# EE-612:
# Lecture 7
# The Ballistic MOSFET

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This lecture (and the next one) are based on the series:

"Physics of Nanoscale MOSFETs"

by Mark Lundstrom

http://www.nanoHUB.org/resources/5306

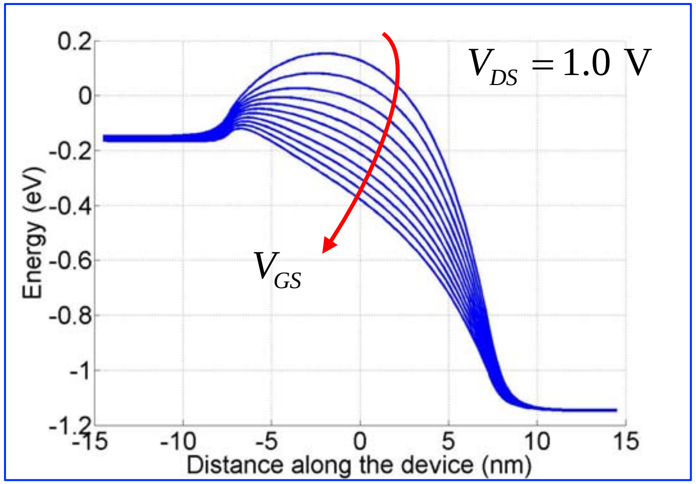
which discusses this material in more depth.

# outline

1) **Introduction**
2) Ballistic theory of the MOSFET
3) Discussion
4) Summary

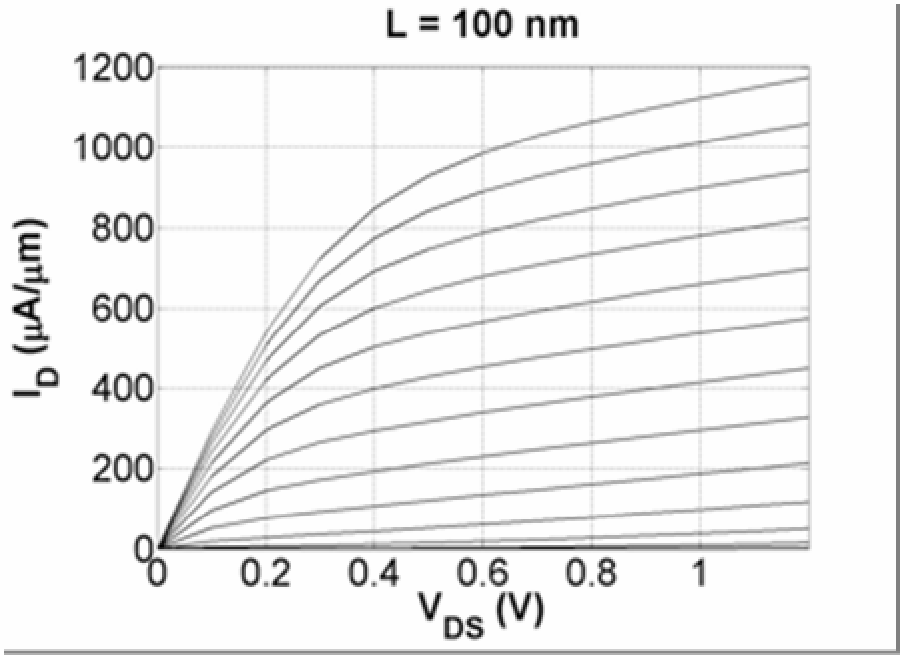
# transistors

$I^+$ $\qquad$ $I^-$




$$I_D \propto I^+ - I^-$$

2007 N-MOSFET

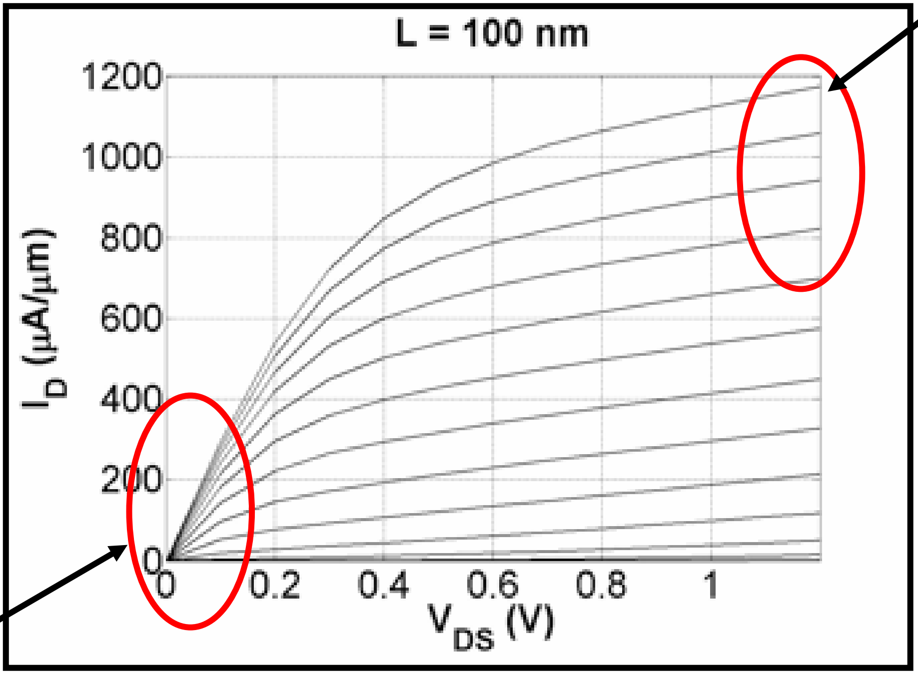



(Courtesy, Shuji Ikeda, ATDF, Dec. 2007)

# traditional MOSFET theory

$$I_D = W C_{ox} \upsilon_{sat} \left(V_{GS} - V_T\right)$$

*2007* N MOSFET




$$I_D = \frac{W}{L} \mu_{eff} C_{ox} \left(V_{GS} - V_T\right) V_{DS}$$

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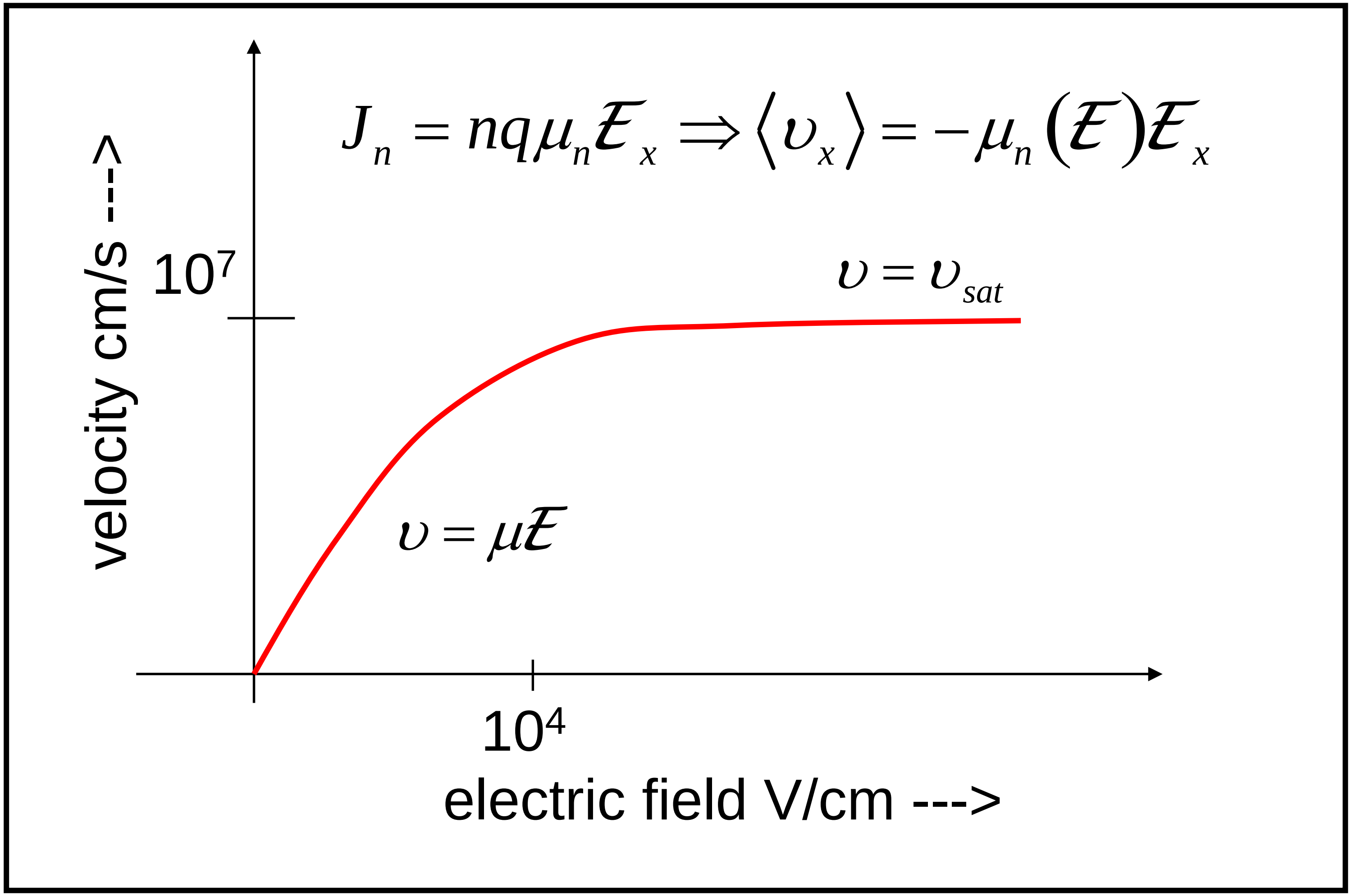
# drift-diffusion ('diffusive') transport

$$J_n = nq\mu_n \mathcal{E}_x + qD_n \frac{dn}{dx}$$

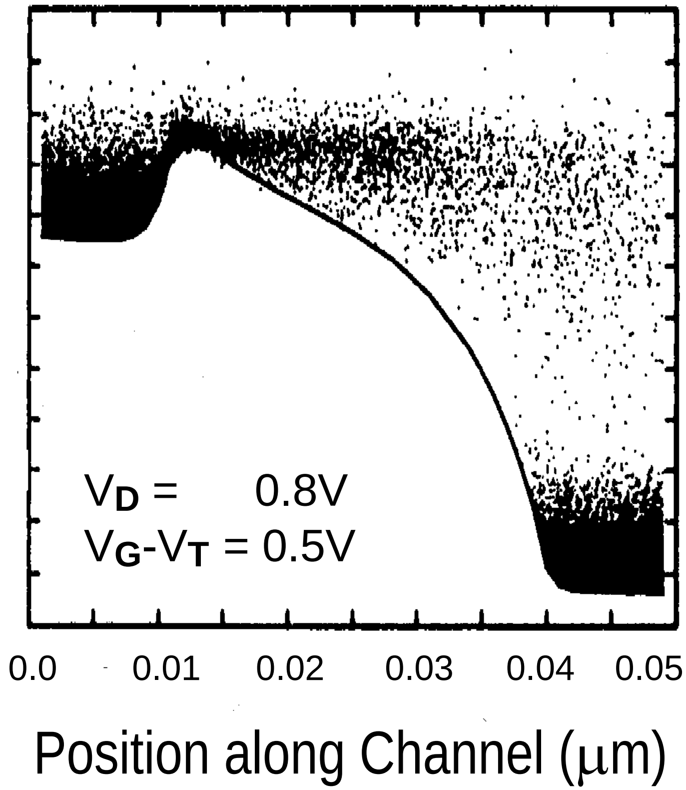
above threshold, drift dominates

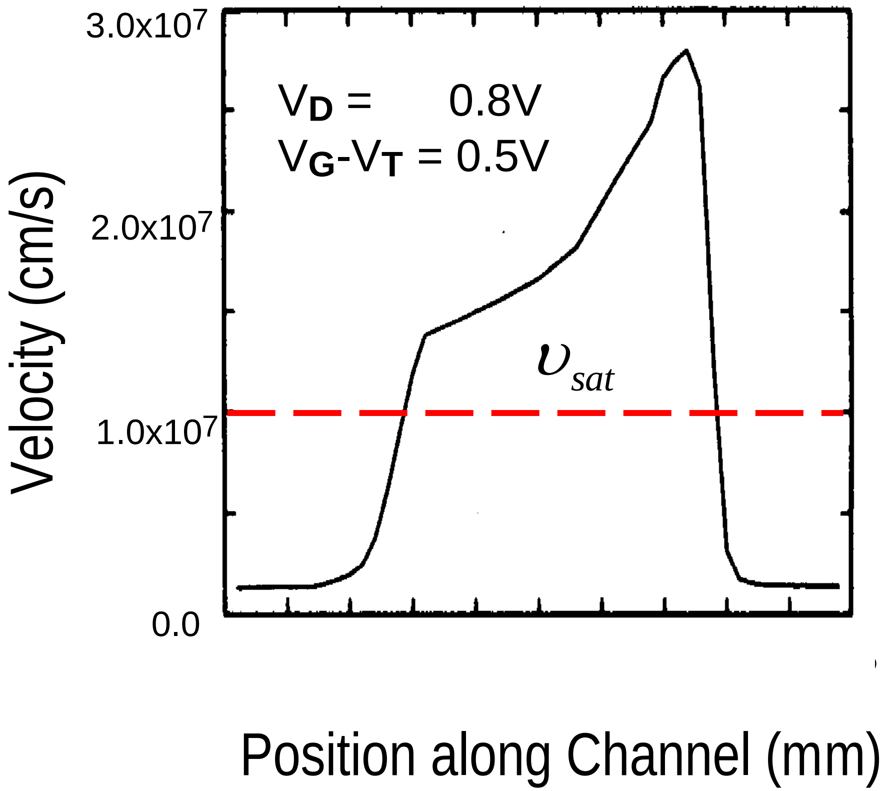
below threshold, diffusion dominates

# velocity saturation



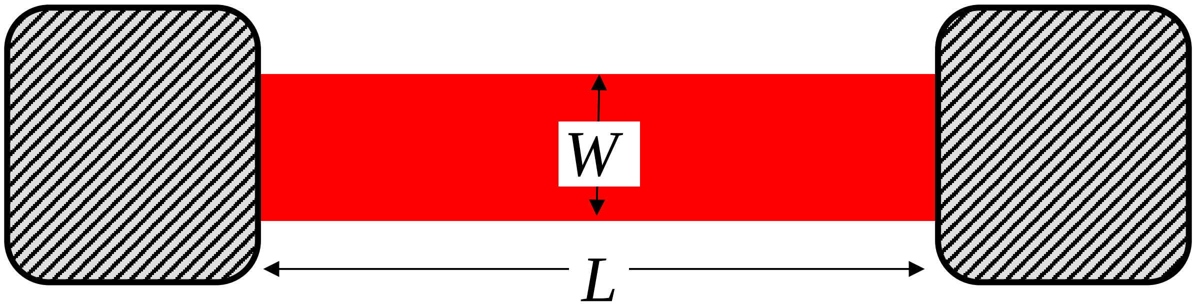
# velocity overshoot in sub-micron MOSFETs




Frank, Laux, and Fischetti, IEDM Tech. Dig., p. 553, 1992

# diffusive vs. ballistic transport




1) diffusive:

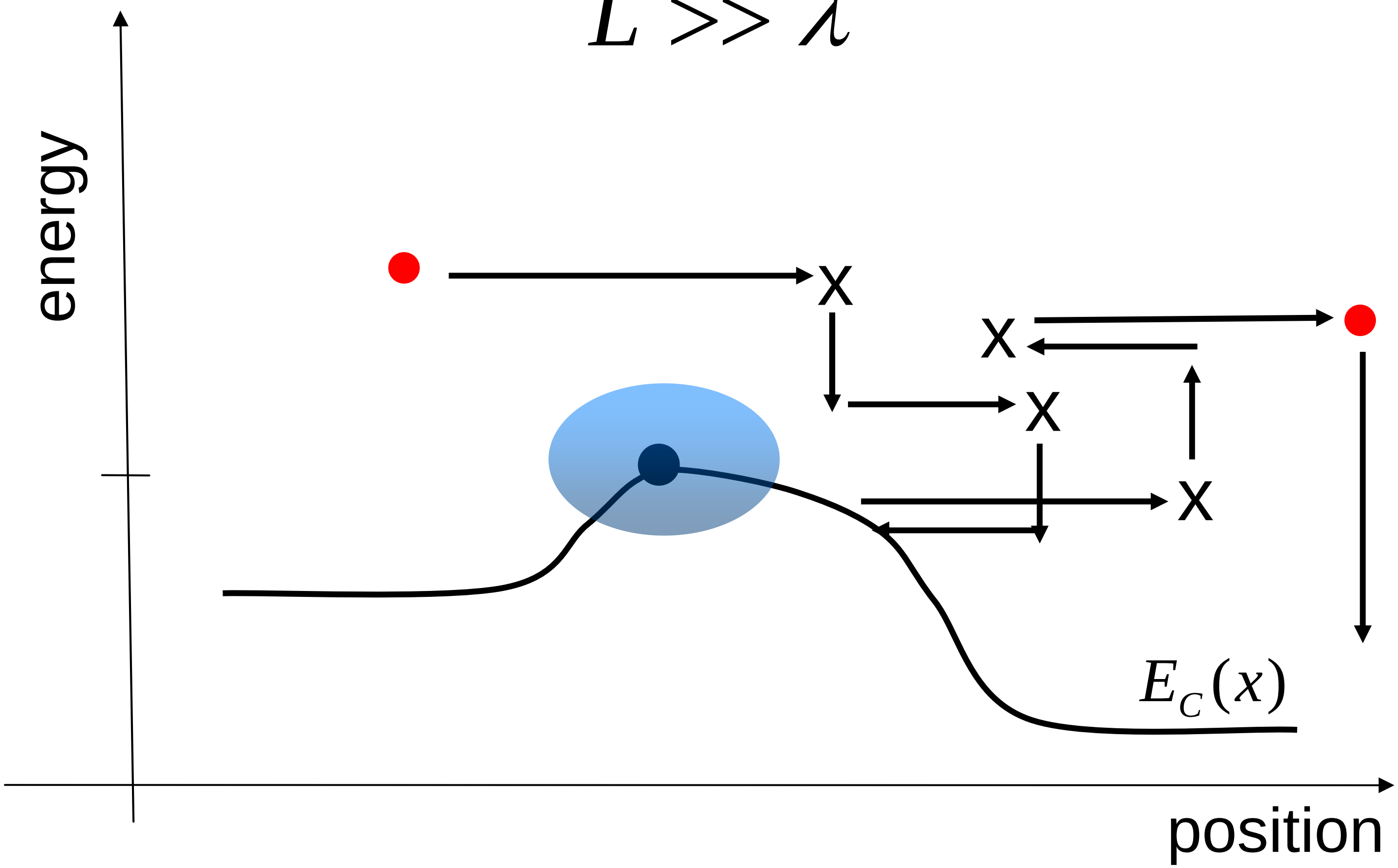
$$R = \rho_S (L/W) \quad \rho_S = 1/n_S q \mu_n \quad \langle \upsilon_x \rangle = -\mu_n \mathcal{E}_x = \mu_n (V/L)$$

2) ballistic:

$$R = \left(M\, 2q^2/h\right)^{-1}$$ ("quantum contact resistance" $T = 0$ K)

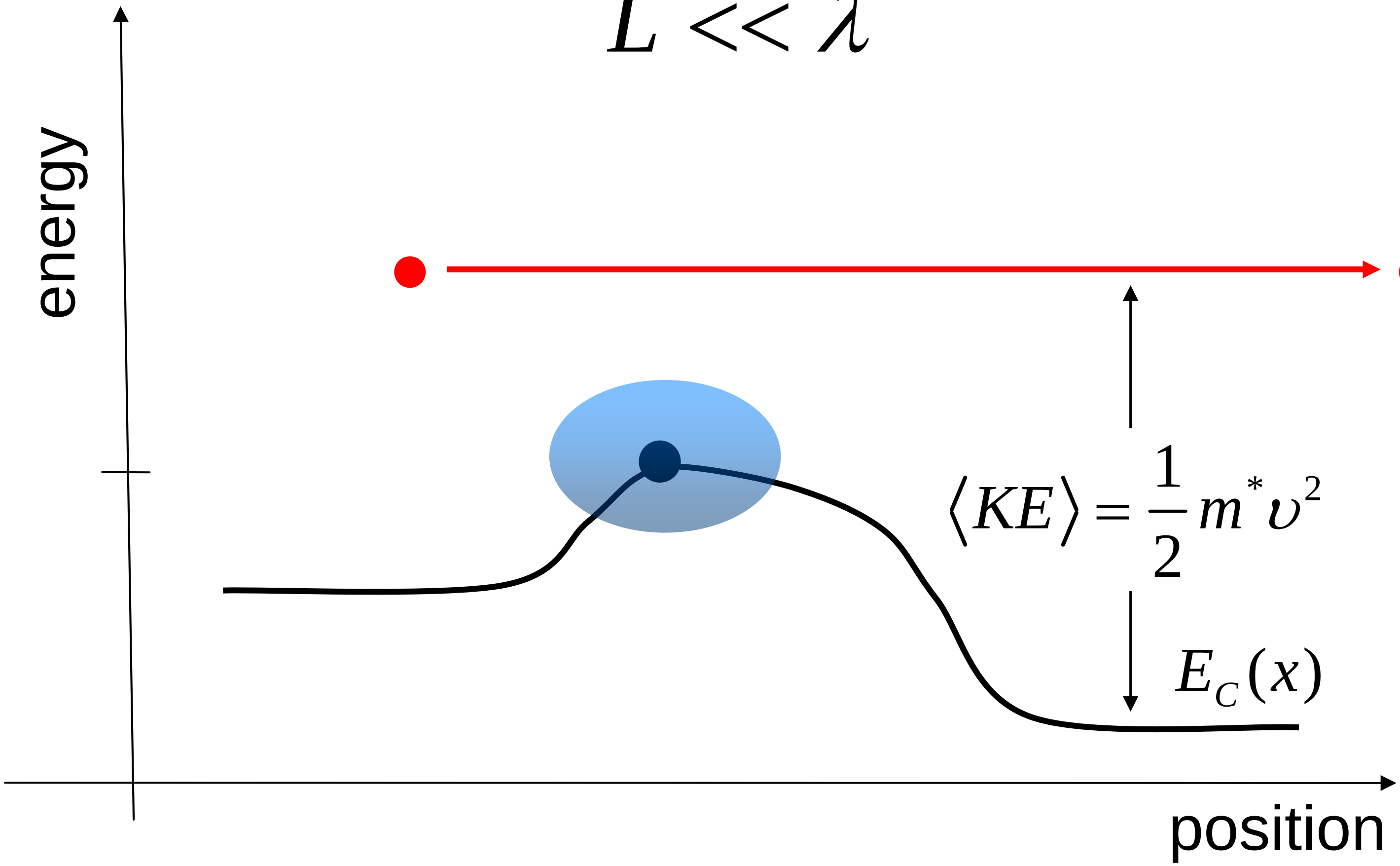
# diffusive transport in a MOSFET

$$L >> \lambda$$



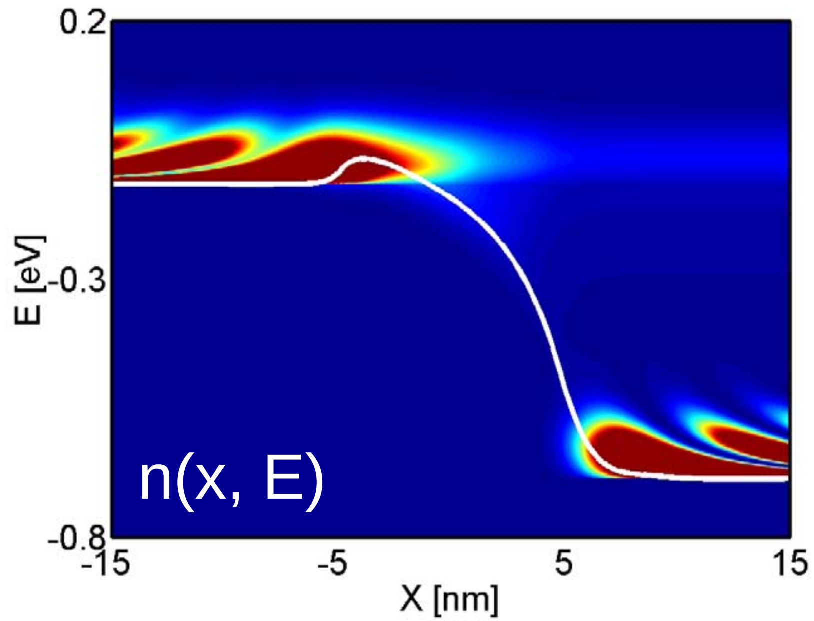
# ballistic transport in a MOSFET

$$L << \lambda$$



# quantum transport in a nano MOSFET

*L* = 10 nm

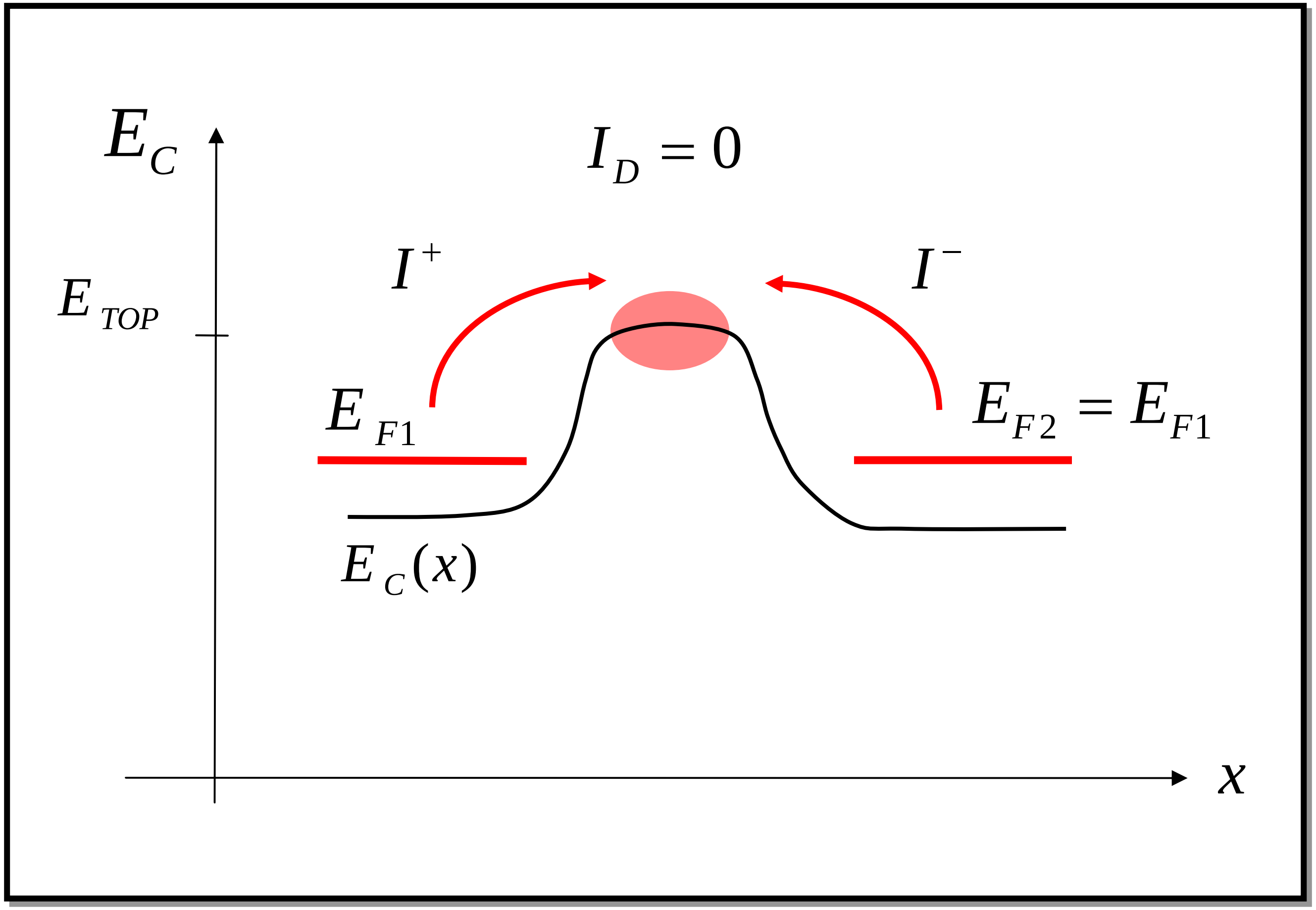



nanoMOS (www.nanoHUB.org)

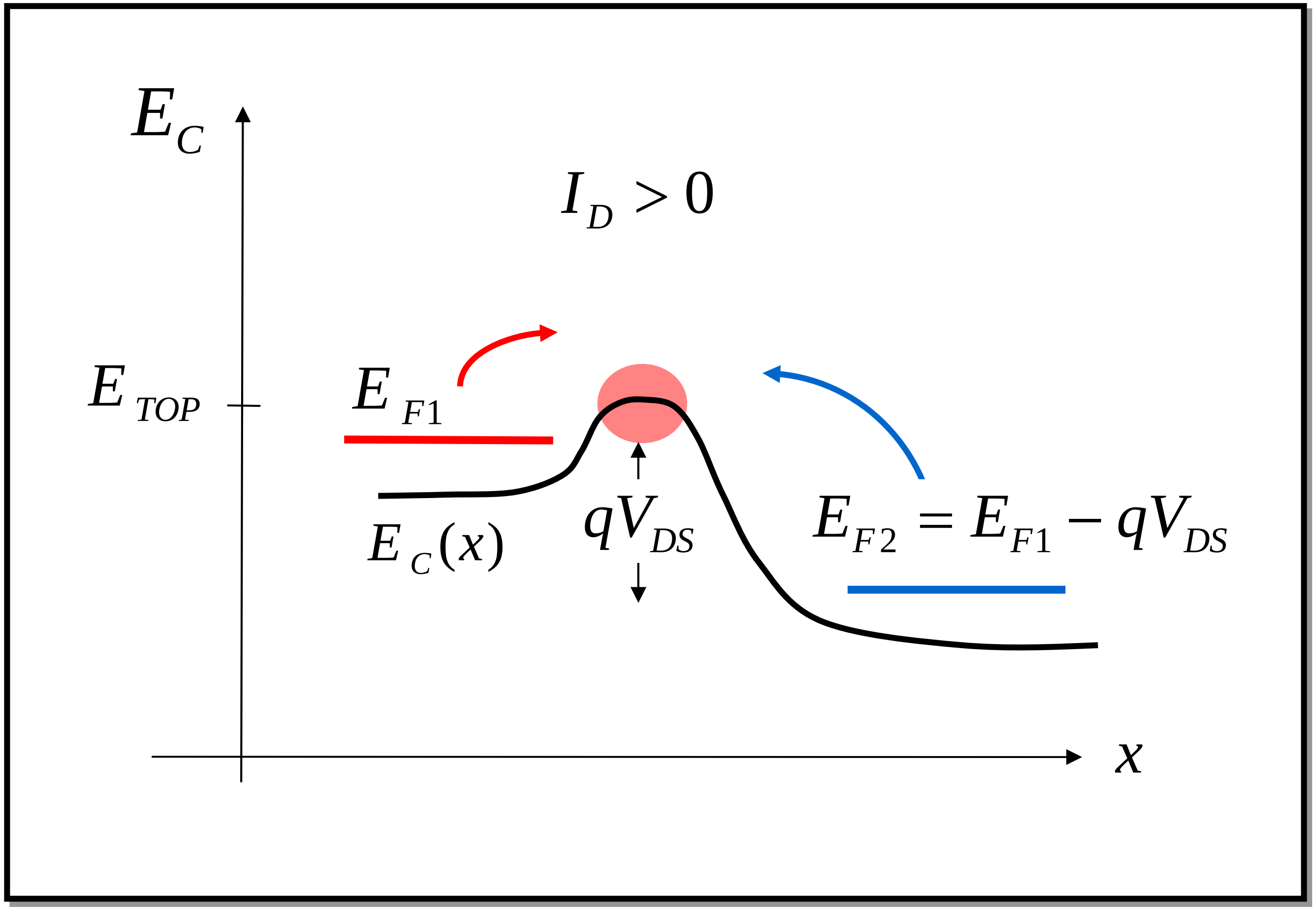
# outline

1) Introduction
2) **Ballistic theory of the MOSFET**
3) Discussion
4) Summary

# MOSFET in equilibrium



# high gate and drain bias

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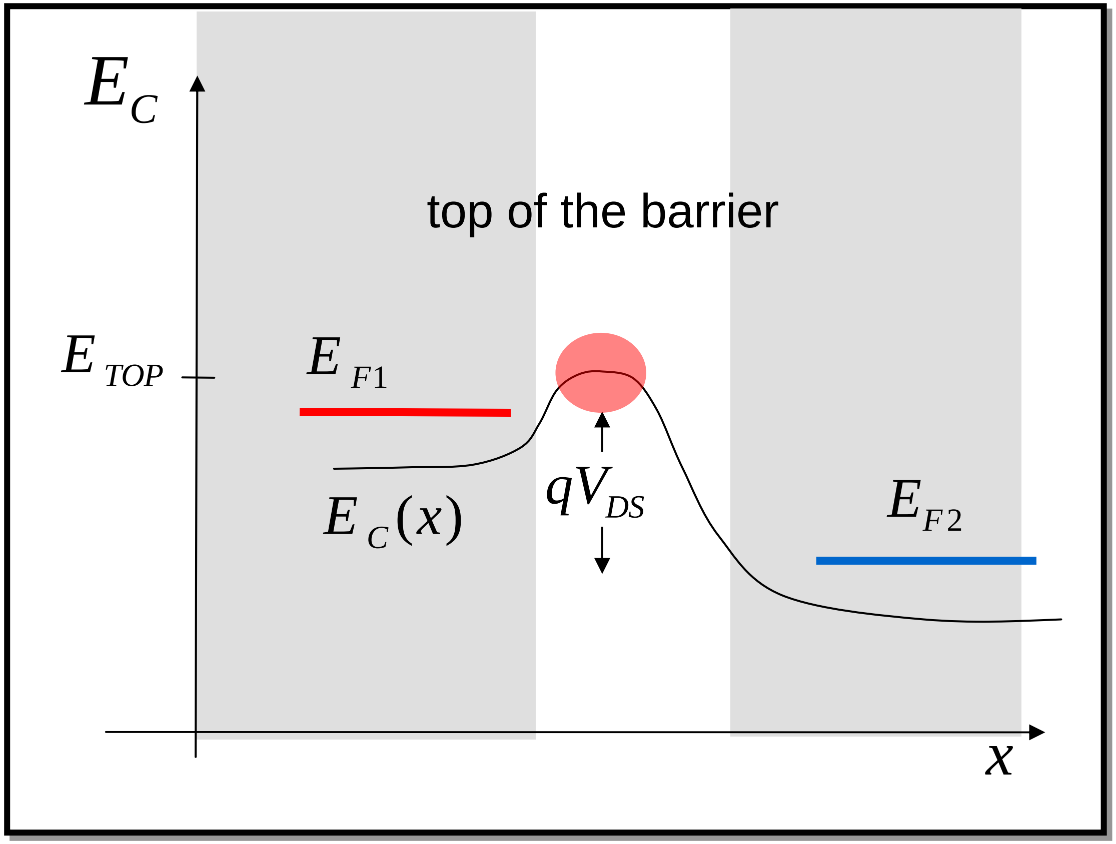
# the ballistic MOSFET




ballistic channel

We will evaluate the current at the top of the barrier ($x$ = 0) under the following **assumptions**.

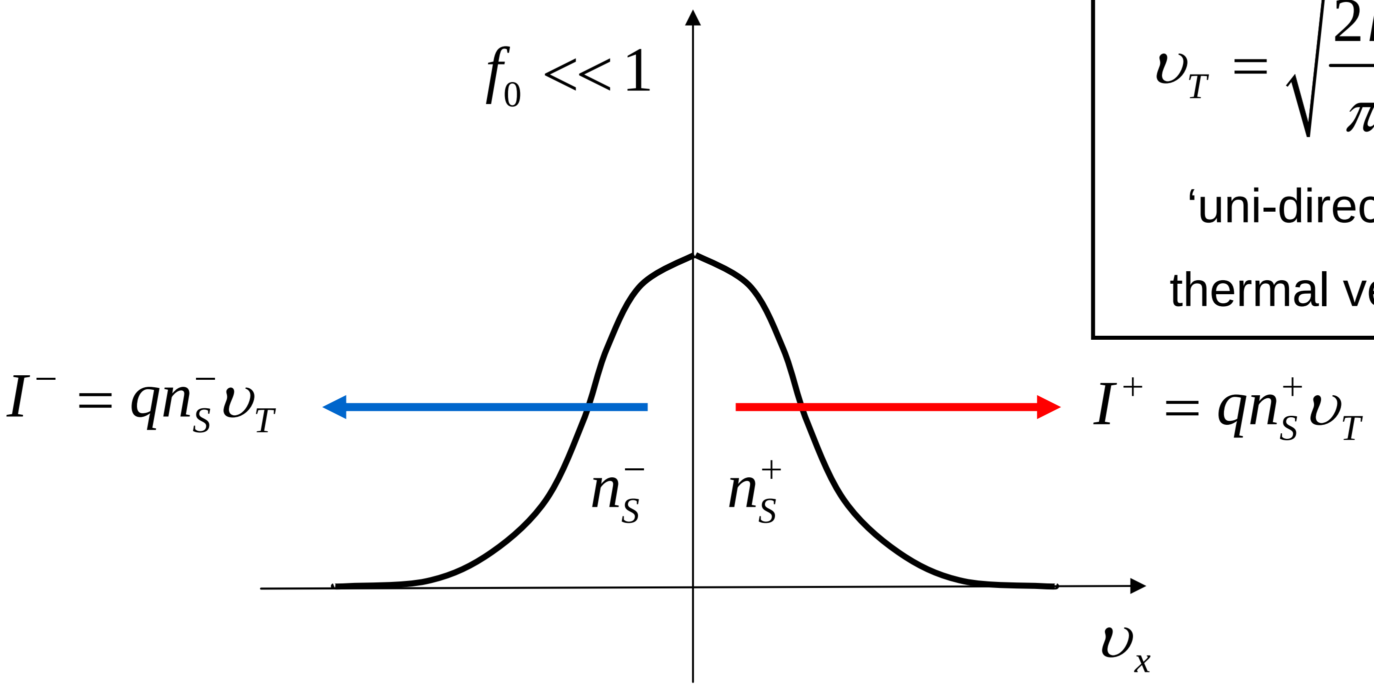
1) ballistic transport in the channel

2) source and drain are reservoirs of thermal equilibrium carriers.

3) nondegenerate carriers

4) $V_{GS} > V_T$

# equilibrium velocity distribution of carriers

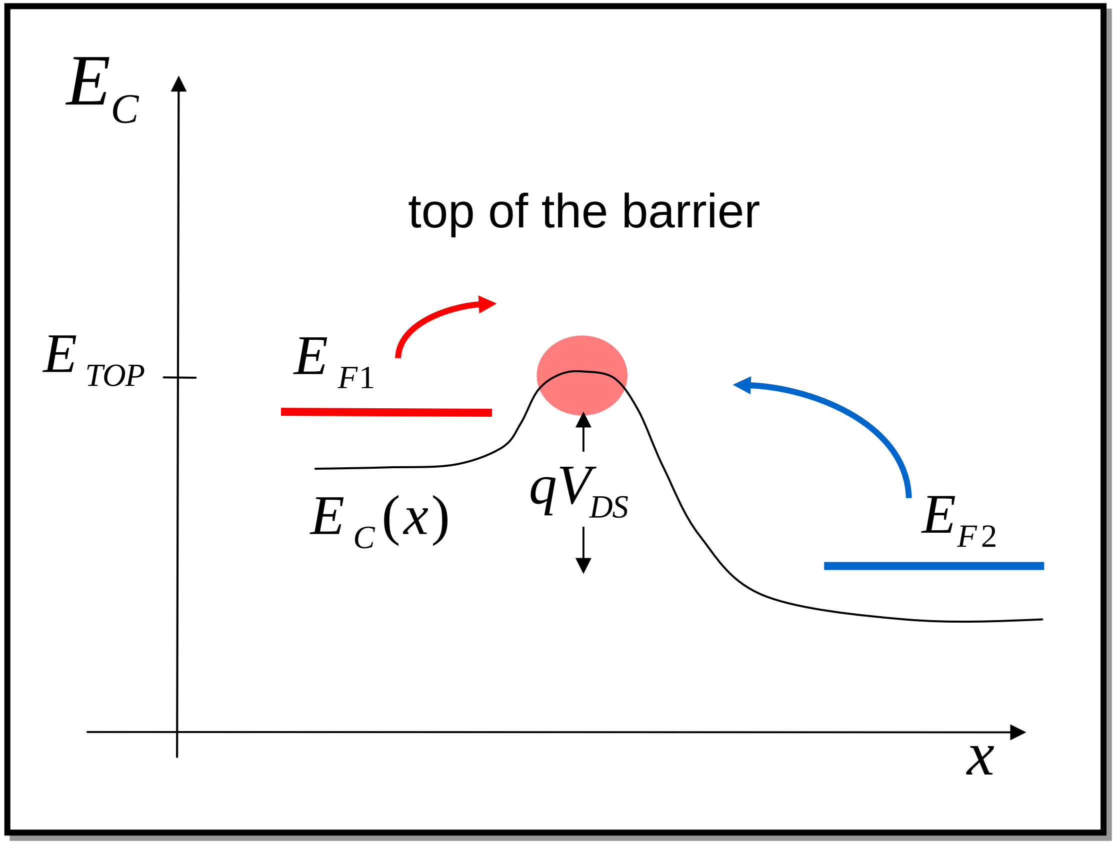



$$\upsilon_T = \sqrt{\frac{2k_BT}{\pi m^*}}$$

'uni-directional thermal velocity'

$$f_0 = \frac{1}{1 + e^{(E - E_F/k_BT)}} \approx e^{(E_F - E)/k_BT} \propto e^{-m^* \upsilon_x^2/2k_BT}$$

# the ballistic MOSFET




$$Q_I(0) \ \text{C/cm}^2$$

$$Q_I = -q\left(n_S^+ + n_S^-\right) \ \text{C/cm}^2$$

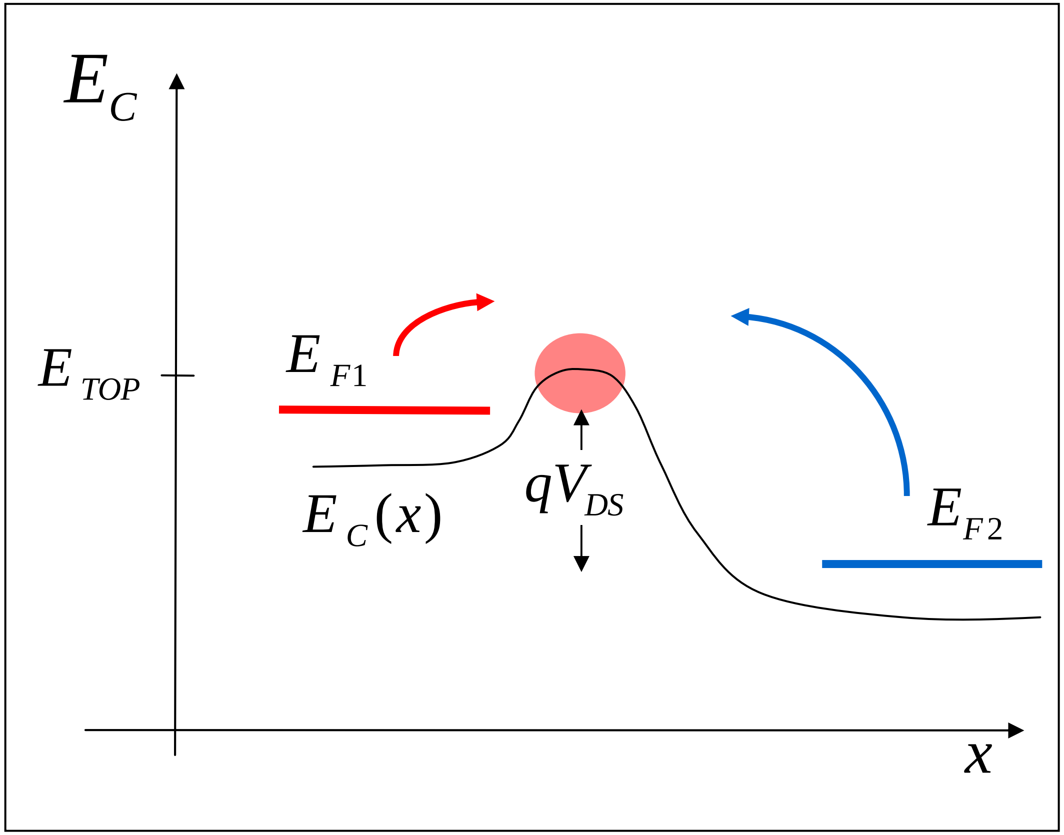
$$n_S^- / n_S^+ = e^{-qV_{DS}/k_BT}$$

$$Q_I = -qn_S^+\left(1 + e^{-qV_{DS}/k_BT}\right)$$

$$qn_S^+ = \frac{-Q_I}{1 + e^{-qV_{DS}/k_BT}}$$

1) ballistic transport
2) nondegenerate carriers

# the ballistic MOSFET:  IV




$$I_D = Wq\left(n_S^+ \upsilon_T - n_S^- \upsilon_T\right)$$

$$I_D = Wqn_S^+ \upsilon_T \left(1 - e^{-qV_{DS}/k_BT}\right)$$

$$qn_S^+ = \frac{-Q_I}{1 + e^{-qV_{DS}/k_BT}}$$

$$I_D = -WQ_I \upsilon_T \left(\frac{1 - e^{-qV_{DS}/k_BT}}{1 + e^{-qV_{DS}/k_BT}}\right)$$

$$Q_I = -C_{ox}\left(V_{GS} - V_T\right)$$

1) ballistic
2) nondegenerate carriers
3) ***$V_{GS} > V_T$***

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# the ballistic MOSFET: IV

$$I_{DS} = WC_{ox}\left(V_{GS} - V_T\right)\upsilon_T\left(\frac{1 - e^{-qV_{DS}/k_BT}}{1 + e^{-qV_{DS}/k_BT}}\right)$$

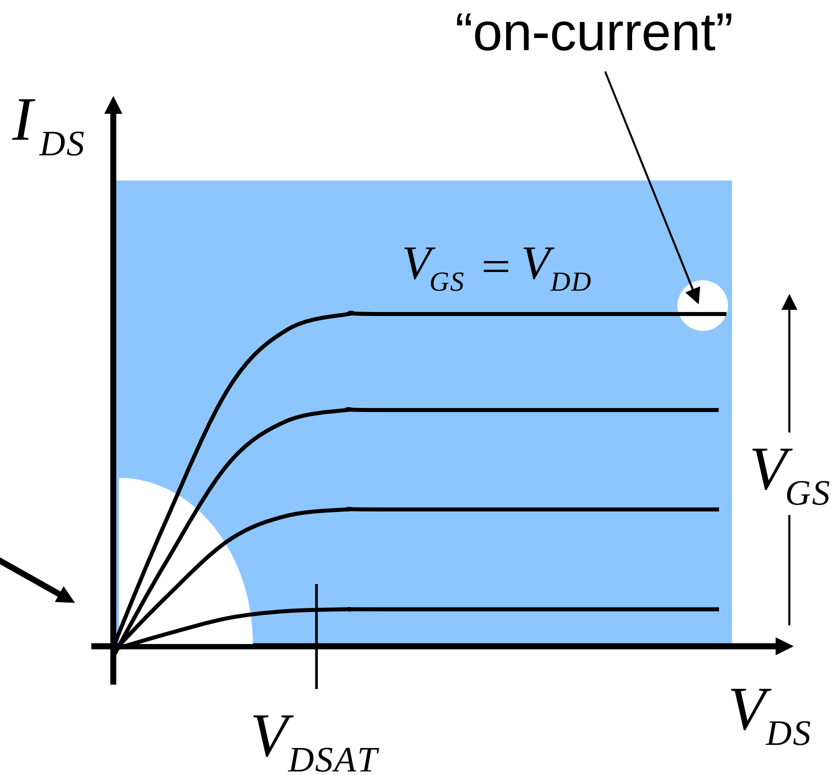
$$I_{ON} = WC_{ox}\upsilon_T\left(V_{DD} - V_T\right)$$

$V_{DS} < k_BT/q$

$$I_{DS} = WC_{OX}\frac{\upsilon_T}{2k_BT/q}\left(V_{GS} - V_T\right)V_{DS}$$

$$I_{DS} = V_{DS}/R_{CH}$$

$$V_{DSAT} \approx k_BT/q$$



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# outline

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2) Ballistic theory of the MOSFET
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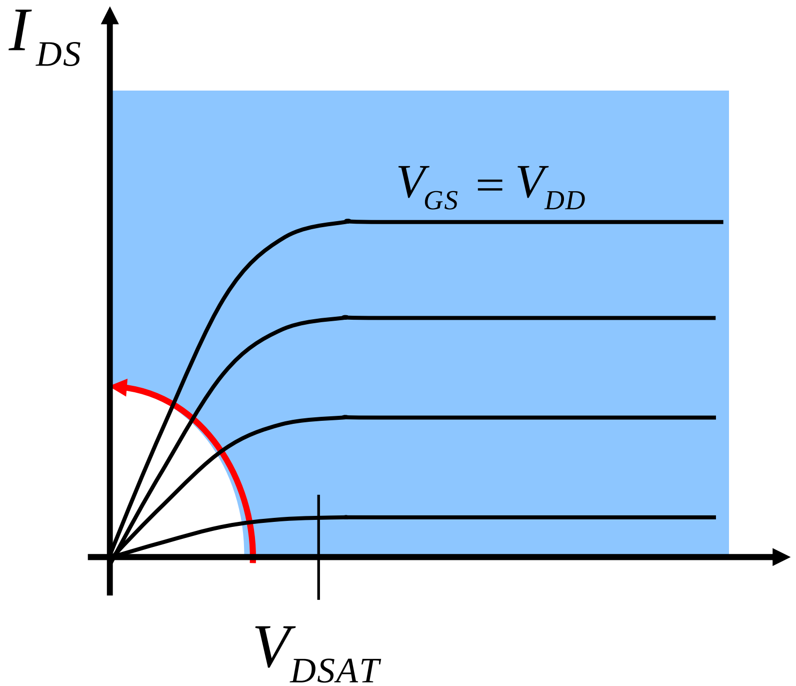
# low $V_{DS}$: diffusive vs. ballistic

**diffusive:**

$$I_D = \frac{W}{L}\mu_{eff}C_{ox}\left(V_{GS} - V_T\right)V_{DS}$$

$$I_D = V_{DS}/R_{CH}$$

$$R_{CH} = \frac{1}{\mu_{eff}C_{ox}\left(V_{GS} - V_T\right)}\left(\frac{L}{W}\right)$$

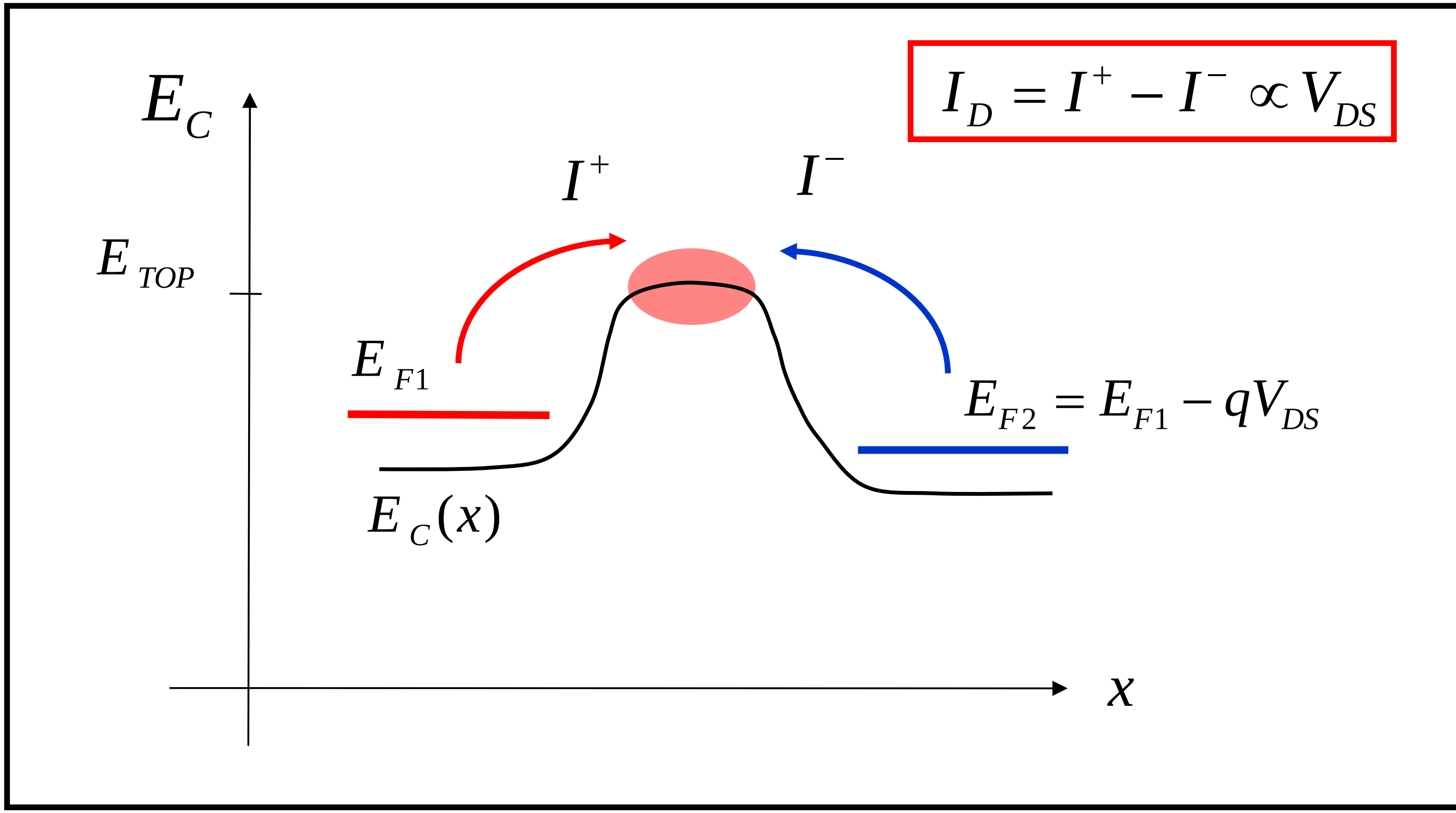



**ballistic:**

$$I_D = WC_{ox}\frac{\upsilon_T}{2k_BT/q}\left(V_{GS} - V_T\right)V_{DS}$$

- a ballistic MOSFET has a <u>finite</u> channel resistance - independent of *L*
- the channel resistance <u>cannot</u> be lower than the ballistic limit

# why is there a finite ballistic channel resistance?

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# relation between diffusive and ballistic models

diffusive:

$$I_D = \frac{W}{L}\mu_{eff} C_{ox}\left(V_{GS} - V_T\right)V_{DS}$$

ballistic:

$$I_D = WC_{ox}\frac{\upsilon_T}{2k_BT/q}\left(V_{GS} - V_T\right)V_{DS}$$

$$I_D = \frac{W}{L}C_{ox}\left[\frac{\upsilon_T L}{2k_BT/q}\right]\left(V_{GS} - V_T\right)V_{DS}$$

$$\mu_B \equiv \left[\frac{\upsilon_T L}{2k_BT/q}\right]$$

'ballistic mobility'

(M.S. Shur, *IEEE Elect. Dev. Lett.*, **3**, 511, 2002.)

$$I_D = \frac{W}{L}\mu_B C_{ox}\left(V_{GS} - V_T\right)V_{DS}$$

# the ballistic mobility

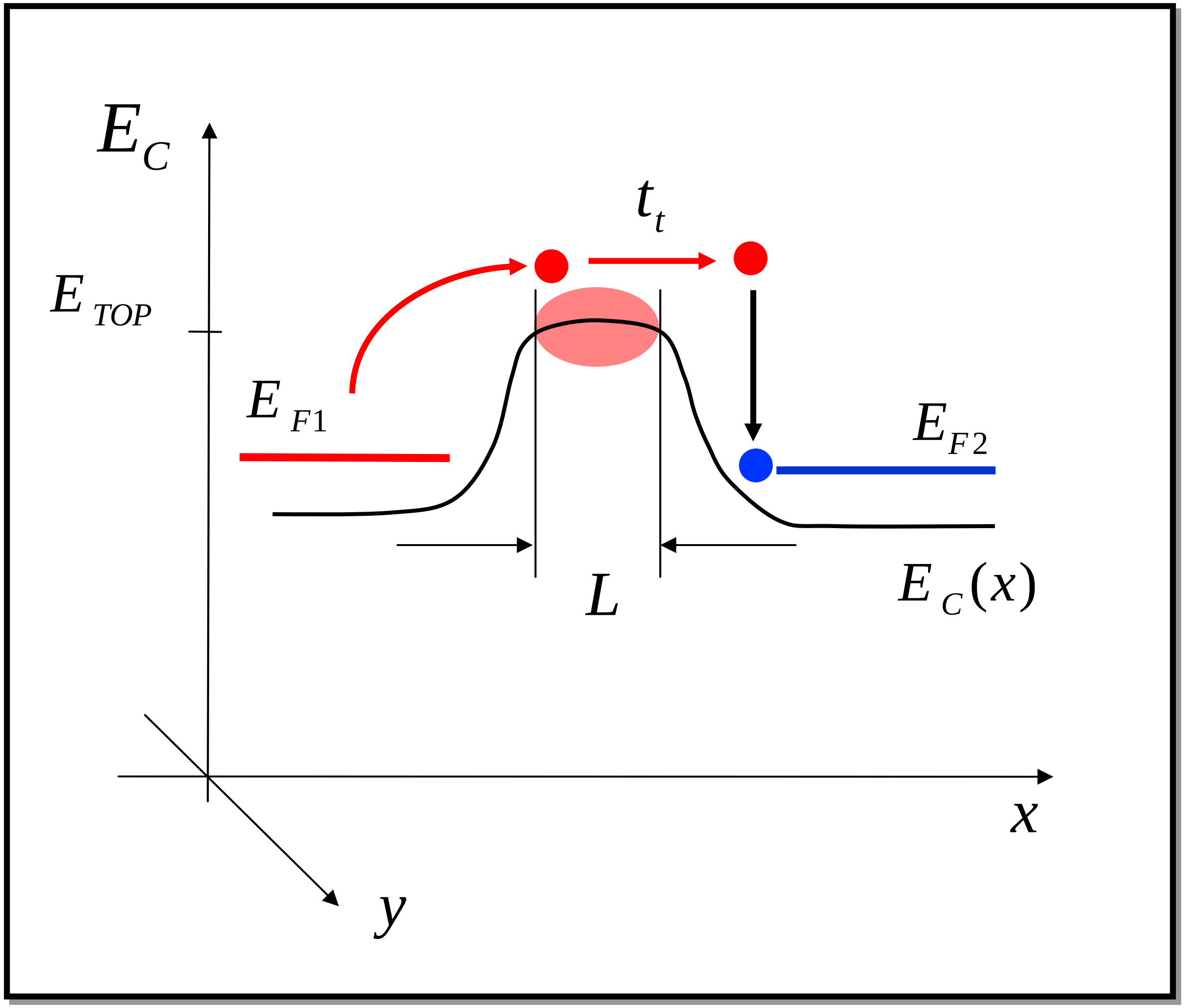
$$\mu_B \equiv \left[ \frac{\upsilon_T L}{2k_B T/q} \right]$$

$$\upsilon_T = \sqrt{2k_B T/\pi m^*}$$

$$\mu_B = \left[ \frac{q\left(L/\pi\upsilon_T\right)}{m^*} \right]$$

$$= \left[ \frac{q\tau_t}{m^*} \right]$$

$$t_t = L/\left(\pi\upsilon_T\right)$$



# quasi-ballistic MOSFET: low $V_{DS}$

**ballistic:**

$$I_D = \frac{W}{L}\mu_B C_{ox}\left(V_{GS} - V_T\right)V_{DS} \qquad \mu_B \equiv \left[\frac{\upsilon_T L}{2k_B T/q}\right]$$

**diffusive:**

$$I_D = \frac{W}{L}\mu_{eff} C_{ox}\left(V_{GS} - V_T\right)V_{DS}$$

**general:**

$$I_D = \frac{W}{L}\left(\frac{1}{\mu_{eff}} + \frac{1}{\mu_B}\right)^{-1} C_{ox}\left(V_{GS} - V_T\right)V_{DS}$$

# the ballistic mobility: example

For $L$ = 100 nm NMOS Si technology: $\mu_{eff} \approx 200 \text{ cm}^2/\text{V-s}$

$$\upsilon_T = \sqrt{2k_BT/\pi m^*} \approx 1\times 10^7 \text{ cm/s}$$

$$\mu_B \equiv \left[\frac{\upsilon_T L}{2k_BT/q}\right] \approx 2000 \text{ cm}^2/\text{V-s}$$

$\mu_{eff} << \mu_B$ so the real mobility limits the current; the actual current is roughly 10% of the ballistic limit.

But what would happen for a high-mobility (e.g. III-V) FET?

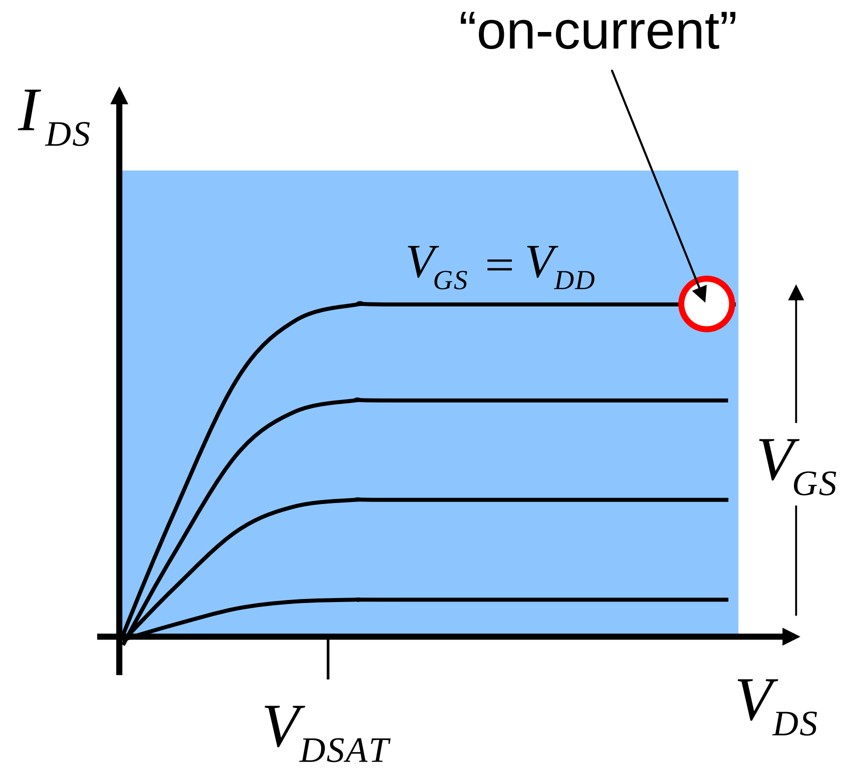
# high $V_{DS}$: diffusive vs. ballistic

**diffusive:**

$$I_D = W C_{ox} \upsilon_{sat} \left(V_{GS} - V_T\right)$$

**ballistic:**

$$I_{ON} = W C_{ox} \upsilon_T \left(V_{DD} - V_T\right)$$




How does the velocity saturate in a ballistic MOSFET?

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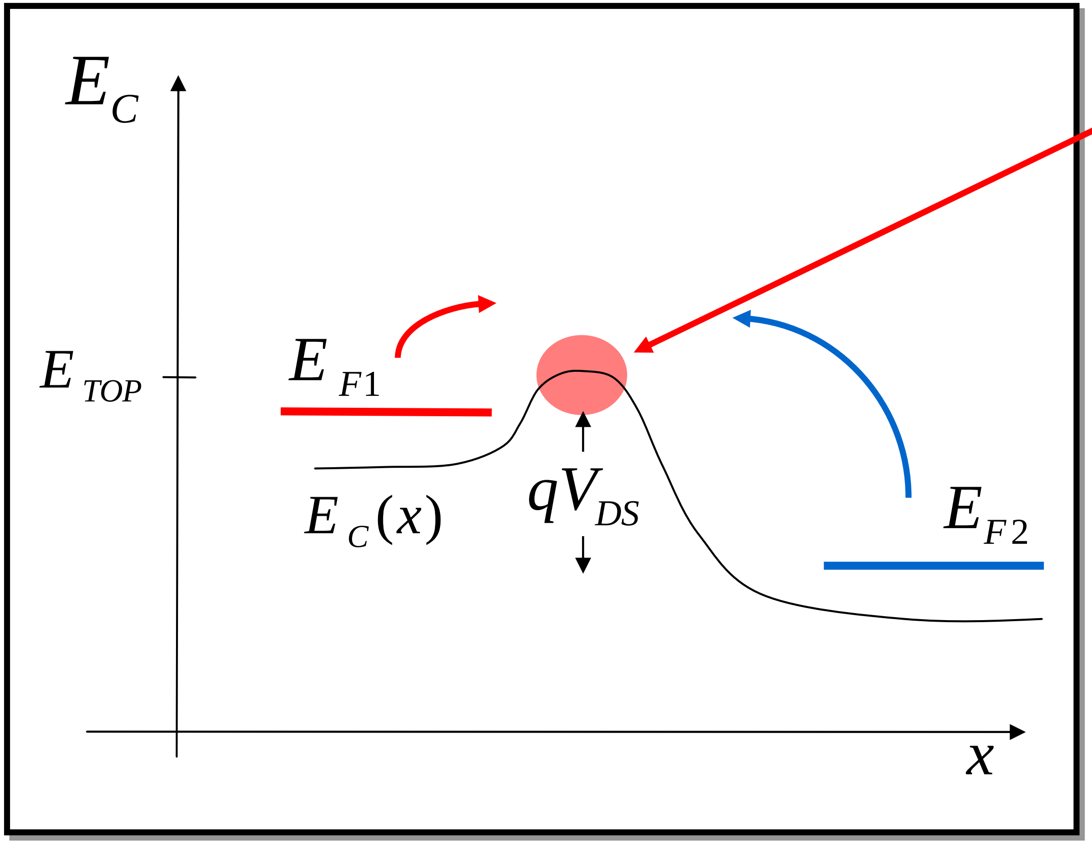
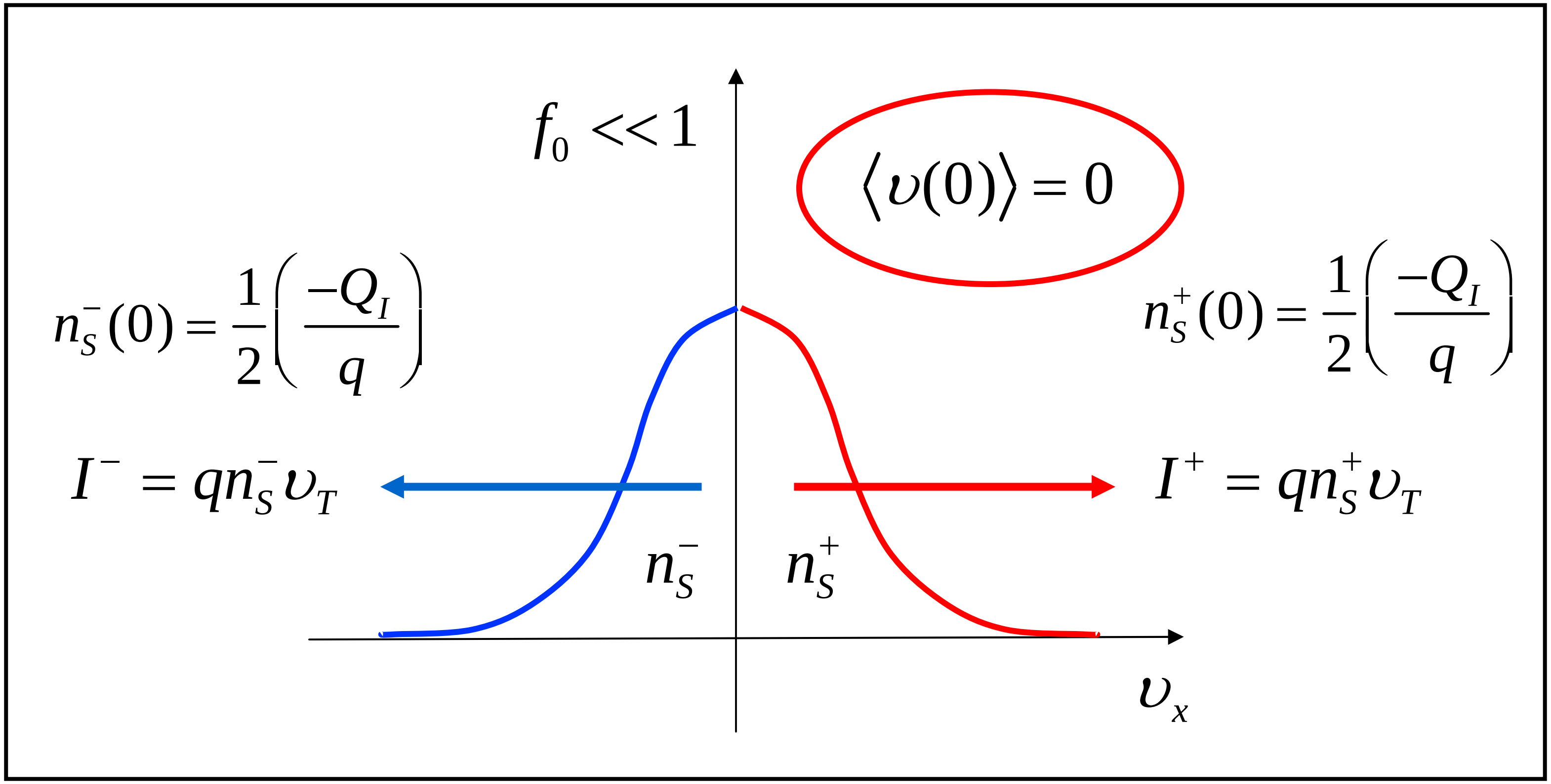
# velocity vs. drain bias in a ballistic MOSFET



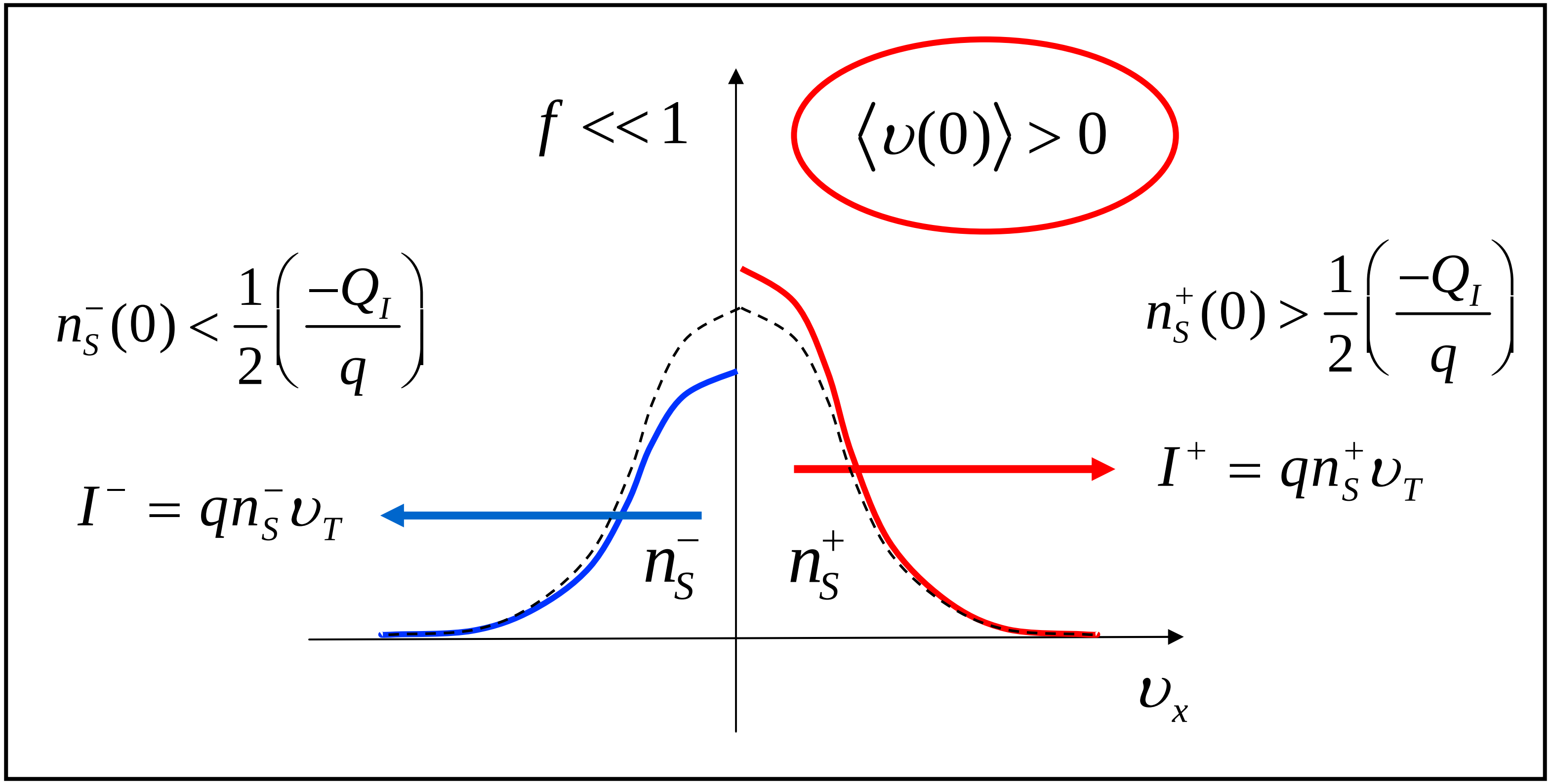

What is the velocity vs. drain bias at the top of the barrier?

# the ballistic MOSFET: $V_{DS} = 0$



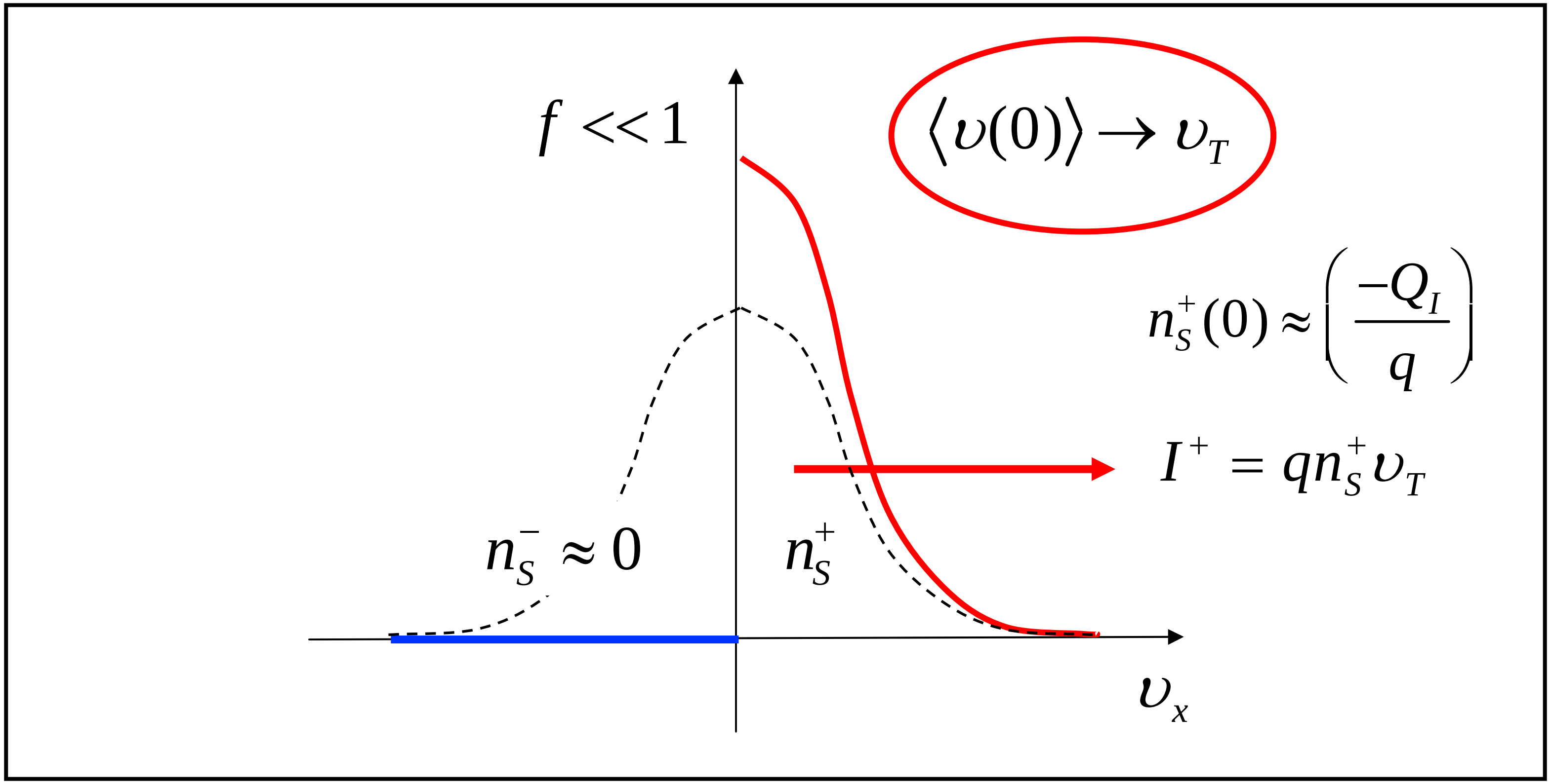

$$Q_I(0) = -q\left[n_S^+(0) + n_S^-(0)\right] = Q_G$$

# the ballistic MOSFET: $V_{DS} > 0$




$$Q_I(0) = -q\left[n_S^+(0) + n_S^-(0)\right] = Q_G$$

# the ballistic MOSFET: $V_{DS} >> 0$




$$Q_I(0) = -q\left[ n_S^+(0) + n_S^-(0) \right] = Q_G$$

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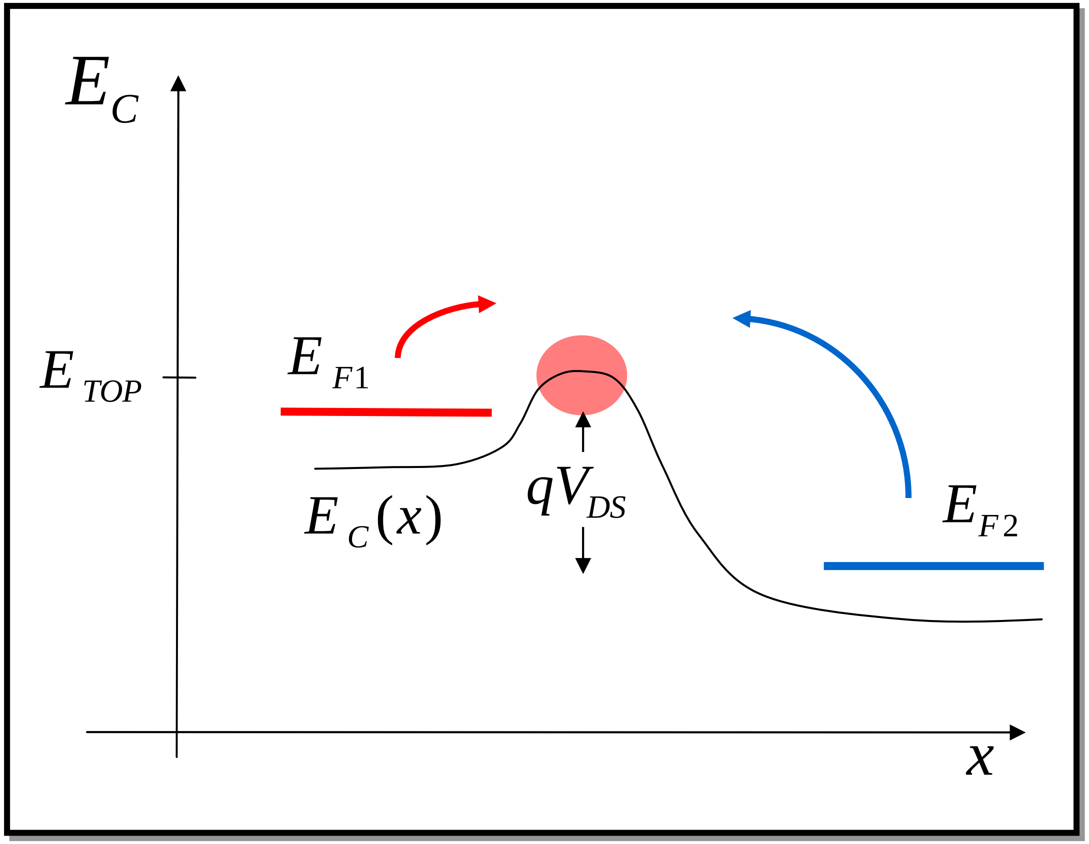
# velocity vs. drain bias in a ballistic MOSFET

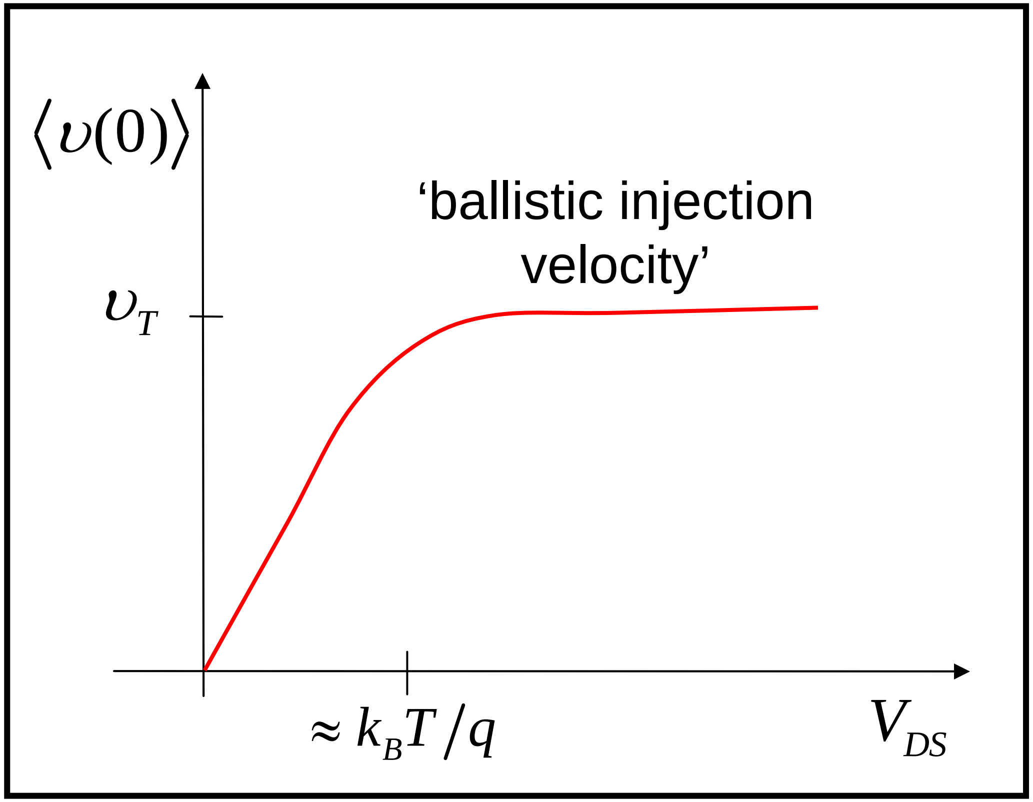



$$\langle \upsilon(0) \rangle = \frac{n_S^+ \upsilon_T - n_S^- \upsilon_T}{n_S^+ + n_S^-}$$

$$= \upsilon_T \frac{1 - n_S^- / n_S^+}{1 + n_S^- / n_S^+}$$

$$= \upsilon_T \frac{1 - e^{-qV_{DS}/k_B T}}{1 + e^{-qV_{DS}/k_B T}}$$

# velocity vs. drain bias in a ballistic MOSFET




$$\langle \upsilon(0) \rangle = \upsilon_T \left( \frac{1 - e^{-qV_{DS}/k_B T}}{1 + e^{-qV_{DS}/k_B T}} \right)$$

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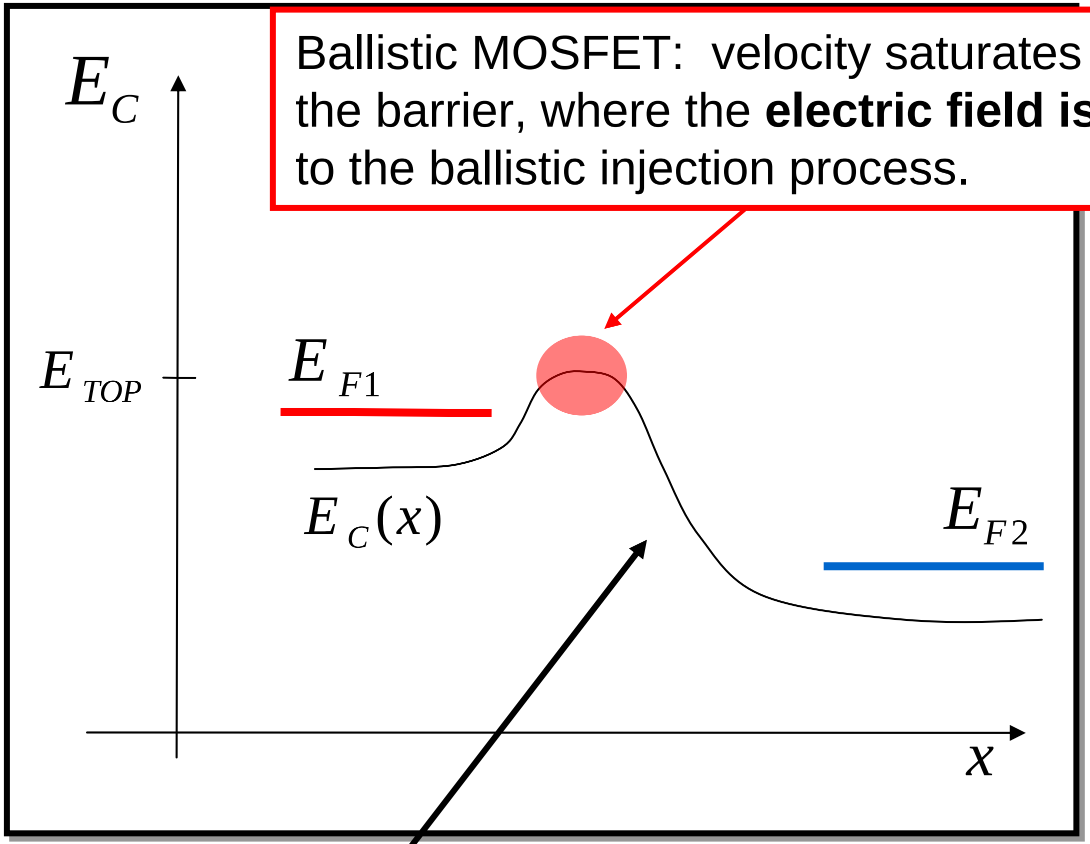
# velocity vs. drain bias in a ballistic MOSFET




Ballistic MOSFET: velocity saturates at the top of the barrier, where the **electric field is zero**, due to the ballistic injection process.

Long channel MOSFET: velocity saturates near the drain due to scattering in the **high electric field** region.

# what have we left out (of the ballistic model?)

1) Fermi-Dirac statistics (above threshold)

2) Treatment of sub-threshold conduction

3) Two-dimensional electrostatics

4) Multiple subbands and other details…

***These topics are discussed in "Physics of Nanoscale MOSFETs" www.nanoHUB.org***

# without assuming Boltzmann statistics

$$I_D = WC_{ox}\left(V_{GS} - V_T\right)\tilde{\upsilon}_T \left[\frac{1 - \mathcal{F}_{1/2}\left(\eta_{F2}\right)/\mathcal{F}_{1/2}\left(\eta_{F1}\right)}{1 + \mathcal{F}_{0}\left(\eta_{F2}\right)/\mathcal{F}_{0}\left(\eta_{F1}\right)}\right]$$

$$\tilde{\upsilon}_T \equiv \sqrt{\frac{2k_BT}{\pi m^*}}\,\frac{\mathcal{F}_{1/2}\left(\eta_{F1}\right)}{\mathcal{F}_{0}\left(\eta_{F1}\right)} = \upsilon_T \frac{\mathcal{F}_{1/2}\left(\eta_{F1}\right)}{\mathcal{F}_{0}\left(\eta_{F1}\right)}$$

See: “Notes on Fermi-Dirac Integrals, 2nd Edition”

by Raseong Kim and Mark Lundstrom

Boltzmann limit: $\left[E_F - \varepsilon_1(0)\right]/k_BT << 0 \qquad \eta << 0 \quad \mathcal{F}_j\left(\eta\right) \rightarrow e^{\eta}$

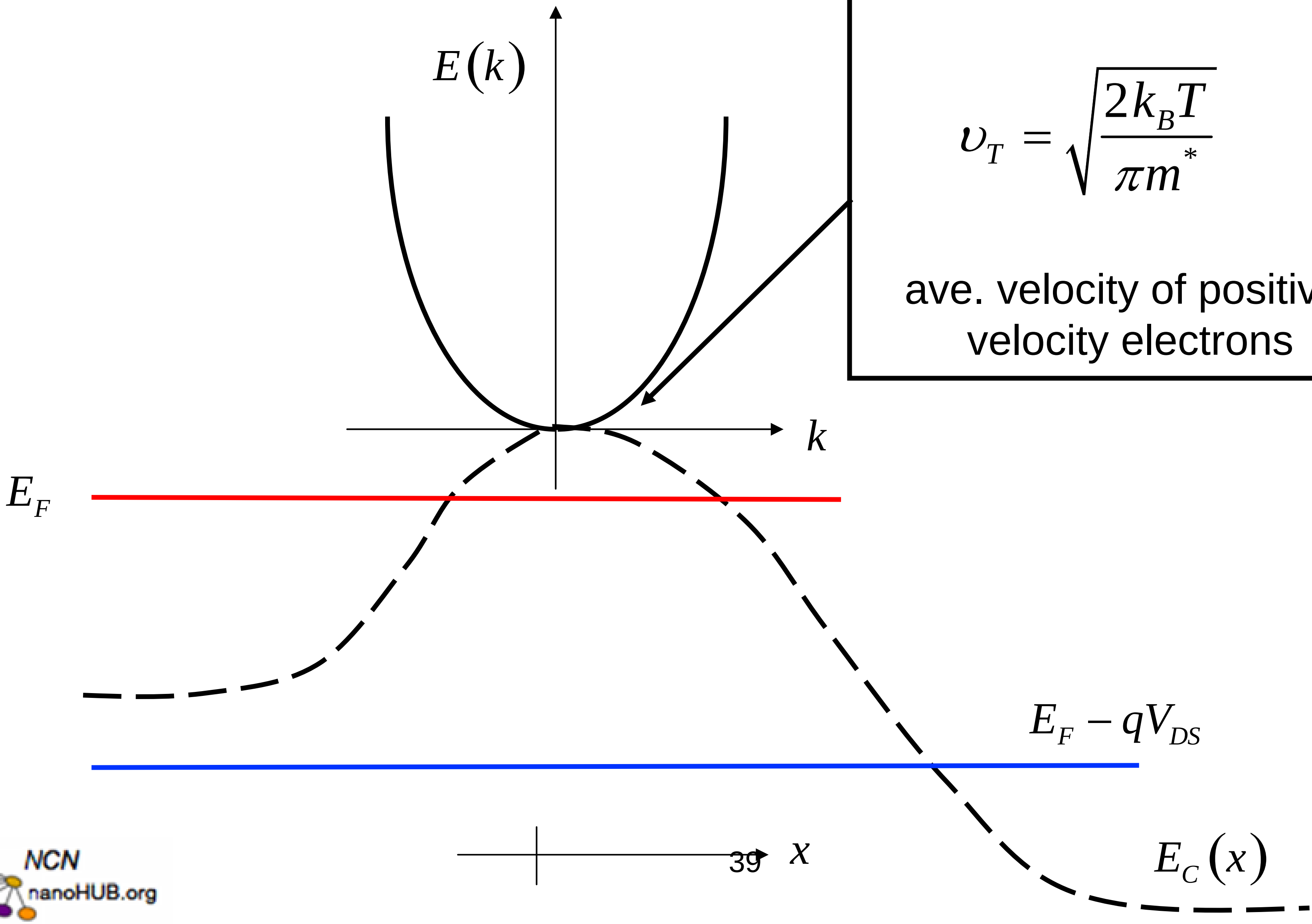
# Boltzmann limit

$$I_D = WC_{ox}\left(V_{GS} - V_T\right)\tilde{\upsilon}_T \left[\frac{1 - \mathcal{F}_{1/2}\left(\eta_{F2}\right)/\mathcal{F}_{1/2}\left(\eta_{F1}\right)}{1 + \mathcal{F}_0\left(\eta_{F2}\right)/\mathcal{F}_0\left(\eta_{F1}\right)}\right]$$

$$\tilde{\upsilon}_T \equiv \sqrt{\frac{2k_BT}{\pi m^*}}\frac{\mathcal{F}_{1/2}\left(\eta_{F1}\right)}{\mathcal{F}_0\left(\eta_{F1}\right)} = \upsilon_T \frac{\mathcal{F}_{1/2}\left(\eta_{F1}\right)}{\mathcal{F}_0\left(\eta_{F1}\right)}$$

$$\tilde{\upsilon}_T \to \upsilon_T = \sqrt{\frac{2k_BT}{\pi m^*}} \qquad I_D \to WC_{ox}\left(V_{GS} - V_T\right)\upsilon_T \left[\frac{1 - e^{-qV_{DS}/k_BT}}{1 + e^{-qV_{DS}/k_BT}}\right]$$

# injection velocity: non-degenerate




$$\upsilon_T = \sqrt{\frac{2k_BT}{\pi m^*}}$$

ave. velocity of positive-velocity electrons

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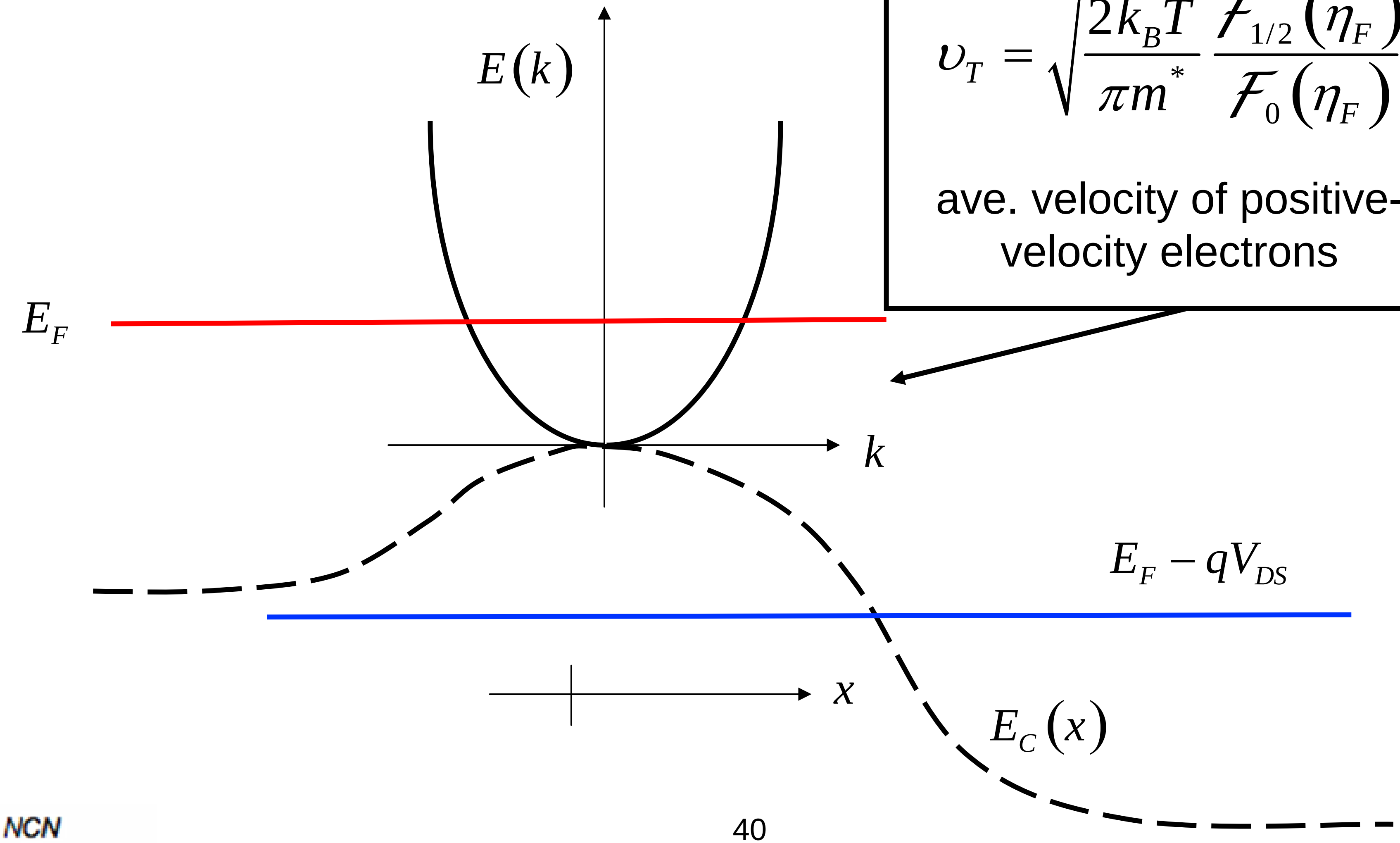
# injection velocity: degenerate

$$\upsilon_T = \sqrt{\frac{2k_BT}{\pi m^*}} \frac{\mathcal{F}_{1/2}(\eta_F)}{\mathcal{F}_0(\eta_F)}$$

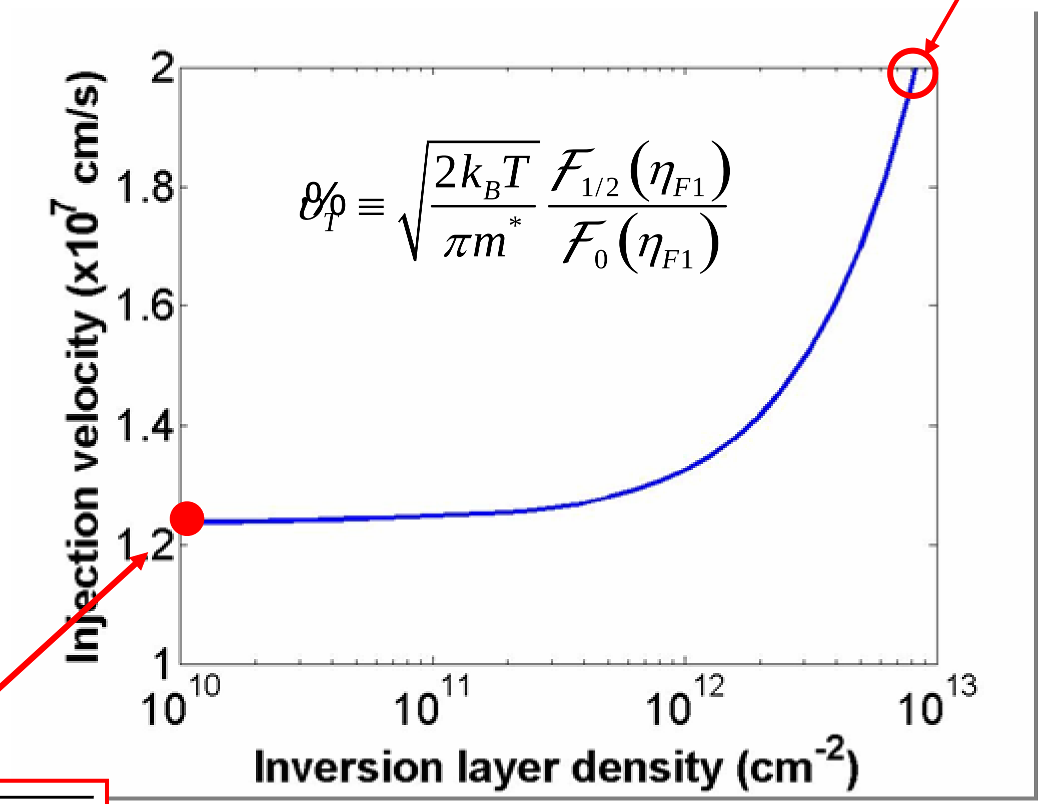
ave. velocity of positive-velocity electrons



# injection velocity vs. $n_S(0)$

$$\tilde{\upsilon}_T \rightarrow (4/3\pi)\upsilon_F$$

$$\upsilon_F = \frac{\hbar k_F}{m^*}$$

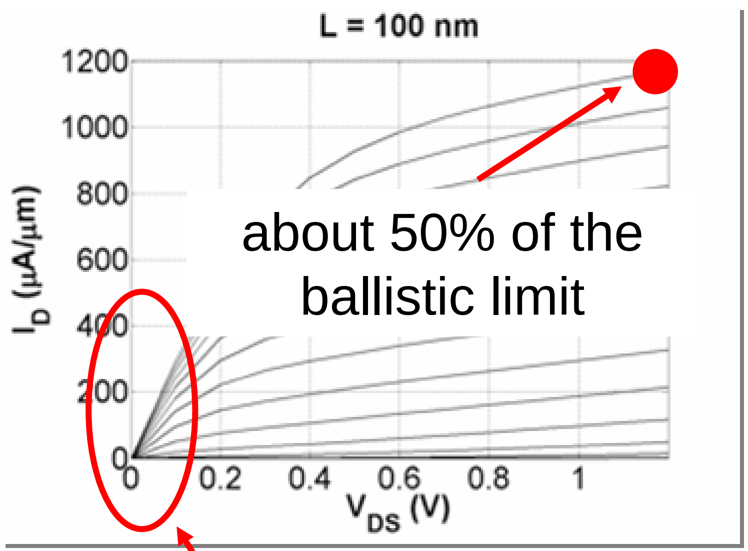



$$\upsilon_T = \sqrt{2k_BT/\pi m^*}$$

# Is a nanoscale MOSFET really ballistic?

Typical N-channel MOSFET:

$$I_{ON} \approx 1\ \text{mA}/\mu\text{m}$$




(Courtesy, Shuji Ikeda, ATDF, Dec. 2007)

about 10% of the ballistic limit.

$$I_{ON}\left(\text{ballistic}\right) = -WQ_I(0)\upsilon_T$$

$$\upsilon_T = \sqrt{2k_BT/\pi m^*}$$

$$\approx 1.2 \times 10^7\ \text{cm/s}$$

$$-Q_I(0)/q = C_{ox}\left(V_{DD} - V_T\right)$$

$$\approx 1 \times 10^{13}\ \text{cm}^{-2}$$

$$I_{ON}/W\left(\text{ballistic}\right) \approx 2\ \text{mA}/\mu\text{m}$$

# outline

1) Introduction
2) Ballistic theory of the MOSFET
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4) **Summary**

# summary

1) MOSFETs are 'barrier-controlled' devices (just like bipolar transistors)

2) A simple, ballistic model is easy to derive (assuming Boltzmann statistics)

3) Connection to diffusive model is clear

$$\mu_{eff} \rightarrow \mu_B \qquad \upsilon_{sat} \rightarrow \upsilon_T$$

4) In practice, degenerate carrier statistics must be used when $V_{GS} > V_T$.

# what have we accomplished?

Developed a very simple, conceptual model that captures the essential physics of a nanoscale MOSFET

- not a replacement for numerical simulation, but helpful in interpreting simulation results.
- provides a ‘sanity check’ for numerical models and for compact circuit models.
- useful for interpreting experiments and guiding device design.