EE-612:
Lecture 8
Scattering Theory of the MOSFET:

Mark Lundstrom
Electrical and Computer Engineering
Purdue University
West Lafayette, IN USA
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This lecture (and the last one) are part of the series:

"Physics of Nanoscale MOSFETs"

by Mark Lundstrom

http://www.nanoHUB.org/resources/5306

which discusses this material in more depth.

outline

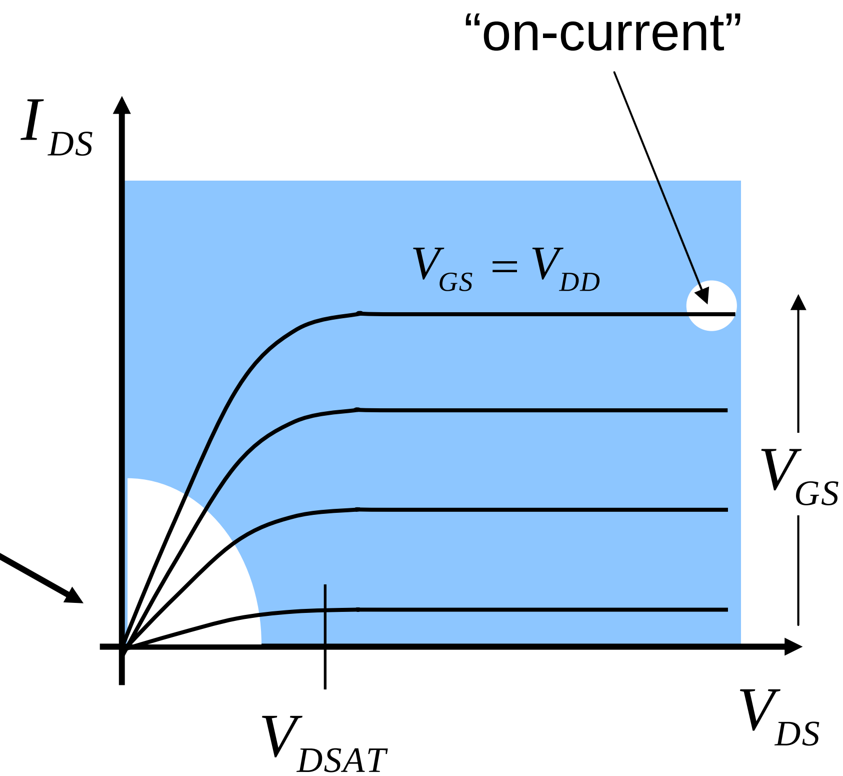
1) **Review and introduction**
2) Scattering theory of the MOSFET
3) Transmission under low V_{DS}
4) Transmission under high V_{DS}
5) Discussion
6) Summary

the ballistic MOSFET: IV

$$I_{DS} = WC_{ox}\left(V_{GS} - V_T\right)\upsilon_T\left(\frac{1 - e^{-qV_{DS}/k_BT}}{1 + e^{-qV_{DS}/k_BT}}\right)$$

$$I_{ON} = WC_{ox}\upsilon_T\left(V_{DD} - V_T\right)$$

$$\begin{cases} V_{DS} < k_BT/q \\ I_{DS} = WC_{OX}\dfrac{\upsilon_T}{2k_BT/q}\left(V_{GS} - V_T\right)V_{DS} \\ I_{DS} = V_{DS}/R_{CH} \end{cases}$$



$$V_{DSAT} \approx k_BT/q$$

review: ballistic I-V

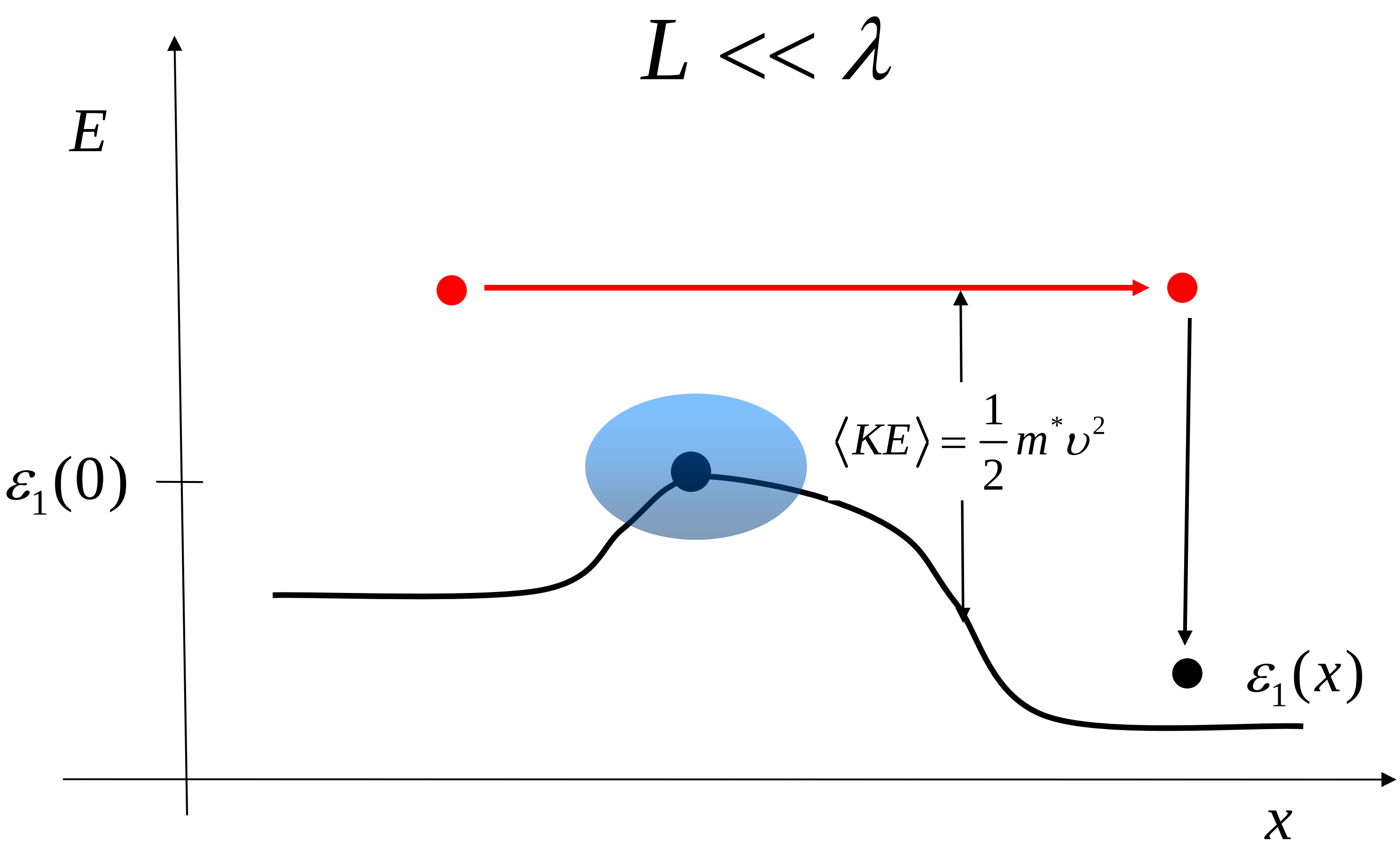
$$I_D = WC_{ox}\left(V_{GS} - V_T\right)\tilde{\upsilon}_T\left[\frac{1 - \mathcal{F}_{1/2}\left(\eta_{F2}\right)/\mathcal{F}_{1/2}\left(\eta_{F1}\right)}{1 + \mathcal{F}_{0}\left(\eta_{F2}\right)/\mathcal{F}_{0}\left(\eta_{F1}\right)}\right]$$

$$\tilde{\upsilon}_T \equiv \sqrt{\frac{2k_BT}{\pi m^*}}\,\frac{\mathcal{F}_{1/2}\left(\eta_{F1}\right)}{\mathcal{F}_{0}\left(\eta_{F1}\right)} = \upsilon_T\,\frac{\mathcal{F}_{1/2}\left(\eta_{F1}\right)}{\mathcal{F}_{0}\left(\eta_{F1}\right)}$$

$$\eta_{F1} = \left(E_F - \varepsilon_1\right)/k_BT \qquad \eta_{F2} = \left(E_F - qV_{DS} - \varepsilon_1\right)/k_BT$$

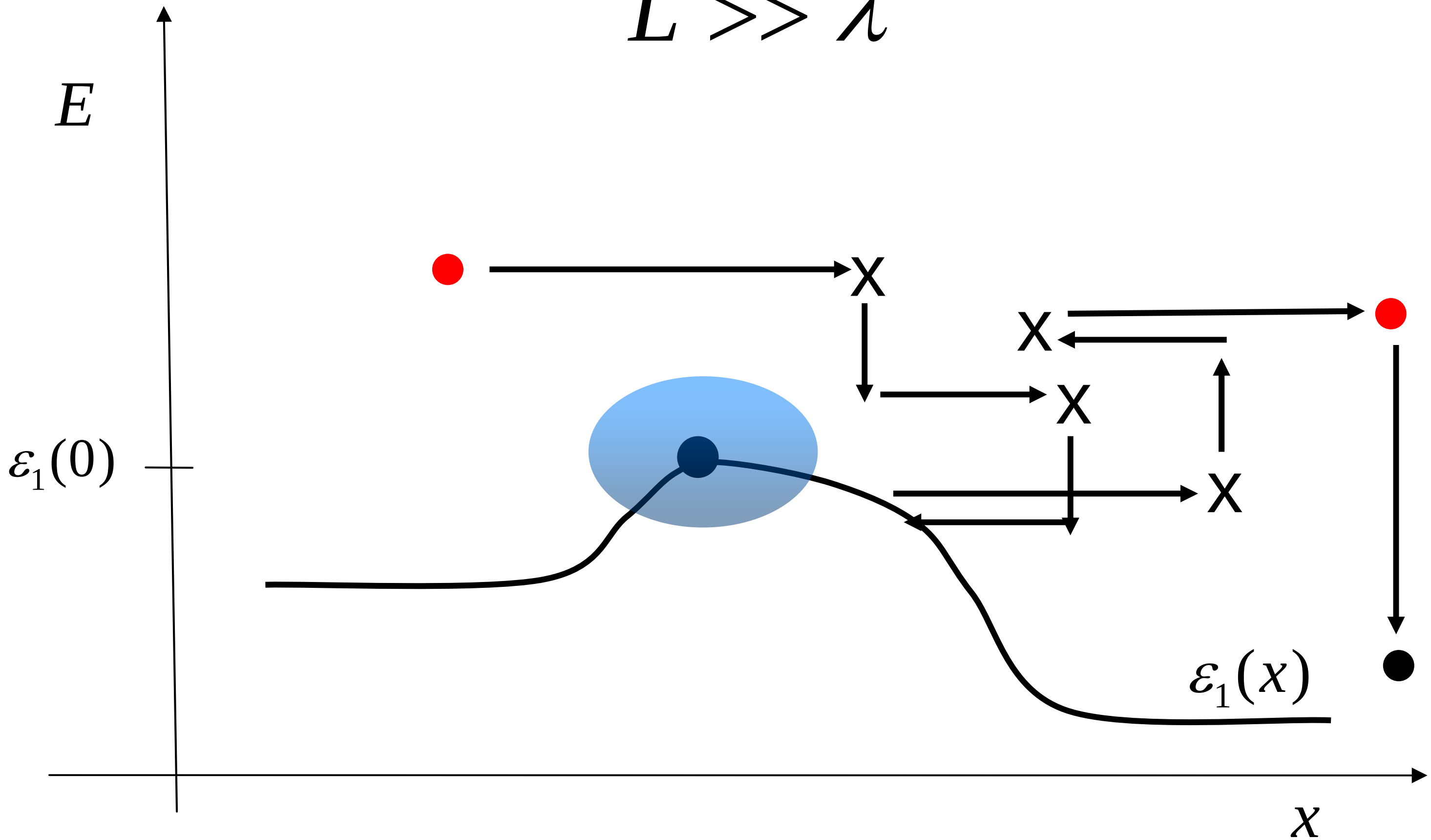
$$\mathcal{F}_{1/2}\left(\eta_F\right) \rightarrow e^{\eta_F} \qquad \left(\eta_F << 0\right)$$

review: ballistic transport in a MOSFET



review: diffusive transport in a MOSFET

$$L >> \lambda$$



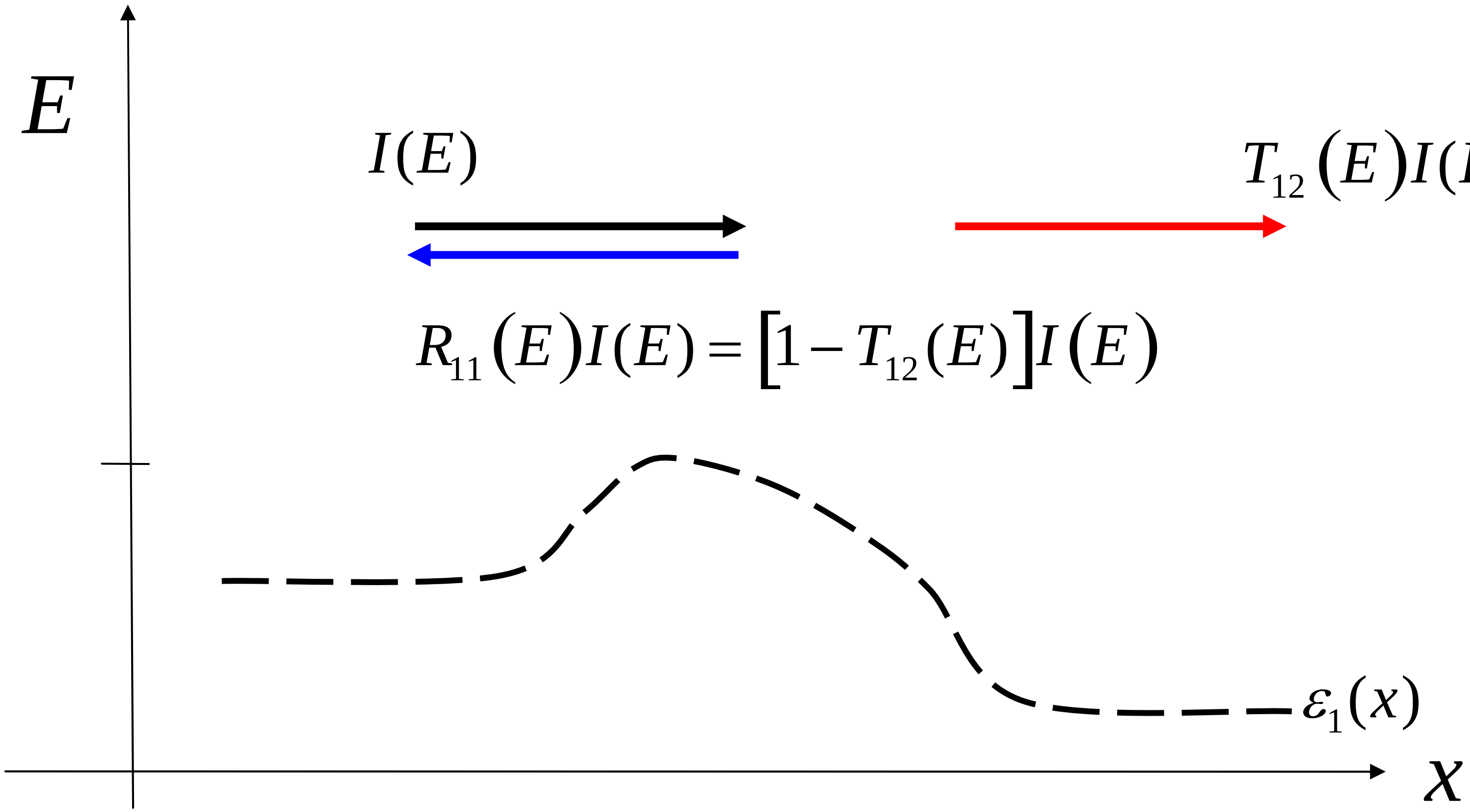
nanoscale MOSFETs

Nanoscale MOSFETs are neither fully ballistic nor fully diffusive; they operate in a ‘quasi-ballistic’ regime.

How do we ***understand*** how carrier scattering affects the performance of a nanoscale MOSFET?

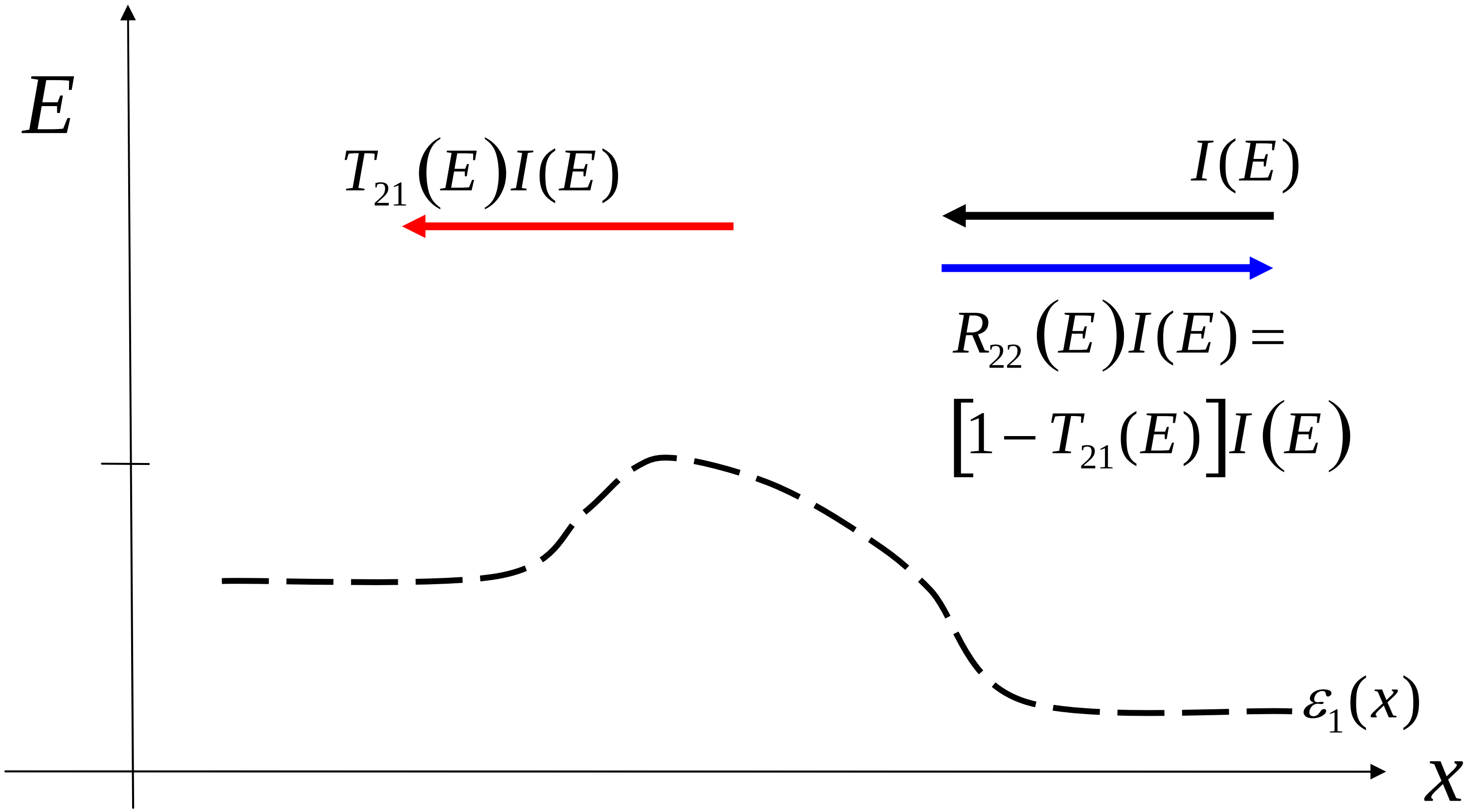
current transmission in a MOSFET

elastic scattering….



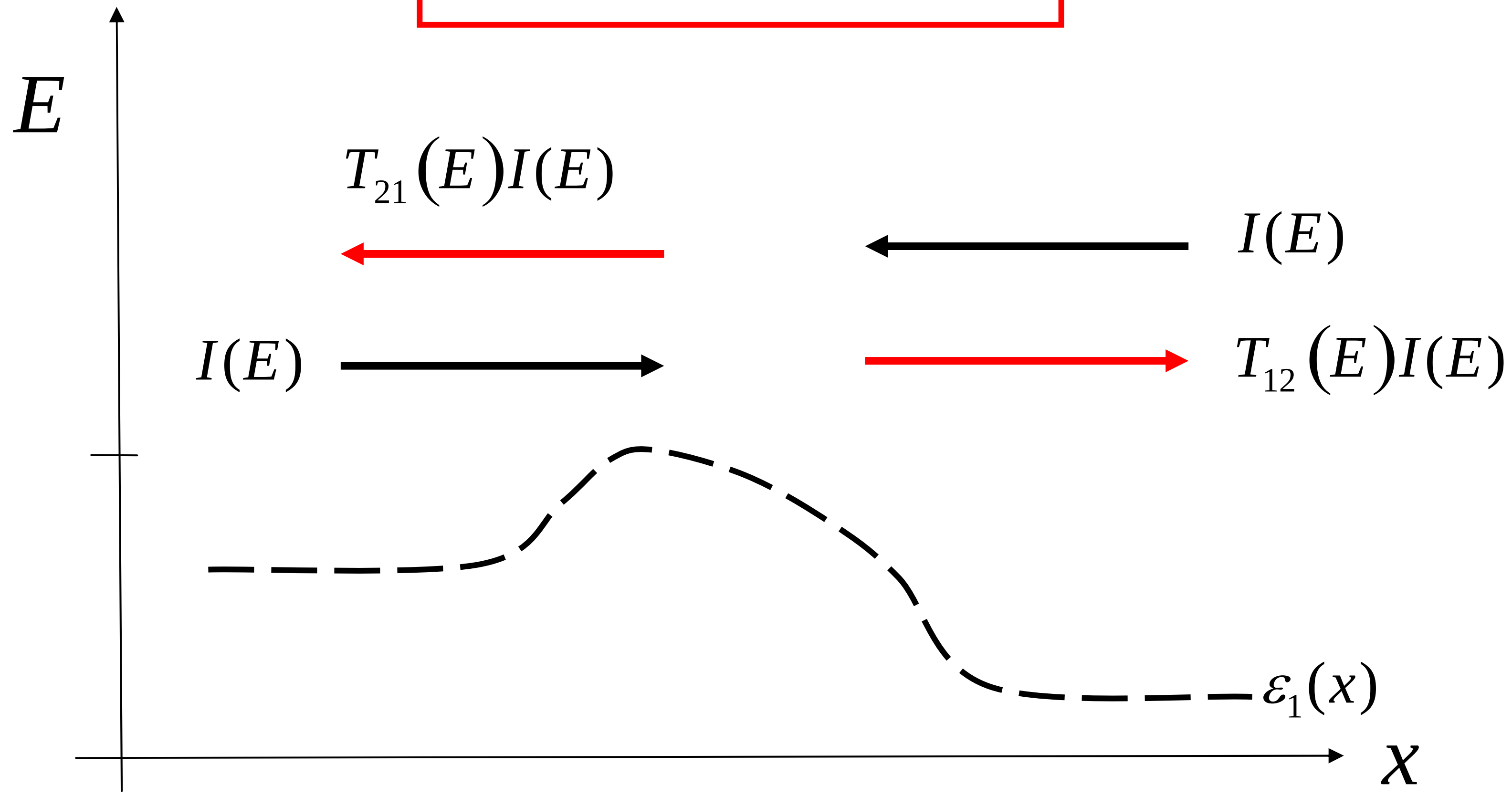
current transmission in a MOSFET

elastic scattering….



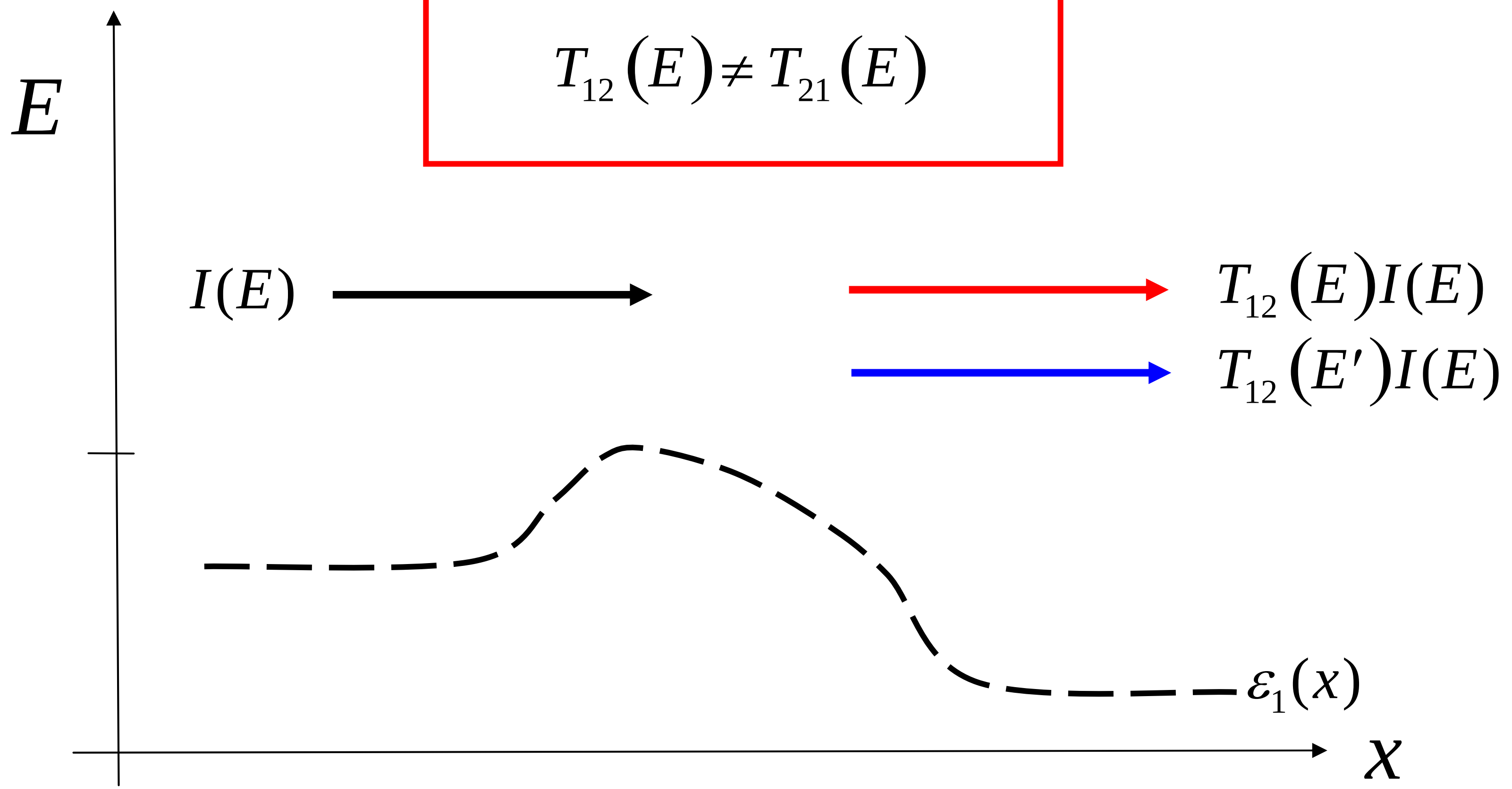
transmisson in the presence of **elastic** scattering

$$T_{12}(E) = T_{21}(E) = T(E)$$



inelastic scattering

$$T_{12}(E) \neq T_{21}(E)$$



S. Datta, *Electronic Transport in Mesoscopic Systems*, Cambridge, 1995.

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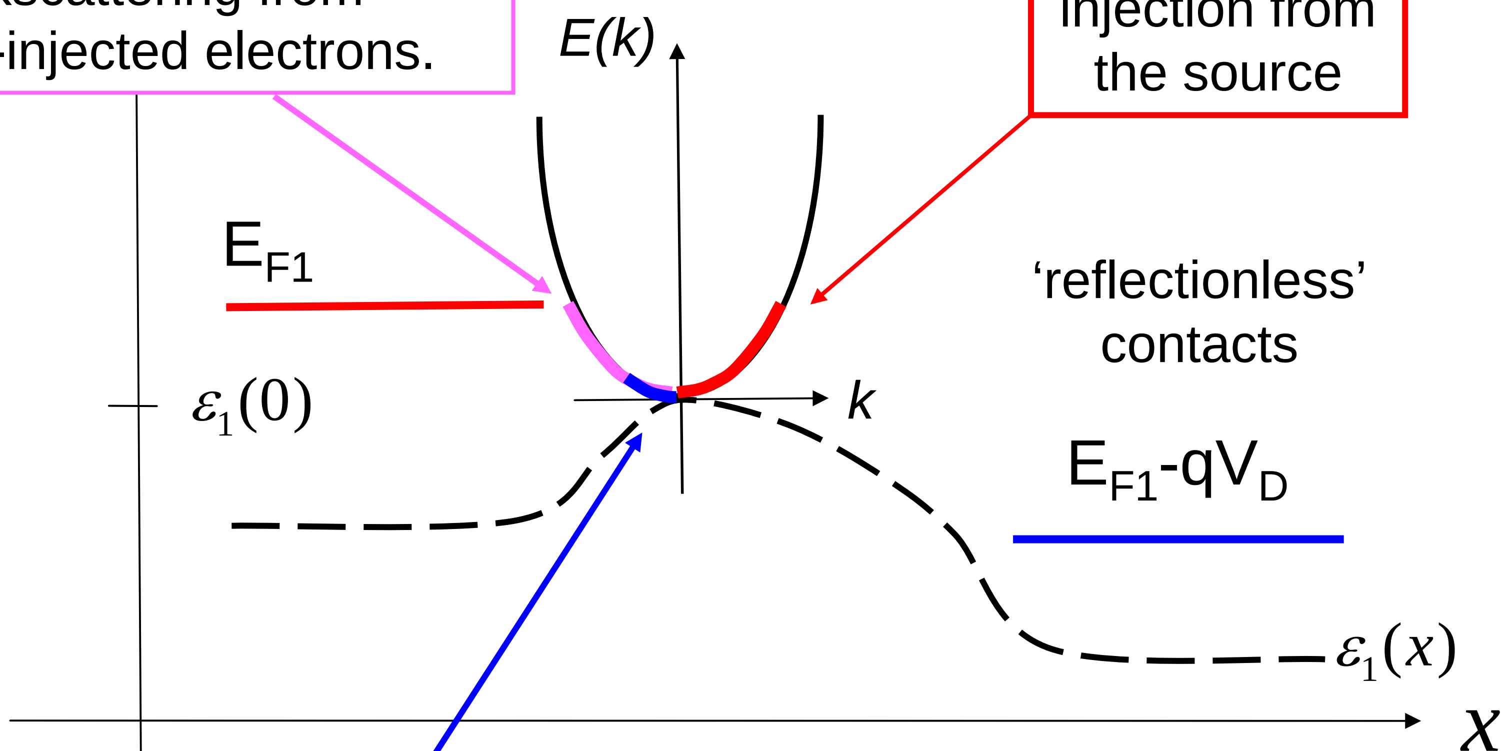
filling states in a quasi-ballistic MOSFET

Some states are now filled by backscattering from source-injected electrons.

filled by injection from the source



some states are still filled from the drain, but the magnitude is reduced by back-scattering.

outline

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scattering theory of the MOSFET

Goal:

To illustrate the influence on scattering on the I-V characteristic of a MOSFET by developing a very simple theory.

Assumptions:

1) Average quantities, not energy-resolved.
2) Boltzmann statistics for carriers
3) $T_{12} = T_{21} = T$
4) *Average velocity of backscattered carriers equals that of the injected carriers.*

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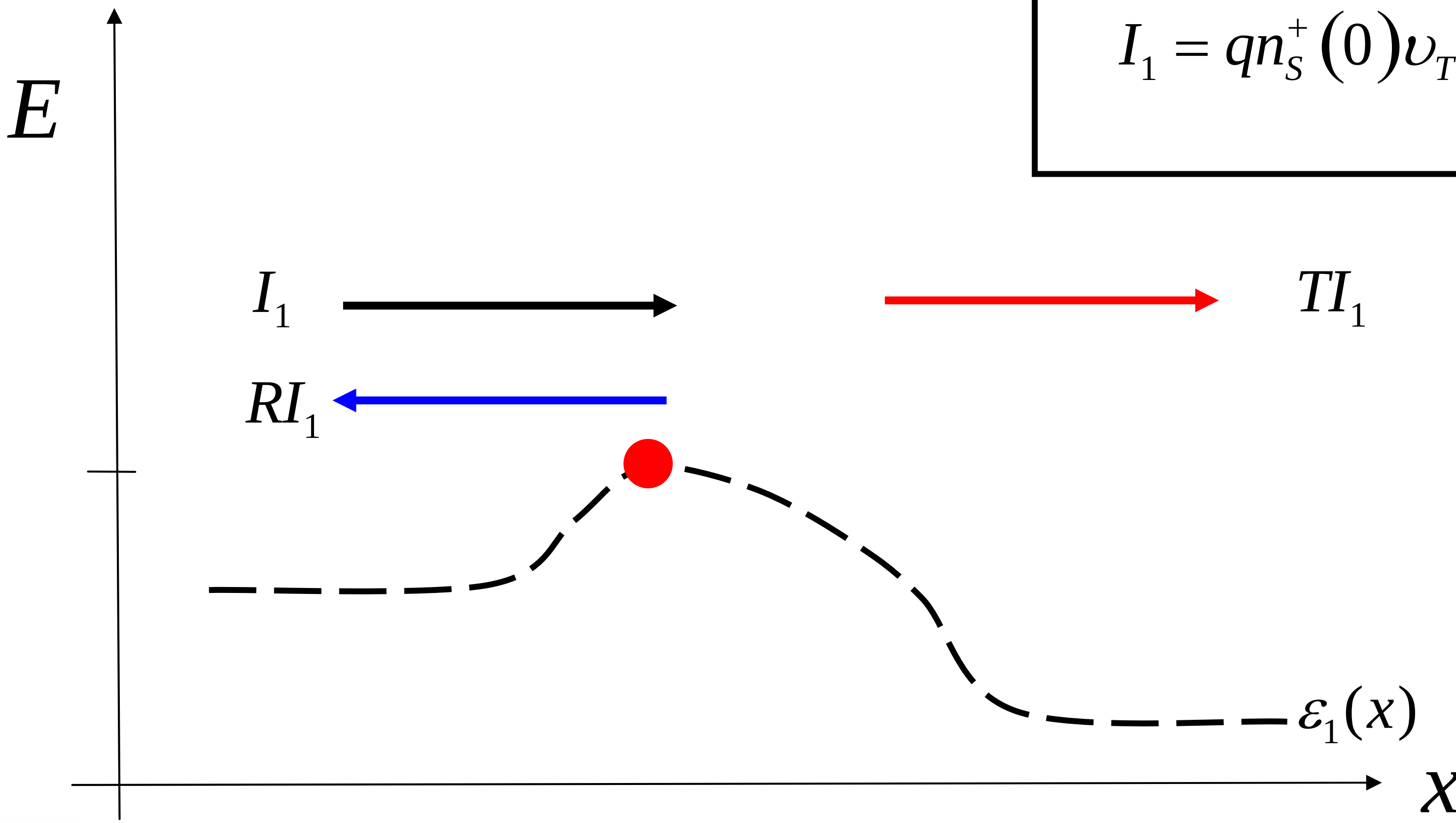
scattering in a nano-MOSFET

$$T + R = 1$$

$$I_1 = qn_S^+(0)\upsilon_T$$



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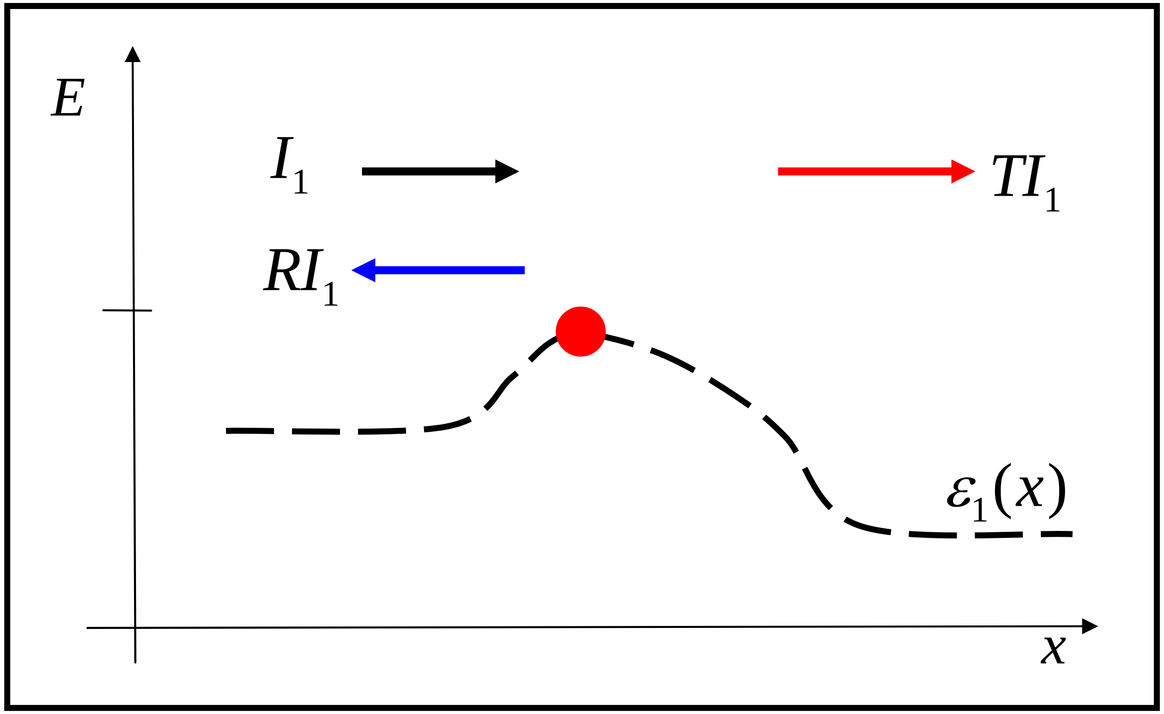
current

$$I_D = W\left(qn_S^+(0)\upsilon_T - qn_S^-(0)\upsilon_T\right) = Wqn_S^+(0)\upsilon_T\left[1 - n_S^-(0)/n_S^+(0)\right] \qquad (1)$$

$$n_S(0) = n_S^+(0) + n_S^-(0) = n_S^+(0)\left[1 + n_S^-(0)/n_S^+(0)\right] \qquad (2)$$



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current

$$I_D = Wqn_S(0)\upsilon_T\left(\frac{1-n_S^-(0)/n_S^+(0)}{1+n_S^-(0)/n_S^+(0)}\right)$$

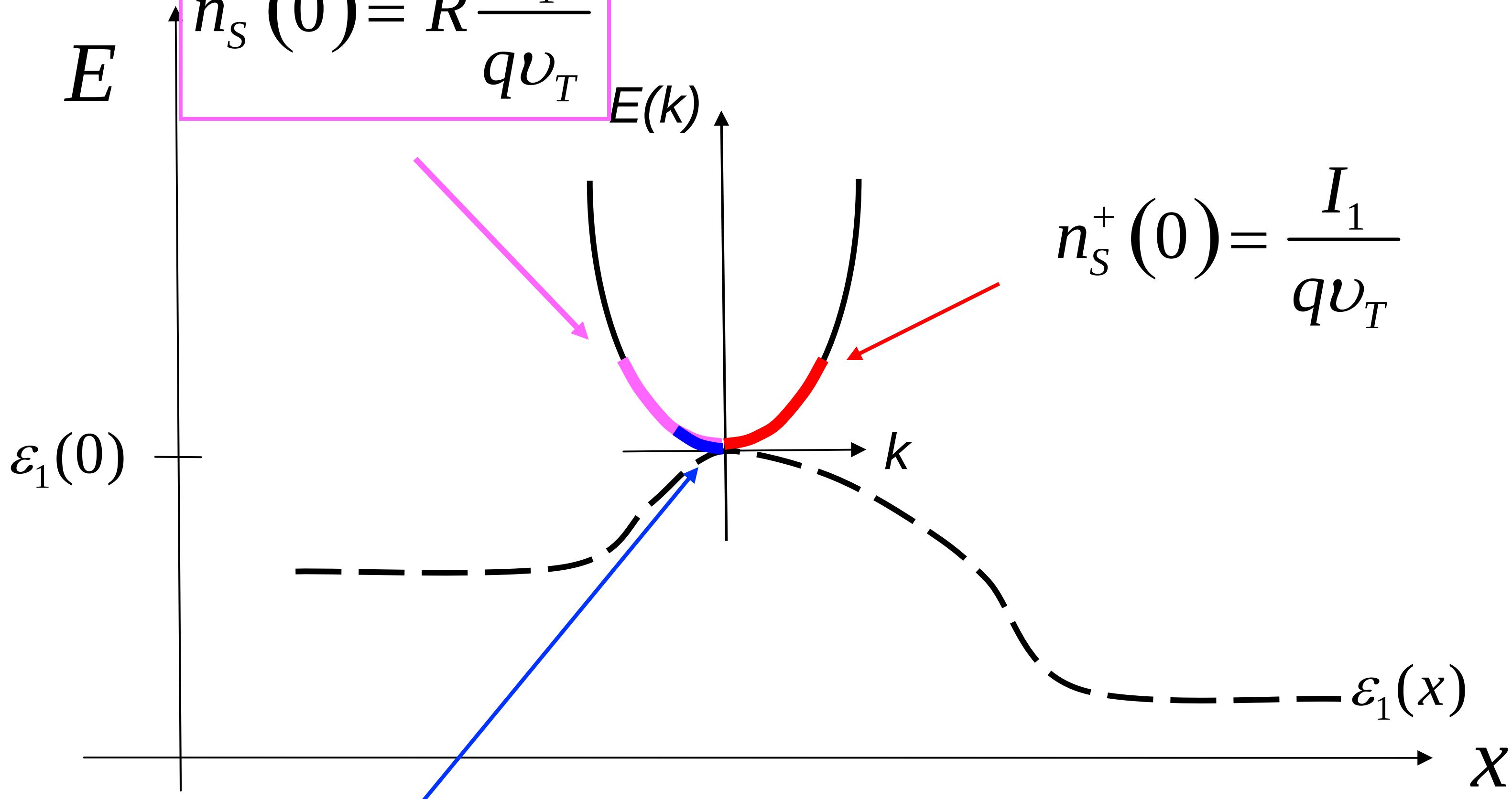
$$I_D = WQ_I(0)\upsilon_T\left(\frac{1-n_S^-(0)/n_S^+(0)}{1+n_S^-(0)/n_S^+(0)}\right)$$

Exactly the same result we had for the ballistic case, but the (- velocity) carrier density at the top of the barrier is altered by scattering.

carrier densities at the top of the barrier

$$n_S^-(0) = R\frac{I_1}{q\upsilon_T}$$

$$n_S^+(0) = \frac{I_1}{q\upsilon_T}$$



$$n_S^-(0) = Tn_S^+(0)e^{-qV_{DS}/k_BT}$$

from carrier densities to drain current

$$n_S^-(0) = Rn_S^+(0) + Tn_S^+(0)e^{-qV_{DS}/k_BT} = n_S^+(0)\left[R + (1-R)e^{-qV_{DS}/k_BT}\right]$$

$$\frac{n_S^-(0)}{n_S^+(0)} = R + (1-R)e^{-qV_{DS}/k_BT}$$

$$I_D = WQ_I(0)\upsilon_T\left(\frac{1 - n_S^-(0)/n_S^+(0)}{1 + n_S^-(0)/n_S^+(0)}\right)$$

$$I_D = WQ_I(0)\upsilon_T\left(\frac{(1-R) - (1-R)e^{-qV_{DS}/k_BT}}{(1+R) + (1-R)e^{-qV_{DS}/k_BT}}\right)$$

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the MOSFET I-V with scattering

$$I_{DS} = WC_{ox}\left(V_{GS} - V_T\right)\upsilon_T\left(\frac{1 - e^{qV_{DS}/k_BT}}{1 + e^{qV_{DS}/k_BT}}\right)$$ (ballistic, Boltzmann statistics)

$$I_D = WC_{ox}\left(V_{GS} - V_T\right)\upsilon_T\left(\frac{(1-R)-(1-R)e^{-qV_{DS}/k_BT}}{(1+R)+(1-R)e^{-qV_{DS}/k_BT}}\right)$$

$$T = (1-R)$$

$$I_D = WC_{ox}\left(V_{GS} - V_T\right)\upsilon_T T\left(\frac{1 - e^{-qV_{DS}/k_BT}}{(2-T)+Te^{-qV_{DS}/k_BT}}\right)$$

$$I_D(\text{scattering}) \neq TI_D(\text{ballistic})$$

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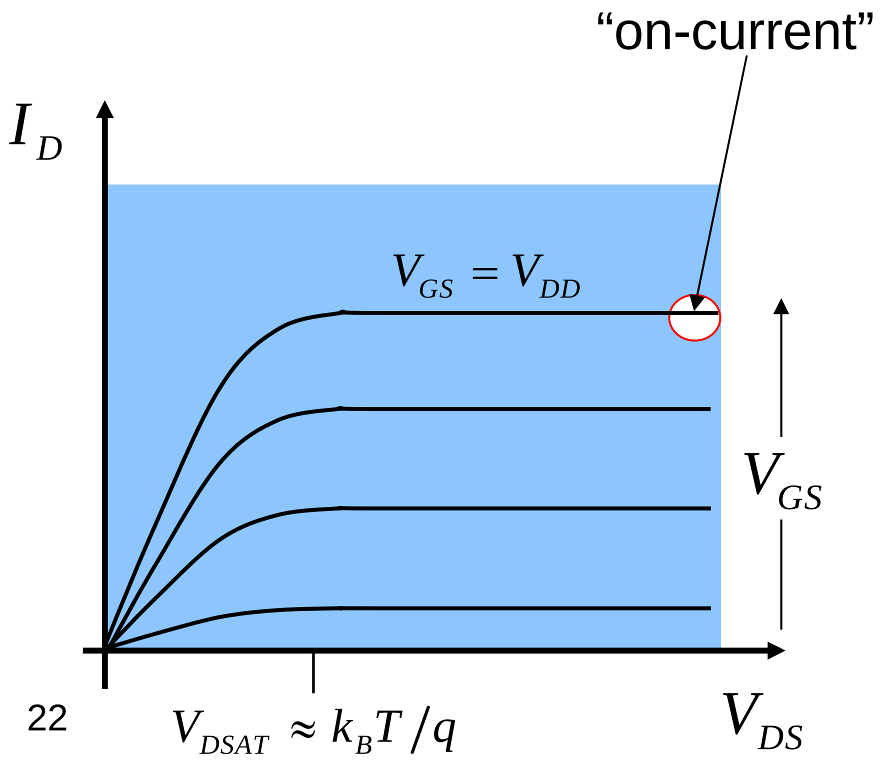
high drain bias

$$I_D = WC_{ox}\left(V_{GS} - V_T\right)\upsilon_T\left(\frac{(1-R)-(1-R)e^{-qV_{DS}/k_BT}}{(1+R)+(1-R)e^{-qV_{DS}/k_BT}}\right)$$

$$I_D = WC_{ox}\left(V_{GS} - V_T\right)\upsilon_T\frac{(1-R)}{(1+R)}$$

$$\langle\upsilon(0)\rangle = \upsilon_T\frac{(1-R)}{(1+R)} \leq \upsilon_T$$

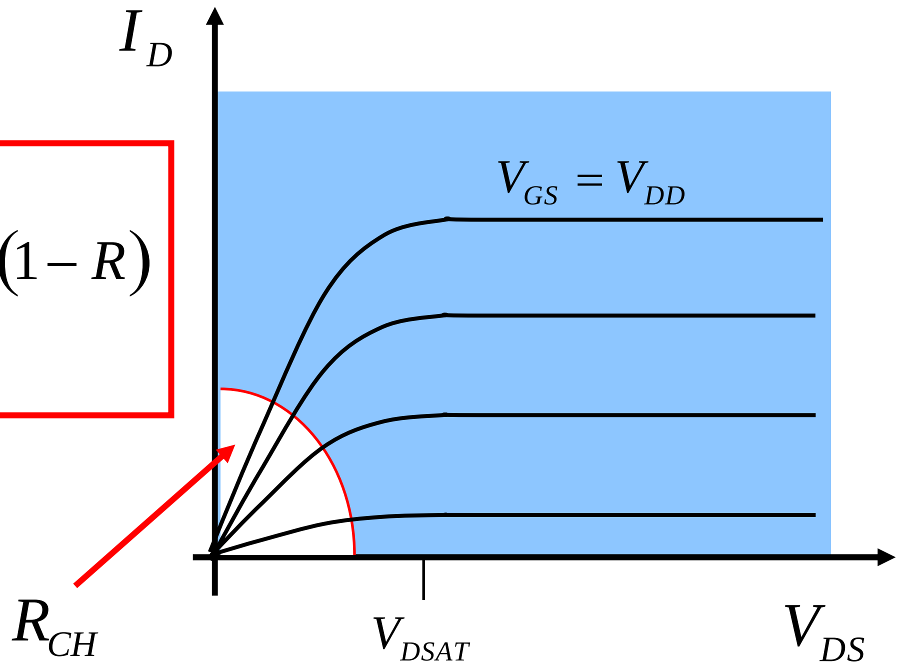


low drain bias

$$I_D = WC_{ox}\left(V_{GS}-V_T\right)\upsilon_T\left(\frac{(1-R)-(1-R)e^{-qV_{DS}/k_BT}}{(1+R)+(1-R)e^{-qV_{DS}/k_BT}}\right)$$

$$G_{CH}=\frac{I_D}{V_{DS}}=WC_{ox}\left(V_{GS}-V_T\right)\left[\frac{\upsilon_T}{2\left(k_BT/q\right)}\right](1-R)$$

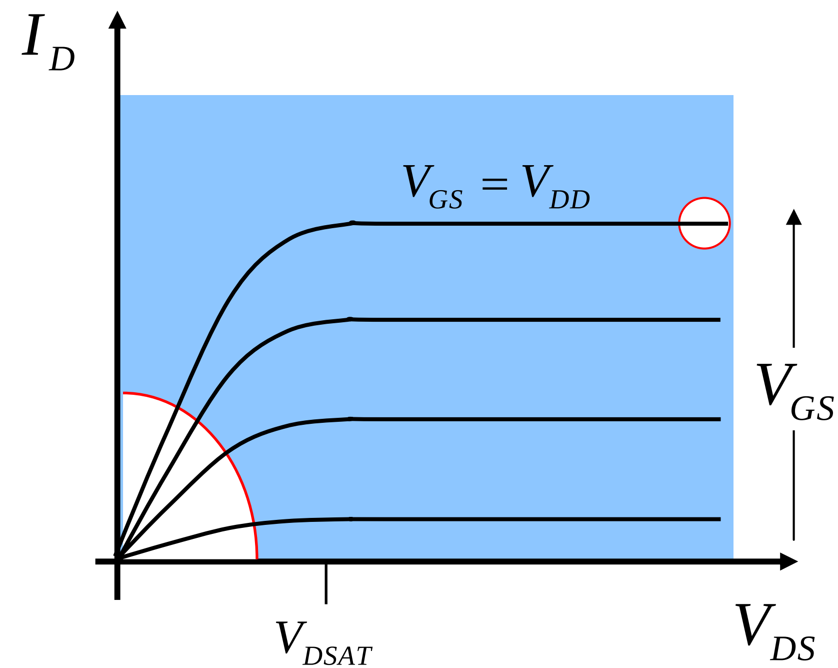
$$G_{CH}\left(\text{scattering}\right)=TG_{CH}\left(\text{ballistic}\right)$$



summary of the scattering model

$$
\begin{cases}
I_D \approx WC_{ox}\left(V_{GS}-V_T\right)\upsilon_T\left(\dfrac{(1-R)-(1-R)\mathcal{F}_{1/2}(\eta_{F2})/\mathcal{F}_{1/2}(\eta_{F1})}{(1+R)+(1-R)\mathcal{F}_{0}(\eta_{F2})/\mathcal{F}_{0}(\eta_{F1})}\right) \\[2ex]
G_{CH} \approx \left(WC_{ox}\left(V_{GS}-V_T\right)\dfrac{\upsilon_T}{(2k_BT/q)}\right)\left[\dfrac{\mathcal{F}_{-1/2}(\eta_{F1})}{\mathcal{F}_{0}(\eta_{F1})}\right](1-R) \\[2ex]
I_{ON} \approx WC_{ox}\left(V_{GS}-V_T\right)\upsilon_T\dfrac{(1-R)}{(1+R)}
\end{cases}
$$

To proceed, we need to understand $R\left(V_{GS},V_{DS}\right)$



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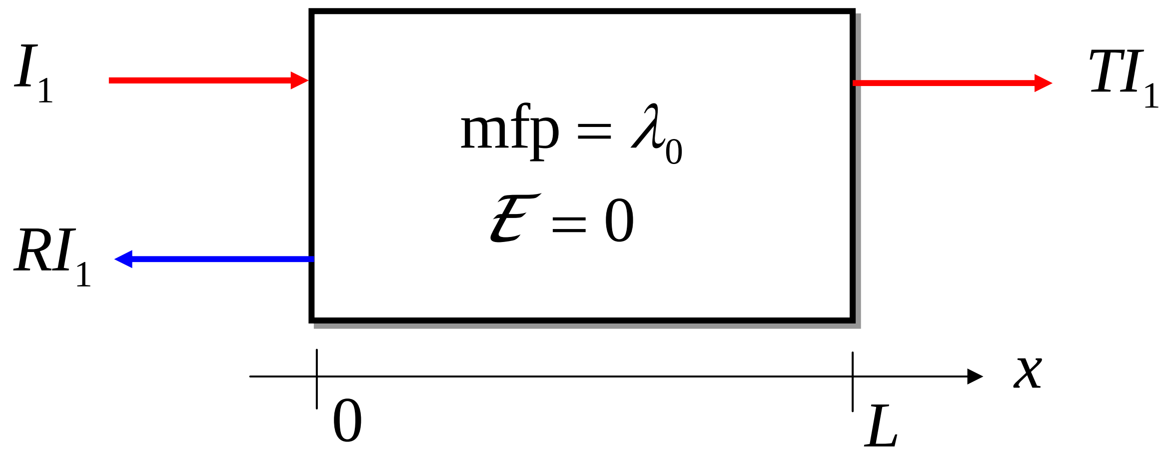
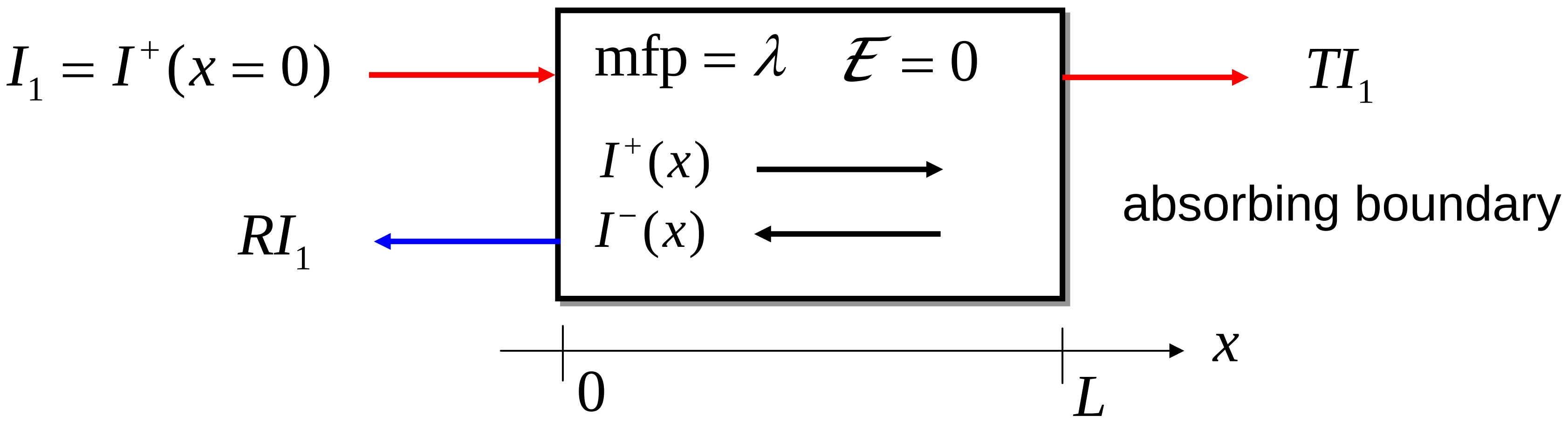
transmission across a field-free slab



Consider a flux of carriers injected into a field-free slab of length, *L*. The flux that emerges at $x = L$ is *T* times the incident flux, where $0 < T < 1$. The flux that emerges from $x = 0$ is *R* times the incident flux, where $T + R = 1$, assuming no carrier recombination-generation.

How is *T* related to the mean-free-path for backscattering within the slab?

transmission (iii)



$$T = \frac{\lambda_0}{\lambda_0 + L} \qquad R = \frac{L}{\lambda_0 + L}$$

$$T \to 0 \quad L >> \lambda_0$$

$$T \to 1 \quad L << \lambda_0$$

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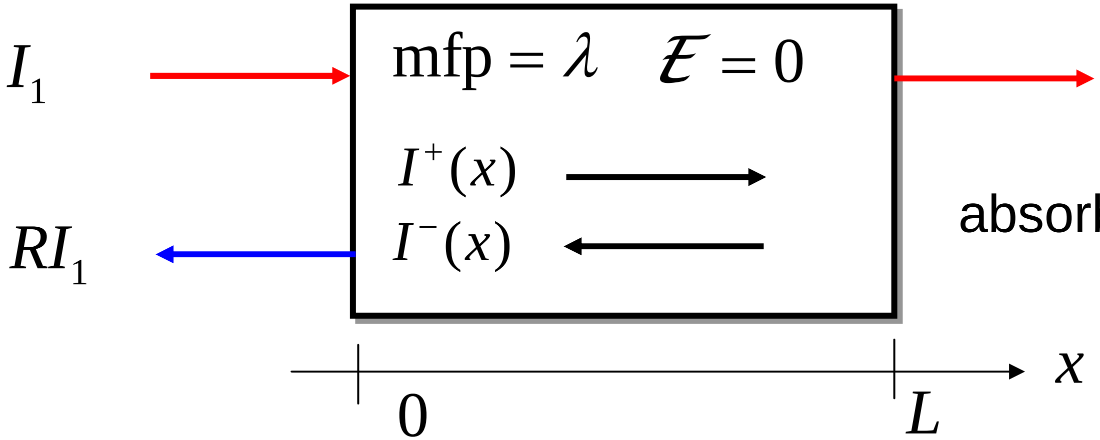
mean-free-path



$$T = \frac{\lambda_0}{\lambda_0 + L}$$

absorbing boundary

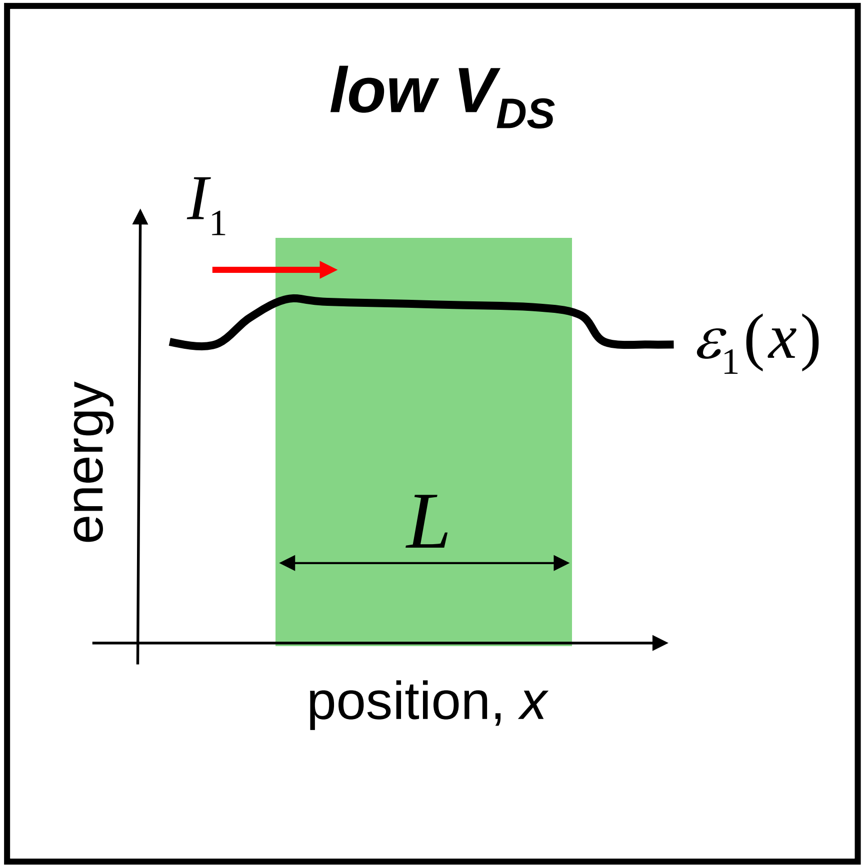
How do we relate λ_0 to known parameters?

If I_1 is a thermal equilibrium injected flux, $I_1 = n^+(0)\upsilon_T$
then, it can be shown that:

$$D_n = \frac{k_B T}{q}\mu_n = \frac{\upsilon_T}{2}\lambda_0$$

(non-degenerate carrier statistics)

example



$$\mu_n \approx 200 \text{ cm}^2/\text{V-s}$$

$$\mu_n = \frac{\upsilon_T}{2(k_B T/q)} \lambda_0$$

$$\lambda_0 \approx 9 \text{ nm}$$

$$L \approx 50 \text{ nm}$$

$$T \approx \frac{\lambda_o}{L + \lambda_o} \approx 0.15$$

relation to conventional theory

$$G_{CH} = \left(WC_{ox}\left(V_{GS} - V_T\right)\frac{\upsilon_T}{\left(2k_BT/q\right)}\right)\left(1 - R\right)$$

$$1 - R = T = \frac{\lambda_0}{\lambda_0 + L} \approx \frac{\lambda_0}{L}$$ (diffusive limit)

$$\lambda_0 = \frac{2k_BT/q}{\upsilon_T}\mu_n$$

$$G_{CH} = \frac{W}{L}\mu_n C_{ox}\left(V_{GS} - V_T\right)$$

(non-degenerate carrier statistics)

The scattering model works in the diffusive limit, as well as the ballistic limit, and in the quasi-ballistic regime in between.

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channel conductance

$$G_{CH} = T\left(WC_{ox}\left(V_{GS} - V_T\right)\frac{\upsilon_T}{\left(2k_BT/q\right)}\right) \qquad T = \frac{\lambda_0}{\lambda_0 + L}$$

one can show that:

$$G_{CH} = \frac{W}{L}\left(\frac{1}{\mu_n} + \frac{1}{\mu_B}\right)^{-1} C_{ox}\left(V_{GS} - V_T\right)$$

$$\begin{cases} \mu_n = \dfrac{\upsilon_T \lambda_0}{2\,k_BT/q} \\ \mu_B = \dfrac{\upsilon_T L}{2\,k_BT/q} \end{cases}$$

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transmission under high drain bias

***scattering model*:**

$$I_{ON} = WC_{ox}\left(V_{GS} - V_T\right)\upsilon_T \frac{(1-R)}{(1+R)} = WC_{ox}\left(V_{GS} - V_T\right)\upsilon_T \frac{T}{(2-T)}$$

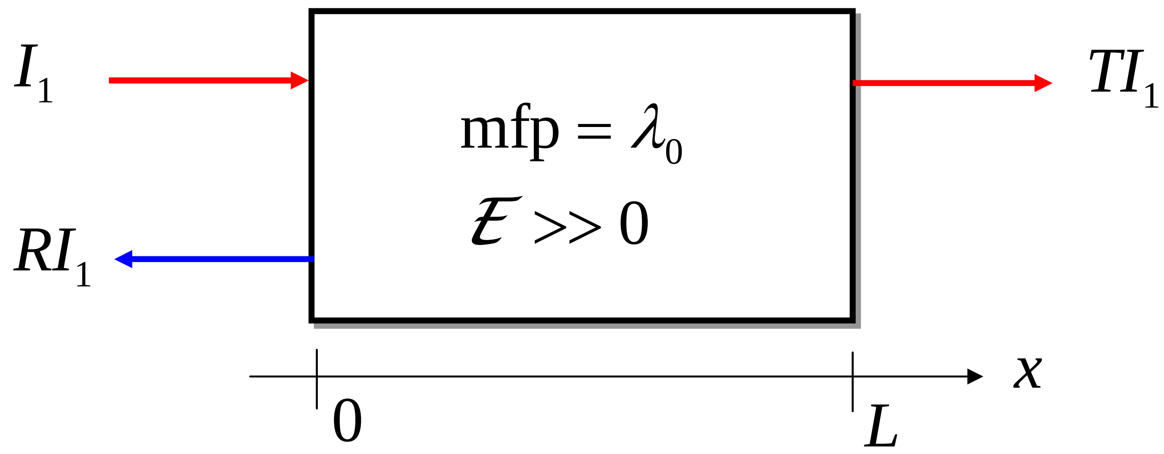
in practice:

$$B \equiv \frac{I_{ON}(\text{measured})}{I_{ON}(\text{ballistic})} \approx 0.50$$

$$B = \frac{T}{(2-T)} \quad \rightarrow \quad T \approx 0.67 >> 0.15$$

Why?

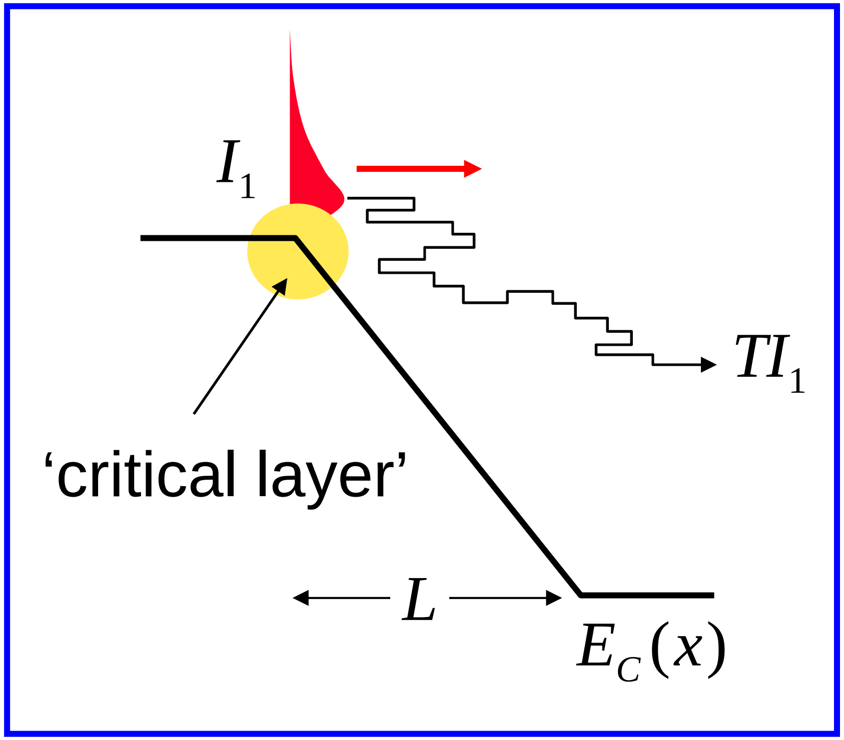
transmission across a slab with an electric field



When the electric field is strong and position-dependent and several scattering mechanisms operate, this turns out to be a difficult problem.

How can we understand the essential physics?

transport "downhill"

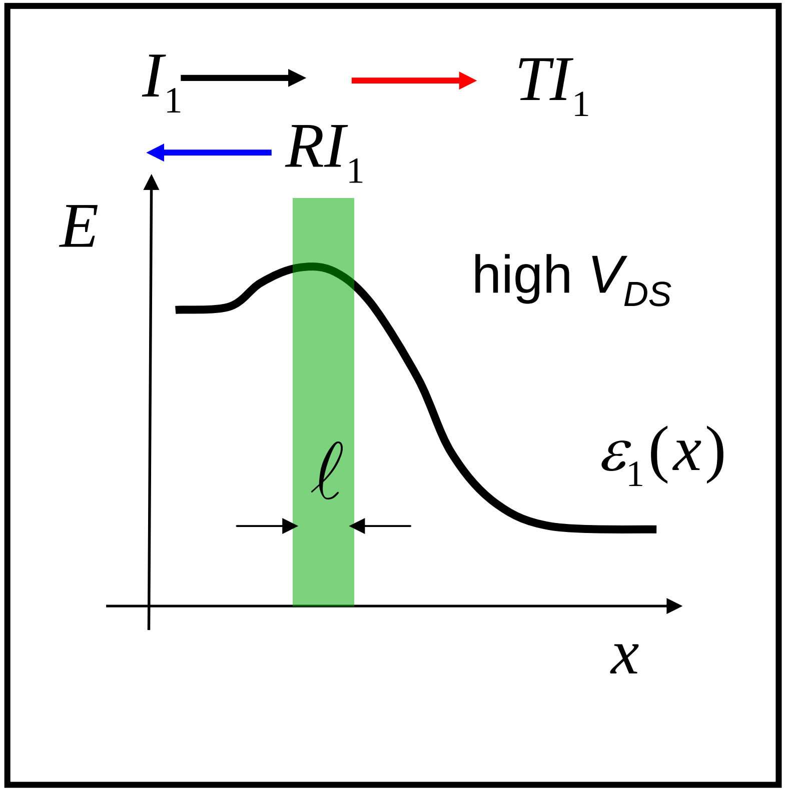


$$T = \frac{\lambda_o}{\ell + \lambda_o} \qquad \ell << L$$

$T \approx 1$:
High field regions are good carrier collectors.

Peter J, Price, "Monte Carlo calculation of electron transport in solids," *Semiconductors and Semimetals*, **14**, pp. 249-334, 1979

transport in a MOS transistor



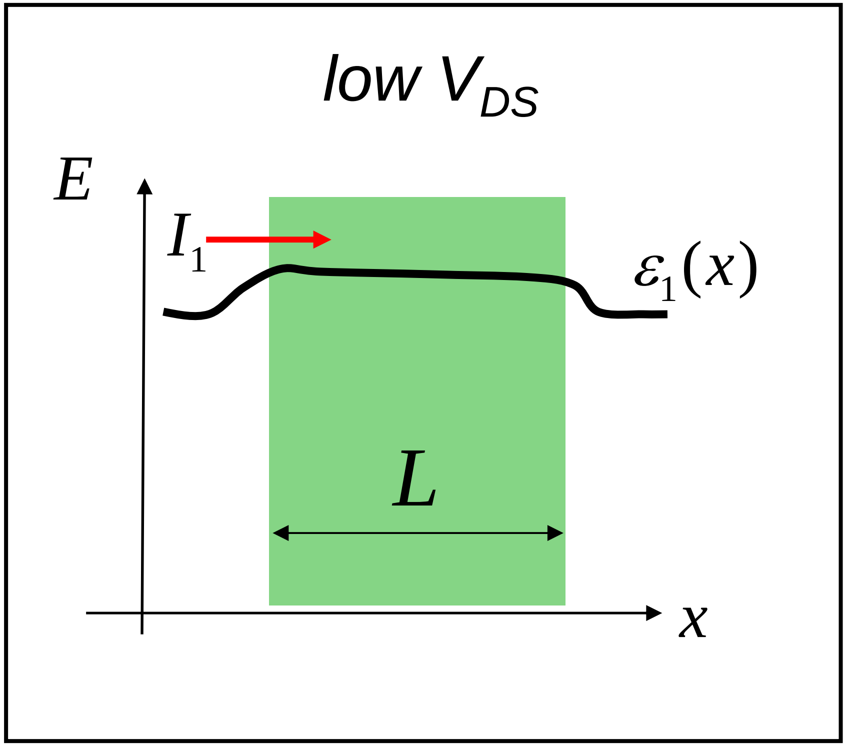
$$T \approx \frac{\lambda_o}{l + \lambda_o}$$

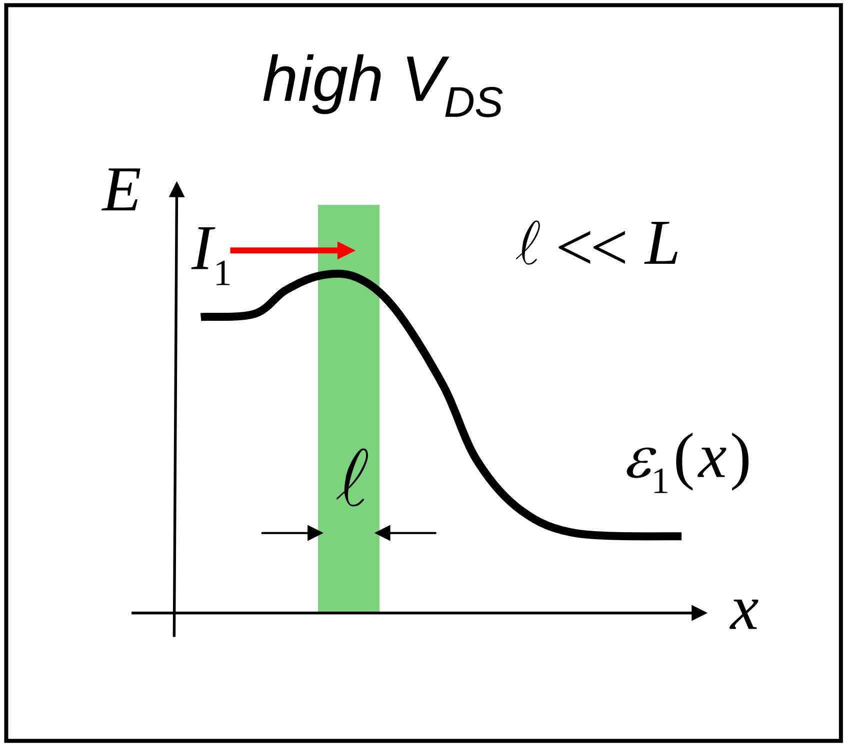
1) A MOSFET consists of a low-field region near the source that is strongly controlled by the gate voltage, and a high-field region near the drain that is strongly controlled by the drain voltage.

2) Transmission is controlled by the low-field region near the source.

3) Scattering near the drain has a smaller effect on backscattering to the source.

bias-dependent transmission





$$T \approx \frac{\lambda_o}{L + \lambda_o} \approx 0.15$$

$$T \approx \frac{\lambda_o}{l + \lambda_o} \approx 0.67$$

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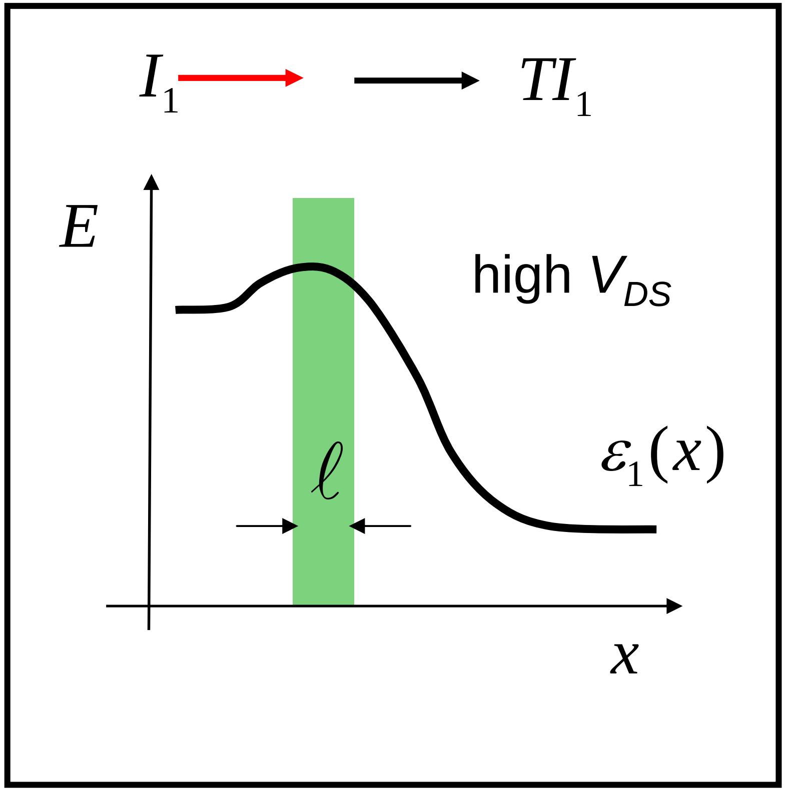
relation to conventional theory (high V_{DS})



$$T = \frac{\lambda_o}{\ell + \lambda_o} \approx \frac{\lambda_o}{\ell} \quad (\lambda_o << \ell)$$

$$I_D = WC_{ox}\frac{T}{2-T}\upsilon_T\left(V_{GS} - V_T\right)$$

$$I_D \approx WC_{ox}\frac{T}{2}\upsilon_T\left(V_{GS} - V_T\right)$$

$$I_D \approx WC_{ox}\frac{\lambda_0}{2\ell}\upsilon_T\left(V_{GS} - V_T\right)$$

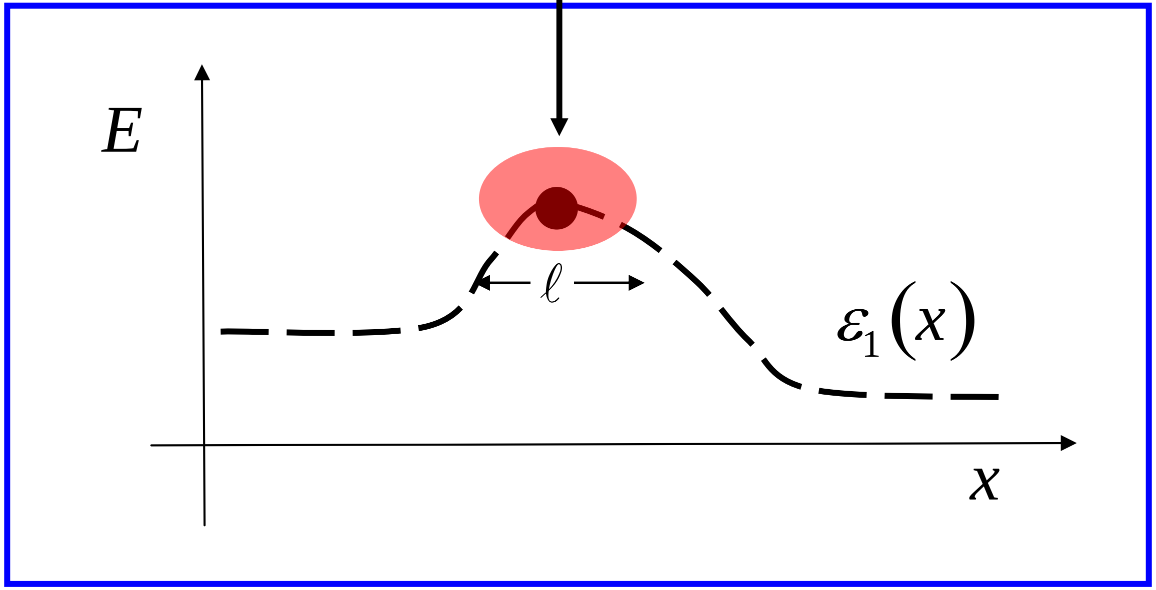
How do we physically interpret this result?

$$I_D \approx WC_{ox}\frac{D_n}{\ell}\left(V_{GS} - V_T\right)$$

drift-diffusion picture

$$I_D = WC_{ox}\frac{D_n}{l}\left(V_{GS} - V_T\right)$$

The top of the barrier is a bottleneck that carriers must diffuse across.



drift-diffusion vs. scattering model

$$I_D = WC_{ox}\left[\frac{1}{\upsilon_T}+\frac{1}{\left(D_n/l\right)}\right]^{-1}\left(V_{GS}-V_T\right)$$

drift-diffusion

$$D_n = \upsilon_T \lambda_0 / 2 \qquad T = \frac{\lambda_0}{\lambda_0 + l}$$

$$I_D = WC_{ox}\frac{T}{2-T}\upsilon_T\left(V_{GS}-V_T\right)$$

scattering

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summary

1) Modern MOSFETs operate between the ballistic and diffusive limits, so we need to understand transport in the quasi-ballistic regime.

2) Transmission (or scattering) theory provides a simple, physical description of quasi-ballistic transport.

3) The same physics can also be understood at the drift-diffusion level.

4) Quantitative treatments require detailed numerical simulation.

Questions & Answers