EE-612: Lecture 8
Scattering Theory of the MOSFET:

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Physics of Nanoscale MOSFETs

This lecture (and the last one) are part of the series:

“Physics of Nanoscale MOSFETs”
by Mark Lundstrom
http://www.nanoHUB.org/resources/5306

which discusses this material in more depth.
1) Review and introduction
2) Scattering theory of the MOSFET
3) Transmission under low $V_{DS}$
4) Transmission under high $V_{DS}$
5) Discussion
6) Summary
the ballistic MOSFET: IV

\[ I_{DS} = WC_{ox} \left( V_{GS} - V_T \right) \nu_T \left( \frac{1 - e^{-qV_{DS}/k_BT}}{1 + e^{-qV_{DS}/k_BT}} \right) \]

\[ I_{ON} = WC_{ox} \nu_T \left( V_{DD} - V_T \right) \]

\[
\begin{cases}
  V_{DS} < k_B T / q \\
  I_{DS} = WC_{OX} \frac{\nu_T}{2k_B T / q} \left( V_{GS} - V_T \right) V_{DS} \\
  I_{DS} = V_{DS} / R_{CH}
\end{cases}
\]

\[ V_{DSAT} \approx k_B T / q \]

“on-current”
review: ballistic I-V

\[ I_D = W C_{ox} (V_{GS} - V_T) \varphi_f \left[ \frac{1 - \mathcal{F}_{1/2}(\eta_{F_2})/\mathcal{F}_{1/2}(\eta_{F_1})}{1 + \mathcal{F}_0(\eta_{F_2})/\mathcal{F}_0(\eta_{F_1})} \right] \]

\[ \varphi_f = \sqrt{\frac{2k_B T}{\pi m^*}} \frac{\mathcal{F}_{1/2}(\eta_{F_1})}{\mathcal{F}_0(\eta_{F_1})} = \nu_T \frac{\mathcal{F}_{1/2}(\eta_{F_1})}{\mathcal{F}_0(\eta_{F_1})} \]

\[ \eta_{F_1} = (E_F - \varepsilon_1)/k_B T \quad \eta_{F_2} = (E_F - qV_{DS} - \varepsilon_1)/k_B T \]

\[ \mathcal{F}_{1/2}(\eta_F) \rightarrow e^{n_F} \quad (\eta_F \ll 0) \]
review: ballistic transport in a MOSFET

\[ L \ll \lambda \]

\[ \langle KE \rangle = \frac{1}{2} m^* \nu^2 \]

\[ \varepsilon_1(0) \]

\[ \varepsilon_1(x) \]
review: diffusive transport in a MOSFET

$L \gg \lambda$

$E$

$\varepsilon_1(0)$

$x$

$\varepsilon_1(x)$
Nanoscale MOSFETs are neither fully ballistic nor fully diffusive; they operate in a ‘quasi-ballistic’ regime.

How do we understand how carrier scattering affects the performance of a nanoscale MOSFET?
current transmission in a MOSFET

elastic scattering....

\[ R_{11}(E)I(E) = [1 - T_{12}(E)]I(E) \]
current transmission in a MOSFET

elastic scattering….
transmission in the presence of elastic scattering

\[ T_{12}(E) = T_{21}(E) = T(E) \]
inelastic scattering

\[ T_{12}(E) \neq T_{21}(E) \]

Some states are now filled by backscattering from source-injected electrons.

Some states are still filled from the drain, but the magnitude is reduced by back-scattering.

Filled by injection from the source.

Reflectionless contacts.

$E_F$ - Fermi level

$\epsilon_1(0)$ - Fermi level at $x=0$

$E_1(x)$ - Energy level at $x$

$E_{F1}-qV_D$ - Energy level at the drain
1) Review and introduction
2) **Scattering theory of the MOSFET**
3) Transmission under low $V_{DS}$
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scattering theory of the MOSFET

Goal:
To illustrate the influence on scattering on the I-V characteristic of a MOSFET by developing a very simple theory.

Assumptions:
1) Average quantities, not energy-resolved.
2) Boltzmann statistics for carriers
3) $T_{12} = T_{21} = T$
4) Average velocity of backscattered carriers equals that of the injected carriers.
scattering in a nano-MOSFET

\[ T + R = 1 \]
\[ I_1 = qn^+_S(0)v_T \]
\[ I_D = W \left( q n_S^+ (0) \nu_T - q n_S^- (0) \nu_T \right) = W q n_S^+ (0) \nu_T \left[ 1 - n_S^- (0) / n_S^+ (0) \right] \quad (1) \]

\[ n_S (0) = n_S^+ (0) + n_S^- (0) = n_S^+ (0) \left[ 1 + n_S^- (0) / n_S^+ (0) \right] \quad (2) \]
\[ I_D = W q n_S(0) \nu_T \left( \frac{1 - n_S^-(0)/n_S^+(0)}{1 + n_S^-(0)/n_S^+(0)} \right) \]

Exactly the same result we had for the ballistic case, but the (- velocity) carrier density at the top of the barrier is altered by scattering.
carrier densities at the top of the barrier

\[ n_S^-(0) = R \frac{I_1}{q\nu_T} \]

\[ n_S^+(0) = \frac{I_1}{q\nu_T} \]

\[ n_S^-(0) = Tn_S^+(0)e^{-qV_{DS}/k_BT} \]
from carrier densities to drain current

\[ n_s^-(0) = Rn_s^+ (0) + Tn_s^+ (0) e^{-qV_{DS}/k_BT} = n_s^+ (0) \left[ R + (1 - R) e^{-qV_{DS}/k_BT} \right] \]

\[ \frac{n_s^-(0)}{n_s^+ (0)} = R + (1 - R) e^{-qV_{DS}/k_BT} \]

\[ I_D = WQ_I (0) \nu_T \left( \frac{1 - n_s^-(0) / n_s^+ (0)}{1 + n_s^-(0) / n_s^+ (0)} \right) \]

\[ I_D = WQ_I (0) \nu_T \left( \frac{(1 - R) - (1 - R) e^{-qV_{DS}/k_BT}}{(1 + R) + (1 - R) e^{-qV_{DS}/k_BT}} \right) \]
the MOSFET I-V with scattering

\[ I_{DS} = WC_{ox} (V_{GS} - V_T) \nu_T \left( \frac{1 - e^{qV_{DS}/k_BT}}{1 + e^{qV_{DS}/k_BT}} \right) \text{ (ballistic, Boltzmann statistics)} \]

\[ I_D = WC_{ox} (V_{GS} - V_T) \nu_T \left( \frac{(1 - R) - (1 - R)e^{-qV_{DS}/k_BT}}{(1 + R) + (1 - R)e^{-qV_{DS}/k_BT}} \right) \]

\[ T = (1 - R) \]

\[ I_D = WC_{ox} (V_{GS} - V_T) \nu_T T \left( \frac{1 - e^{-qV_{DS}/k_BT}}{(2 - T) + Te^{-qV_{DS}/k_BT}} \right) \]

\[ I_D \text{ (scattering)} \neq TI_D \text{ (ballistic)} \]
high drain bias

\[ I_D = W C_{ox} (V_{GS} - V_T) \nu_T \left( \frac{(1 - R) - (1 - R)e^{-q V_{DS}/k_B T}}{(1 + R) + (1 - R)e^{-q V_{DS}/k_B T}} \right) \]

\[ I_D = W C_{ox} (V_{GS} - V_T) \nu_T \frac{1 - R}{1 + R} \]

\[ \langle \nu(0) \rangle = \nu_T \frac{1 - R}{1 + R} \leq \nu_T \]
low drain bias

\[ I_D = WC_{ox} \left( V_{GS} - V_T \right) \nu_T \left\{ \frac{(1 - R) - (1 - R)e^{-qV_DS/k_BT}}{(1 + R) + (1 - R)e^{-qV_DS/k_BT}} \right\} \]

\[ G_{CH} = \frac{I_D}{V_{DS}} = WC_{ox} \left( V_{GS} - V_T \right) \left[ \frac{\nu_T}{2\left(k_B T/q\right)} \right] (1 - R) \]

\[ G_{CH} \text{ (scattering)} = TG_{CH} \text{ (ballistic)} \]
To proceed, we need to understand $R(V_{GS}, V_{DS})$.
1) Review and introduction
2) Scattering theory of the MOSFET
3) **Transmission under low** $V_{DS}$
4) Transmission under high $V_{DS}$
5) Discussion
6) Summary
transmission across a field-free slab

Consider a flux of carriers injected into a field-free slab of length, \( L \). The flux that emerges at \( x = L \) is \( T \) times the incident flux, where \( 0 < T < 1 \). The flux that emerges from \( x = 0 \) is \( R \) times the incident flux, where \( T + R = 1 \), assuming no carrier recombination-generation.

How is \( T \) related to the mean-free-path for backscattering within the slab?
transmission (iii)

\[ I_1 = I^+(x = 0) \]

\[ RI_1 \]

\[ mfp = \lambda \quad \mathcal{E} = 0 \]

\[ I^+(x) \quad I^-(x) \]

absorbing boundary

\[ T \rightarrow 0 \quad L \gg \lambda_0 \]

\[ T \rightarrow 1 \quad L \ll \lambda_0 \]

\[ T = \frac{\lambda_0}{\lambda_0 + L} \quad R = \frac{L}{\lambda_0 + L} \]
How do we relate $\lambda_0$ to known parameters?

If $I_1$ is a thermal equilibrium injected flux, $I_1 = n^+(0)v_T$ then, it can be shown that:

$$D_n = \frac{k_B T}{q} \mu_n = \frac{v_T}{2} \lambda_0$$

(non-degenerate carrier statistics)
example

\[ \varepsilon_1(x) \]

\[ \text{low } V_{DS} \]

\[ I_1 \]

\[ L \]

\[ \mu_n \approx 200 \text{ cm}^2/\text{V-s} \]

\[ \mu_n = \frac{\nu_T}{2 \left( k_B T / q \right)} \lambda_0 \]

\[ \lambda_0 \approx 9 \text{ nm} \]

\[ L \approx 50 \text{ nm} \]

\[ T \approx \frac{\lambda_0}{L + \lambda_0} \approx 0.15 \]
relation to conventional theory

\[ G_{CH} = \left( WC_{ox} (V_{GS} - V_T) \frac{\nu_T}{(2k_B T / q)} \right) (1 - R) \]

\[ 1 - R = T = \frac{\lambda_0}{\lambda_0 + L} \approx \frac{\lambda_0}{L} \] (diffusive limit)

\[ \lambda_0 = \frac{2k_B T / q}{\nu_T} \mu_n \]

The scattering model works in the diffusive limit, as well as the ballistic limit, and in the quasi-ballistic regime in between.

(non-degenerate carrier statistics)
channel conductance

\[ G_{CH} = T \left( WC_{ox} (V_{GS} - V_T) \frac{\nu_T}{(2k_B T / q)} \right) \]

\[ T = \frac{\lambda_0}{\lambda_0 + L} \]

one can show that:

\[ G_{CH} = \frac{W}{L} \left( \frac{1}{\mu_n} + \frac{1}{\mu_B} \right)^{-1} \quad C_{ox} (V_{GS} - V_T) \]

\[
\begin{align*}
\mu_n &= \frac{\nu_T \lambda_0}{2k_B T / q} \\
\mu_B &= \frac{\nu_T L}{2k_B T / q}
\end{align*}
\]
4) Transmission under high $V_{DS}$
transmission under high drain bias

**scattering model:**

\[ I_{ON} = WC_{ox} (V_{GS} - V_T) \eta_p \frac{(1 - R)}{(1 + R)} = WC_{ox} (V_{GS} - V_T) \eta_p \frac{T}{(2 - T)} \]

**in practice:**

\[ B \equiv \frac{I_{ON} \text{ (measured)}}{I_{ON} \text{ (ballistic)}} \approx 0.50 \]

\[ B = \frac{T}{(2 - T)} \rightarrow T \approx 0.67 >> 0.15 \quad \text{Why?} \]
transmission across a slab with an electric field

When the electric field is strong and position-dependent and several scattering mechanisms operate, this turns out to be a difficult problem.

How can we understand the essential physics?
transport “downhill”

\[
T = \frac{\lambda_o}{1 + \lambda_o} \quad \ell \ll L
\]

\(T \approx 1:\) High field regions are good carrier collectors.

transport in a MOS transistor

1) A MOSFET consists of a low-field region near the source that is strongly controlled by the gate voltage, and a high-field region near the drain that is strongly controlled by the drain voltage.

2) Transmission is controlled by the low-field region near the source.

3) Scattering near the drain has a smaller effect on backscattering to the source.
bias-dependent transmission

\[ T \approx \frac{\lambda_o}{L + \lambda_o} \approx 0.15 \]

\[ T \approx \frac{\lambda_o}{1 + \lambda_o} \approx 0.67 \]
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relation to conventional theory (high $V_{DS}$)

\[ T = \frac{\lambda_o}{1 + \lambda_o} \approx \frac{\lambda_o}{1} \quad (\lambda_o << 1) \]

\[ I_D = WC_{ox} \frac{T}{2 - T} \nu_T (V_{GS} - V_T) \]

\[ I_D \approx WC_{ox} \frac{T}{2} \nu_T (V_{GS} - V_T) \]

\[ I_D \approx WC_{ox} \frac{\lambda_0}{2l} \nu_T (V_{GS} - V_T) \]

\[ I_D \approx WC_{ox} \frac{D^n}{l} (V_{GS} - V_T) \]

How do we physically interpret this result?
The top of the barrier is a bottleneck that carriers must diffuse across.

\[ I_D = W C_{ox} \frac{D_n}{l} (V_{GS} - V_T) \]
drift-diffusion vs. scattering model

\[ I_D = WC_{ox} \left[ \frac{1}{\nu_T} + \frac{1}{(D_n/l)} \right]^{-1} (V_{GS} - V_T) \]

\[ D_n = \nu_T \lambda_0 / 2 \quad T = \frac{\lambda_0}{\lambda_0 + 1} \]

\[ I_D = WC_{ox} \frac{T}{2 - T} \nu_T (V_{GS} - V_T) \]

drift-diffusion

scattering
outline

1) Review and introduction
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summary

1) Modern MOSFETs operate between the ballistic and diffusive limits, so we need to understand transport in the quasi-ballistic regime.

2) Transmission (or scattering) theory provides a simple, physical description of quasi-ballistic transport.

3) The same physics can also be understood at the drift-diffusion level.

4) Quantitative treatments require detailed numerical simulation.
Questions & Answers