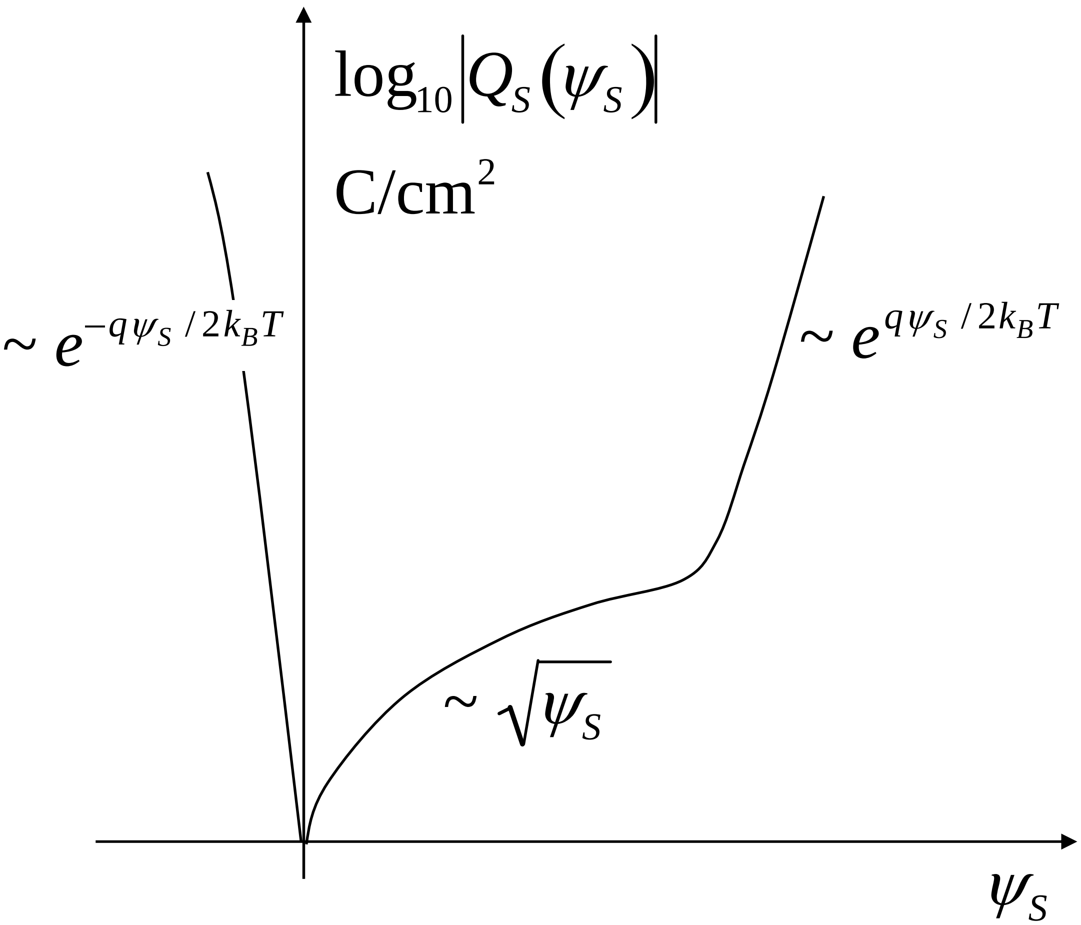
1D MOS electrostatics



need:

$Q_S(\psi_S)$ for capacitance

$Q_i(\psi_S)$ for current

Can we understand the essential features of the PB solution simply?

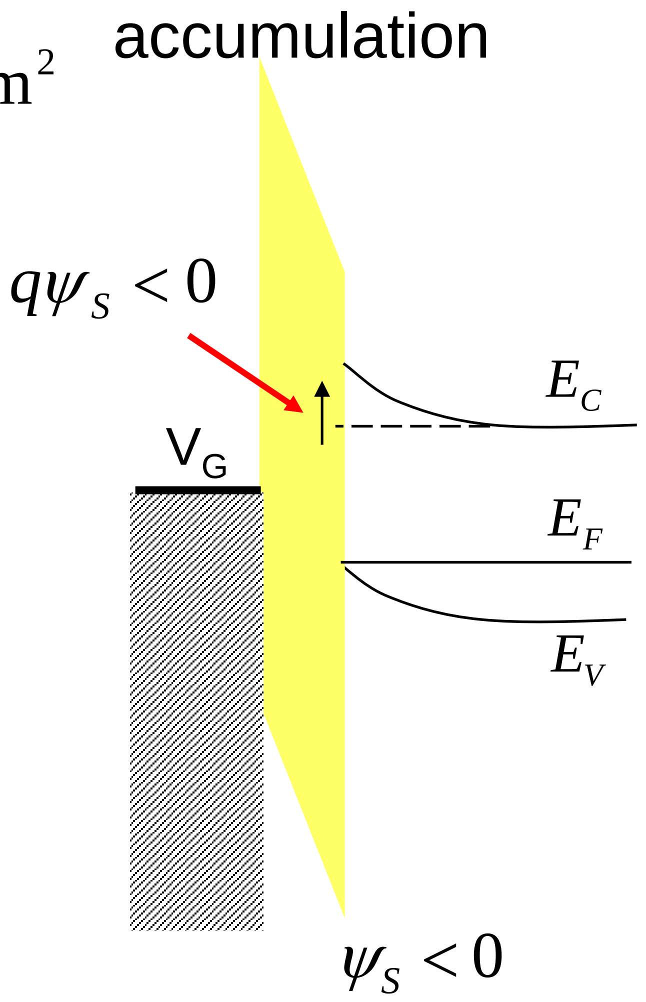
i) accumulation

$$Q_S = q\int_0^\infty \left[(p(x) - p_B) - (n(x) - n_B)\right] dx \ \ \text{C/cm}^2$$

$$\approx q\int_0^\infty \Delta p(x)\, dx$$

$$\Delta p(x) = \left(p(x) - p_B\right) = p_B\left(e^{-q\psi/k_B T} - 1\right)$$

$$Q_S = q\int_0^\infty \Delta p(x) dx \ ; \ \ q p_B \int_0^\infty e^{-q\psi/k_B T} dx$$



i) accumulation

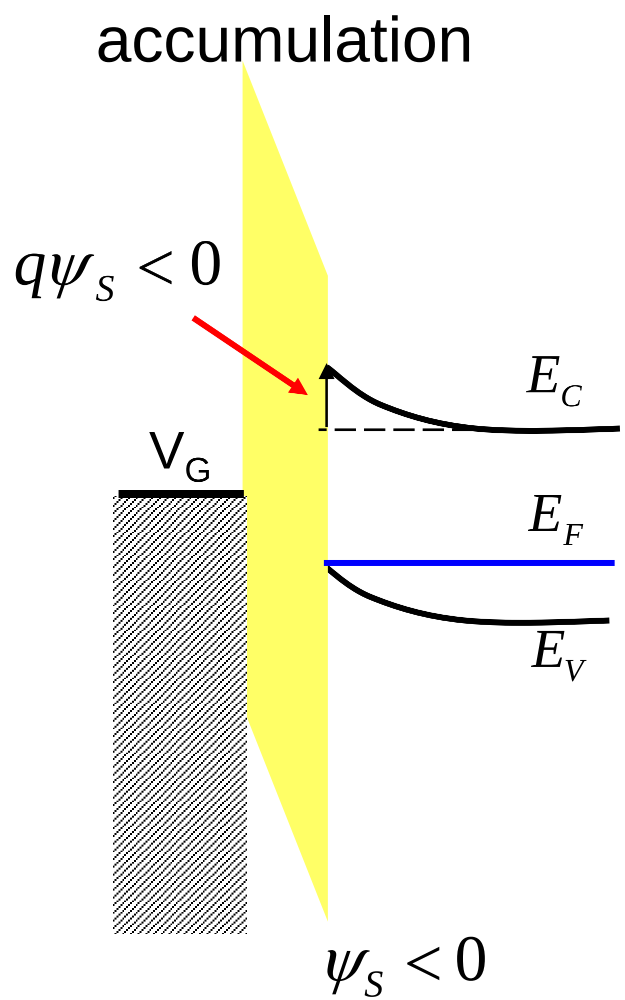
$$Q_S = qp_B \int_0^{\infty} e^{-q\psi/k_BT} dx$$

$$Q_S \; ; \; qp_B \int_{\psi_S}^{0} \frac{e^{-q\psi/k_BT}}{d\psi/dx} d\psi$$

$$; \; \frac{qp_B}{E_S} \int_0^{\psi_S} e^{-q\psi/k_BT} d\psi$$

$$Q_S \; ; \; \frac{qp_B}{-E_S} e^{-q\psi_S/k_BT} \frac{k_BT}{q}$$

$$= qp(0)\left(\frac{k_BT/q}{|E_S|}\right) = qp(0)W_{acc}$$



a note added after the lecture

When performing the integral: $Q_S \simeq qp_B \int_{\psi_S}^{0} \frac{e^{-q\psi/k_BT}}{d\psi/dx} d\psi$

I assumed that: $d\psi/dx = -E_S \quad (\sim\text{constant})$

so that I could move it outside of the integral. This is fine, since I am just trying to explain the form of the exact solution in strong accumulation, but….

a better approximation is: $d\psi/dx = -E_S/2 \quad (\sim\text{constant})$

because the electric field goes from a large magnitude to ~ 0 over the accumulation layer, so the average electric field is lower than E_S. With this assumption, we would get <u>exactly</u> the strong accumulation result on slide 12.

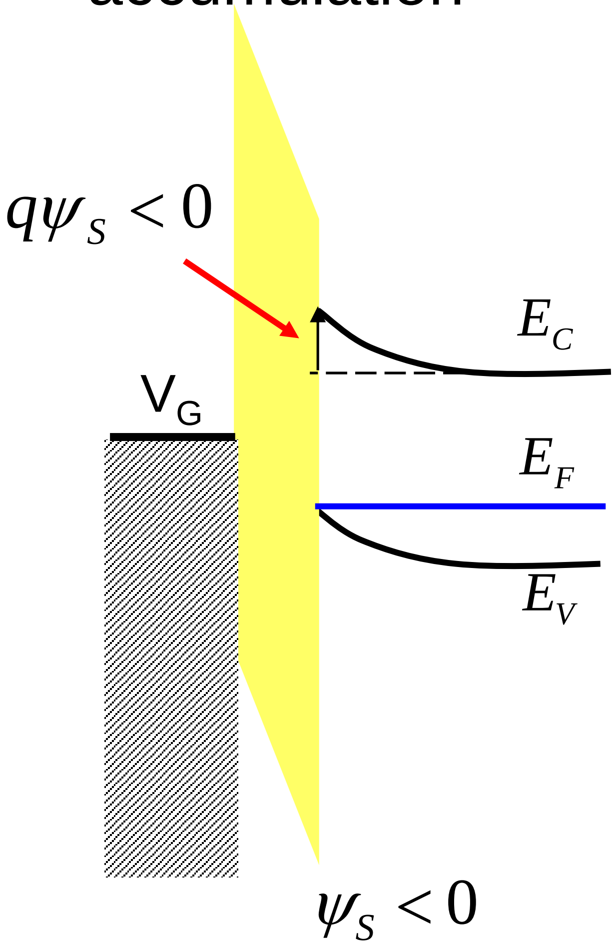
an aside on electrostatics

$$\frac{d\left(\varepsilon_{Si}E\right)}{dx} = \rho(x)$$

$$\int_{E_S}^{0} dE = \frac{1}{\varepsilon_{Si}} \int_{0}^{\infty} \rho(x)dx$$

$$E_S = \frac{-Q_S}{\varepsilon_{Si}}$$

accumulation



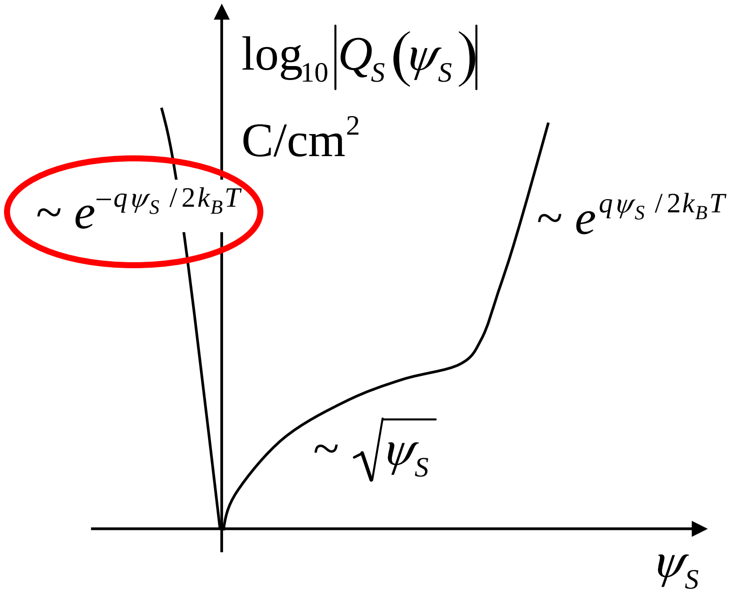
i) accumulation

$$Q_S \; ; \; \frac{qp_B}{-E_S} e^{-q\psi_S/k_BT} \frac{k_BT}{q} \qquad (1)$$

$$E_S = -Q_S/\varepsilon_{Si}$$

$$Q_S \; ; \; \frac{q\varepsilon_{Si}p_B}{Q_S} e^{-q\psi_S/k_BT} \frac{k_BT}{q}$$

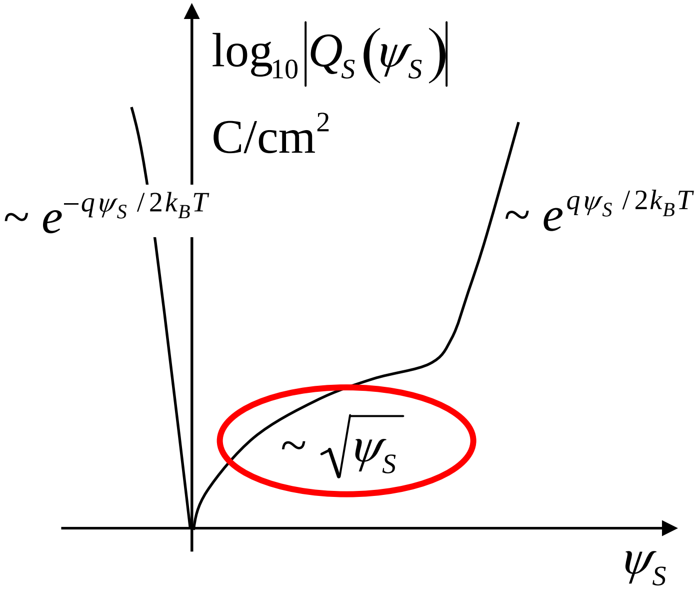
$$Q_S \; ; \; \sqrt{\varepsilon_{Si}N_Ak_BT}\, e^{-q\psi_S/2k_BT}$$



$$\left\{ Q_S \; ; \; qp(0)\left(\frac{k_BT/q}{|E_S|}\right) \right\}$$

depletion

$$Q_S(\psi_S) = -qN_AW = \sqrt{2qN_A\varepsilon_{Si}\psi_S}$$



iii) inversion

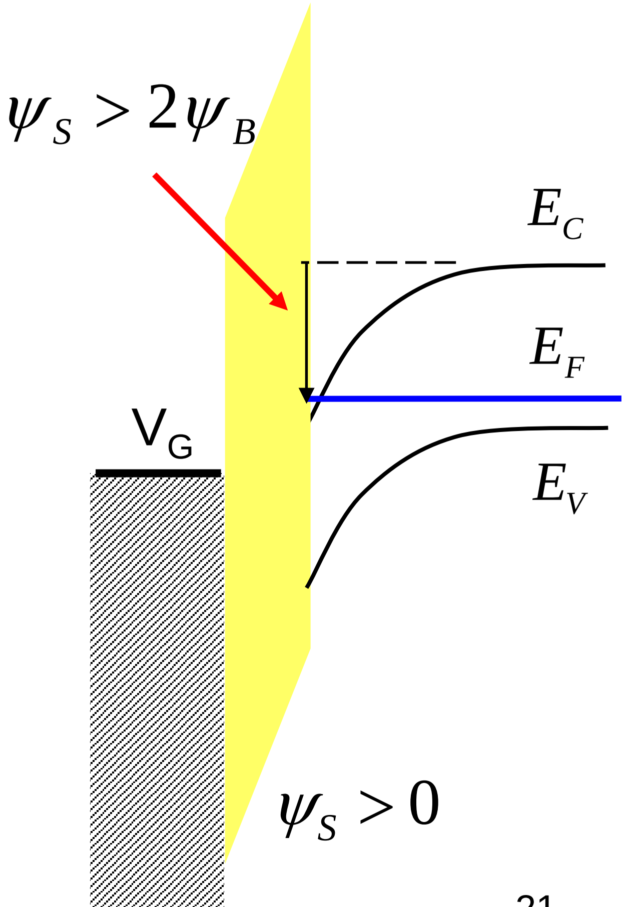
$$Q_S(\psi_S) = Q_D(\psi_S) + Q_i(\psi_S)$$

$$Q_D(\psi_S) = \sqrt{2qN_A\varepsilon_{Si}\psi_S}$$

$$Q_i(\psi_S) = -qn(0)\frac{k_BT/q}{E_S} = -qn(0)W_{inv}$$

$$\left(\begin{array}{l} n(0) = n_B e^{q\psi_S/k_BT} \\ W_{inv} = \dfrac{k_BT/q}{E_S} \end{array}\right)$$

inversion

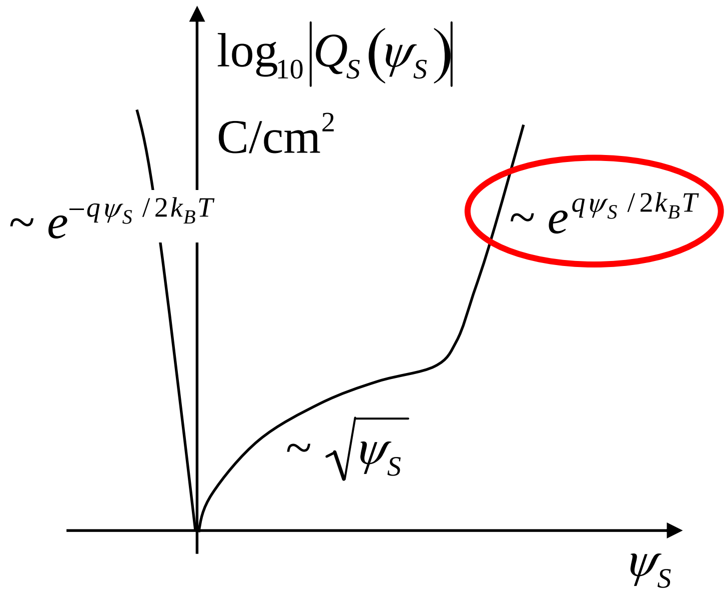


strong inversion (above threshold)

$$Q_S(\psi_S) \approx Q_i(\psi_S)$$

$$Q_i(\psi_S) = -qn(0)\frac{k_B T/q}{E_S}$$

$$E_S(\psi_S) = Q_i/\varepsilon_{Si}$$



$$Q_i(\psi_S) = -\sqrt{\varepsilon_{Si}\, k_B T\, n_B}\, e^{q\psi_S/2k_BT}$$

$$\left\{ Q_i(\psi_S) = -qn(0)\frac{k_B T/q}{E_S} \right\}$$

strong inversion criterion

$$n(0) \approx p_B$$

$$n_B e^{q\psi_S/k_BT} = \frac{n_i^2}{N_A} e^{q\psi_S/k_BT} \approx p_B = N_A$$

$$\psi_S = 2\frac{k_BT}{q}\ln\left(\frac{N_A}{n_i}\right) = 2\psi_B$$

weak inversion (sub-threshold)

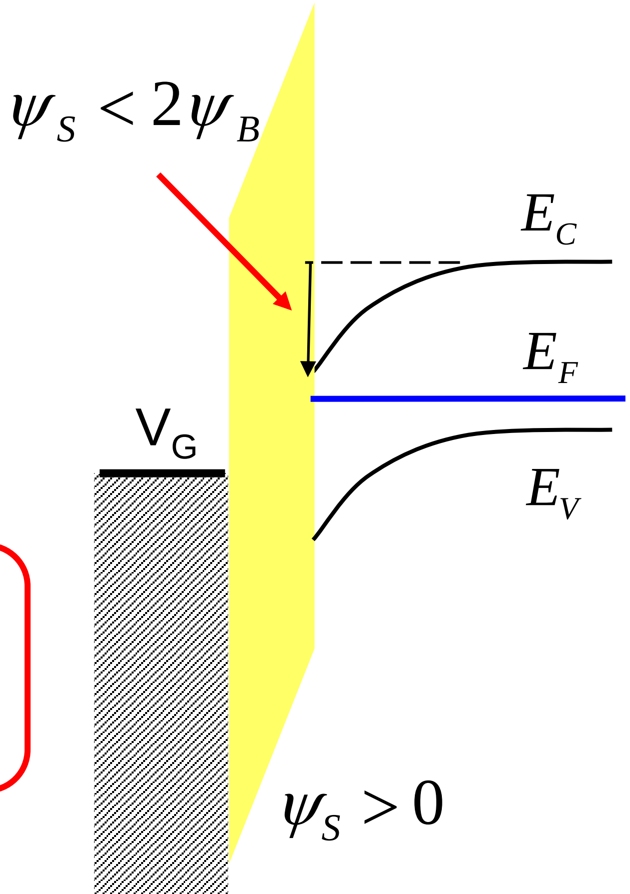
$$Q_S(\psi_S) \approx Q_D(\psi_S)$$

$$Q_i(\psi_S) = -qn(0)\frac{k_B T/q}{E_S}$$

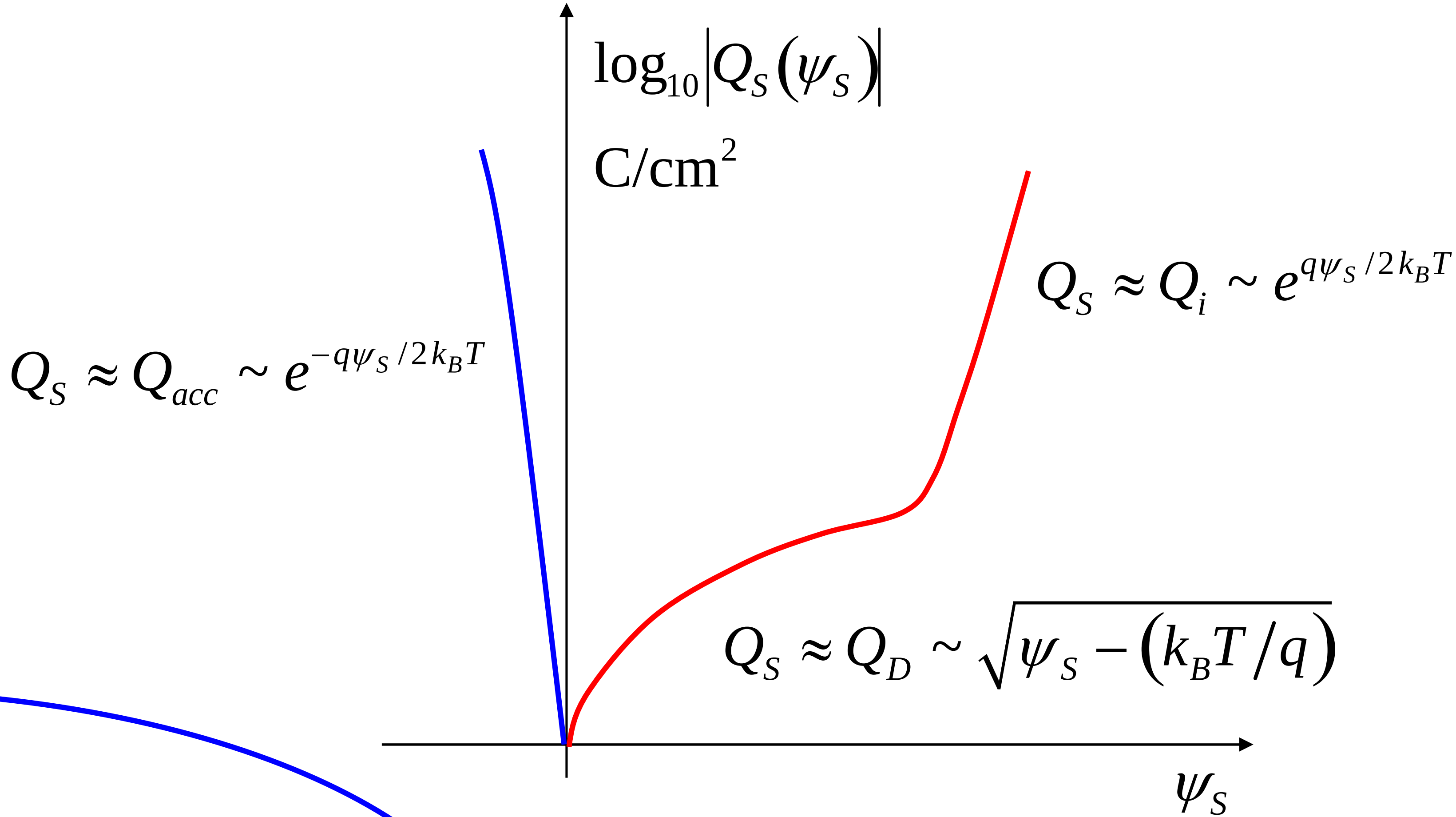
$$E_S(\psi_S) = \sqrt{2qN_A\varepsilon_{Si}\psi_S}\Big/\varepsilon_{Si}$$

$$Q_i(\psi_S) = -q\,n_B e^{q\psi_S/k_B T}\frac{k_B T/q}{\sqrt{2q\varepsilon_{Si}N_A\psi_S}\Big/\varepsilon_{Si}}$$

weak inversion



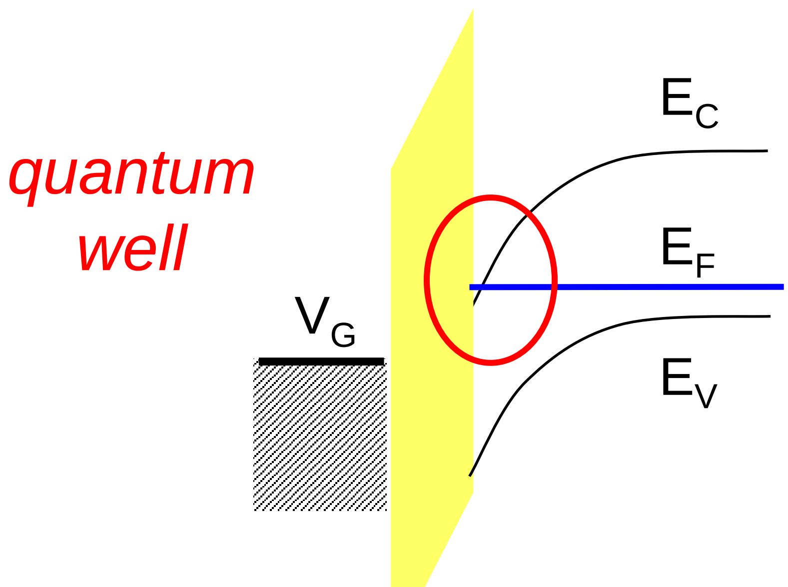
summary



$$W_{acc/inv} = (2k_BT/q)/E_S$$

assumptions

1) Boltzmann statistics (not valid above threshold)
2) Uniform doping (not valid in practice)
3) No quantum confinement (not valid above threshold)



question

What is E_F - E_C at the SiO_2:Si interface under on-current conditions?

$$\left.\begin{aligned} Q_S &\approx Q_I \approx -\sqrt{2\varepsilon_{Si} k_B T\, n(0)} \\ Q_I &= -qN_I \approx -q\times 10^{13}\ \text{C/cm}^2 \end{aligned}\right\}\quad n(0) \approx 3\times 10^{20}\ \ /\text{cm}^3$$

$$n(0) = N_C e^{(E_F - E_C)/k_B T}$$

$$N_C = 3.2\times 10^{19}\ \text{cm}^{-3}$$ (Taur and Ning, Table 2.1)

$$(E_F - E_C)/k_B T = \ln\left[n(0)/N_C\right] \approx \ln 10 = 2.3$$

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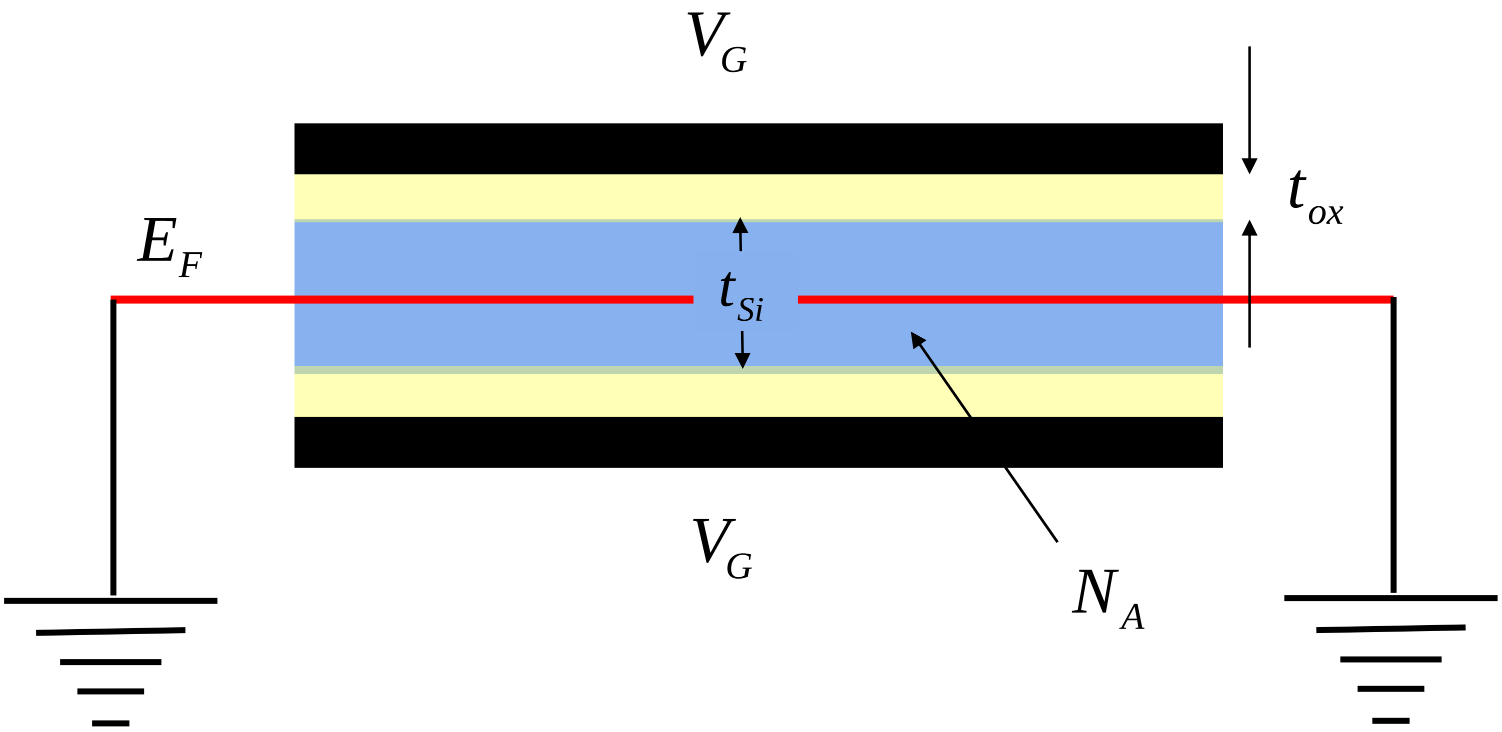
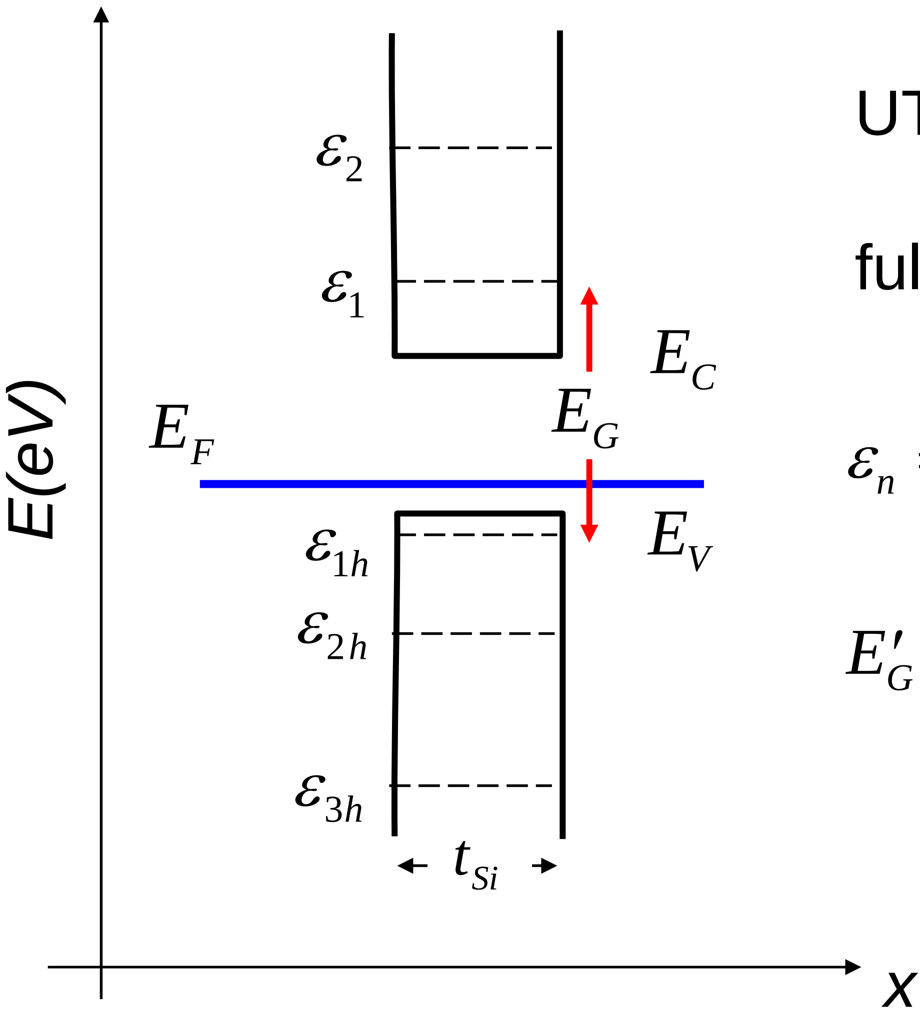
ultra-thin body double gate MOSFET



UTB energy band diagram



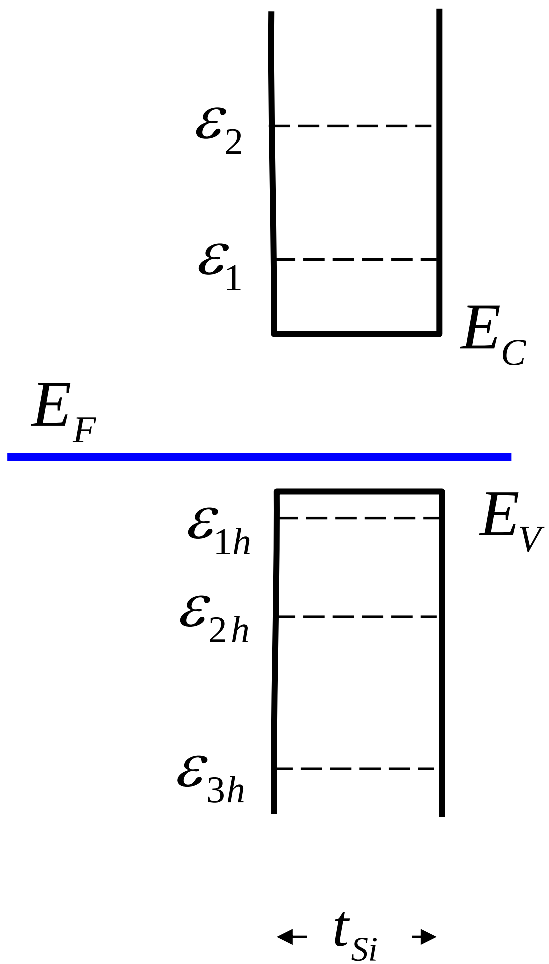
UTB (neglect band bending)

fully depleted (for ψ > 0)

$$\varepsilon_n = \frac{\hbar^2 n^2 \pi^2}{2m^* t_{Si}^2}$$

$$E_G' = E_G + \varepsilon_1 + \varepsilon_{1h}$$

2D carrier densities



$$n_{S1} = \int_0^\infty g_{2D}(E) f_0(E)\, dE$$

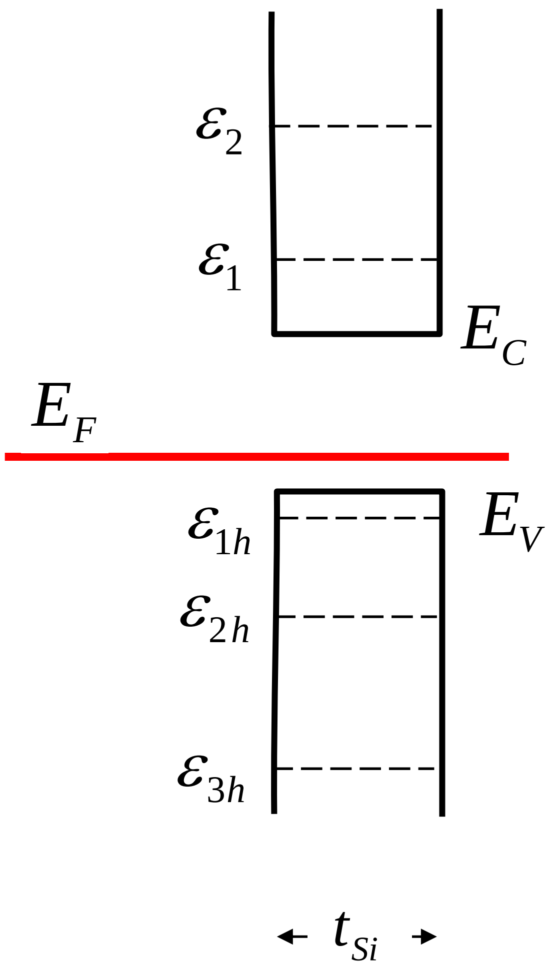
$$= N_C^{2D} \ln\left(1 + e^{(E_F - E_C - \varepsilon_1 / k_B T)}\right) \text{cm}^{-2}$$

$$N_C^{2D} = \frac{m_n^* k_B T}{\pi \hbar^2}$$

Boltzmann statistics:

$$n_{S1} = N_C^{2D} e^{(E_F - E_C - \varepsilon_1)/k_B T} \ \text{cm}^{-2}$$

2D carrier densities



$$n_S = N_C^{2D} e^{(E_F - E_C - \varepsilon_1)/k_B T} \text{ cm}^{-2}$$

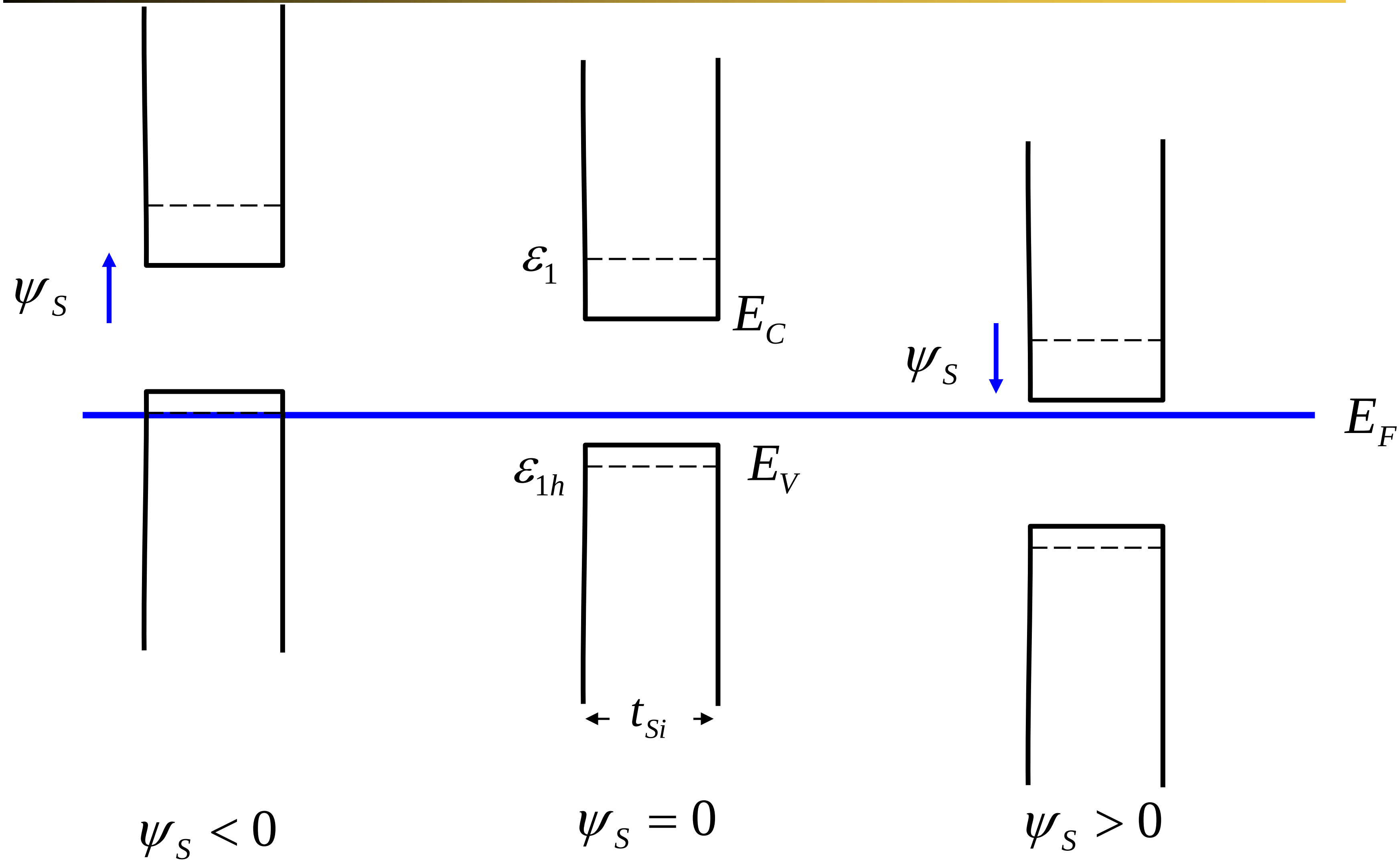
$$N_C^{2D} = \frac{m_n^* k_B T}{\pi \hbar^2}$$

$$p_S = N_V^{2D} e^{(E_V - \varepsilon_{1h} - E_F)/k_B T} \text{ cm}^{-2}$$

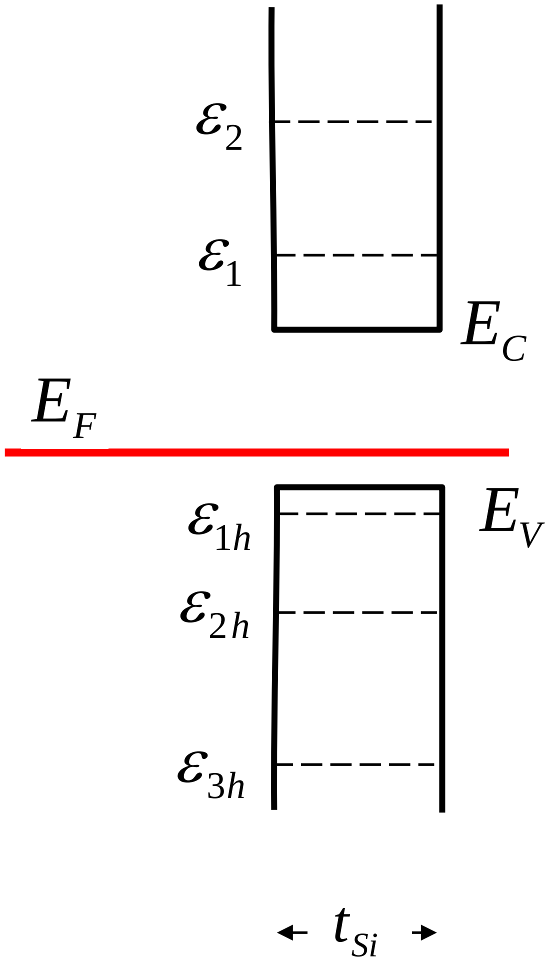
$$N_V^{2D} = \frac{m_p^* k_B T}{\pi \hbar^2}$$

(these eqns. assume that only 1 subband is occupied)

UTB ('surface' potential)



2D carrier densities



$$n_S = N_C^{2D} e^{(E_F - E_C - \varepsilon_1)/k_B T} \text{ cm}^{-2}$$

$$E_C = E_{C0} - q\psi_S$$

$$n_S = N_C^{2D} e^{(E_F - E_{C0} - \varepsilon_1 + q\psi_S)/k_B T}$$

$$n_S = n_{S0} e^{q\psi_S/k_B T}$$

$$p_S = p_{S0} e^{-q\psi_S/k_B T}$$

(these eqns. assume that only 1 subband is occupied)

$Q_S(\psi_S)$ for UTB MOS

$$Q_S = q\left[\left(p_S(\psi_S) - p_{S0}\right) - \left(n_S(\psi_S) - n_{S0}\right)\right] \text{ C/cm}^2$$

$$Q_S = q\left[p_{S0}\left(e^{-q\psi_S/k_B T} - 1\right) - n_{S0}\left(e^{q\psi_S/k_B T} - 1\right)\right] \qquad (2)$$

$$p_{S0} \approx N_A t_{Si} \text{ cm}^{-2}$$

$$n_{S0} = \left(n_i^{2D}\right)^2 / p_{S0} \text{ cm}^{-2}$$

From equation (2), we can readily plot $Q_S(\psi_S)$

$Q_S(\psi_S)$ for UTB MOS

$$Q_S = q\left[p_{S0}\left(e^{-q\psi_s/k_BT} - 1\right) - n_{S0}\left(e^{q\psi_s/k_BT} - 1\right)\right] \qquad (2)$$

1) strong accumulation ($\psi_S << 0$)

$$Q_S = q\, p_{S0}\, e^{-q\psi_s/k_BT}$$

2) depletion ($\psi_S > 0$)

$$Q_S = -q\, N_A\, t_{Si}$$

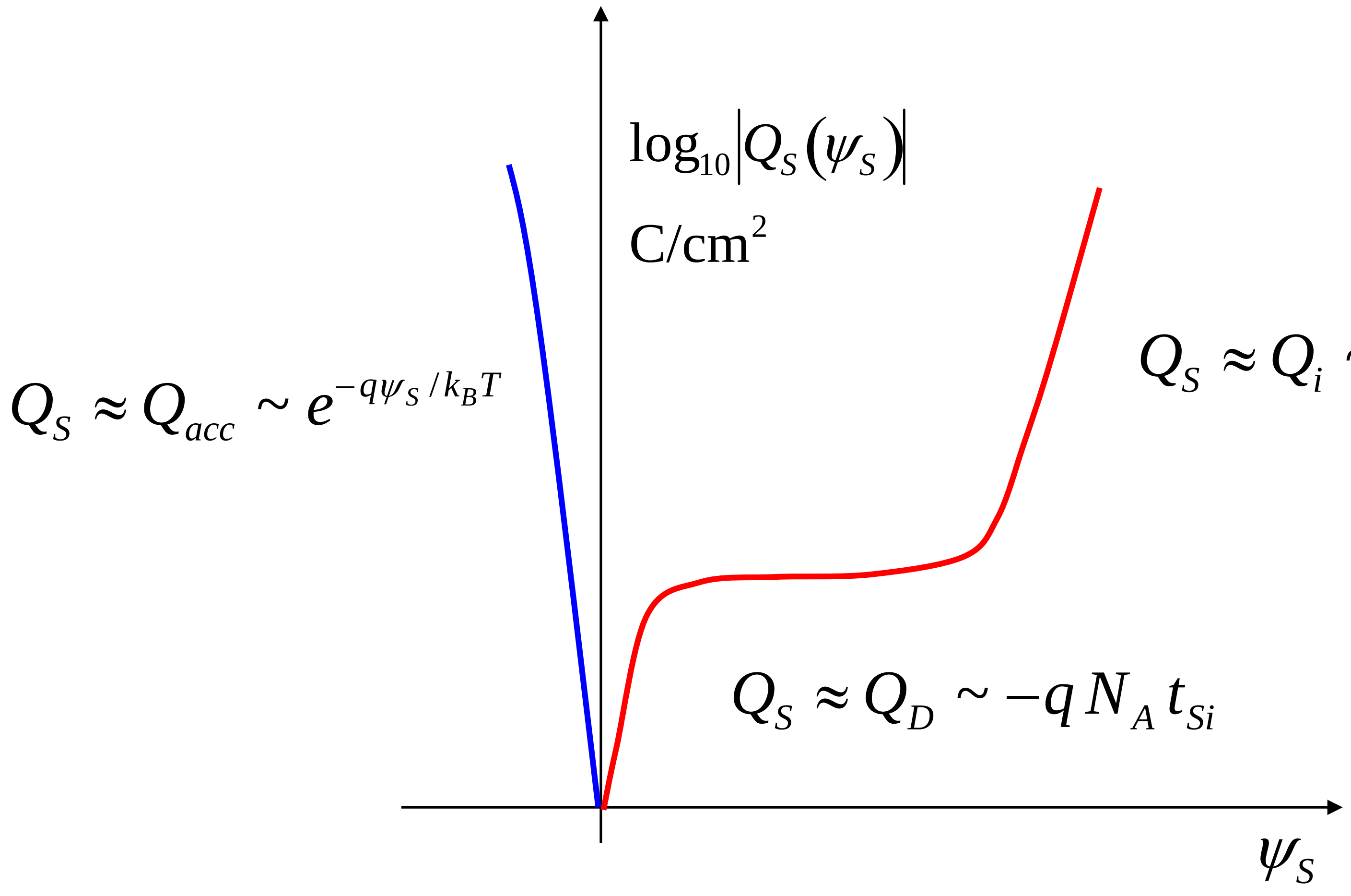
Note: We are not resolving band bending within the Si body.
We assume that the film is fully depleted when $\psi_S > 0$).

3) inversion ($\psi_S >> 0$)

$$Q_S = -q\, n_{S0}\, e^{q\psi_s/k_BT}$$

$Q_S(\psi_S)$ for UTB MOS

$$Q_S = q\left[p_{S0}(e^{-q\psi_s/k_BT} - 1) - n_{S0}(e^{q\psi_s/k_BT} - 1)\right] \qquad (2)$$



$Q_S(\psi_S)$ for UTB MOS: summary

1) The UTB case is easier to solve than the bulk Si case.

2) Results are qualitatively similar - except for the depletion charge and acc and inv layers that vary as $\exp(\psi_S/k_BT)$ rather than $\exp(\psi_S/2k_BT)$.

   Can you explain why this difference occurs?

3) We have included quantum mechanics (without self-consistent electrostatics inside the silicon film) ***but not Fermi-Dirac statistics***.

   Fermi-Dirac statistics are important above threshold.

$Q_S(\psi_S)$ for UTB MOS: exercise

1) Repeat the derivation, but include Fermi-Dirac statistics. (This can be done analytically for the UTB.)

2) Plot Q_S *vs.* ψ_S from accumulation to inversion for both Boltzmann and Fermi-Dirac statistics and compare the results.

summary

1) Understanding $Q_S(\psi_S)$ and $Q_i(\psi_S)$ are essential for understanding MOS C-V and MOSFETs.

2) Both $Q_S(\psi_S)$ and $Q_i(\psi_S)$ are readily computed for simple, model structures.

3) The general features of the $Q_S(\psi_S)$ and $Q_i(\psi_S)$ vs. ψ_S are readily understood with simple calculations.

4) Before we proceed to MOS-C's and MOSFETs, we need to relate ψ_S to the V_G that produced it.