



# **ECE606: Solid State Devices**

## **Lecture 14: Bulk Recombination**

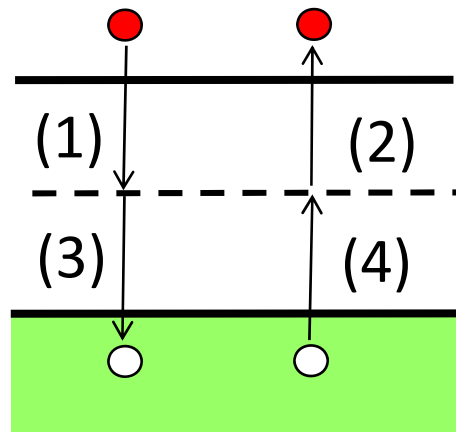
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# Outline

- 1) **Derivation of SRH formula**
- 2) Application of SRH formula for special cases
- 3) Direct and Auger recombination
- 4) Conclusion

Ref. ADF, Chapter 5, pp. 141-154

# Sub-processes of SRH Recombination

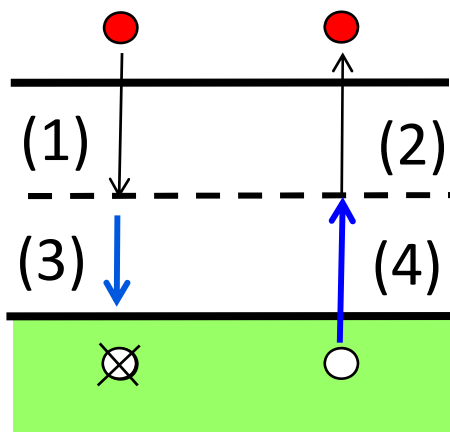


(1)+(3): one electron reduced from Conduction-band & one-hole reduced from valence-band

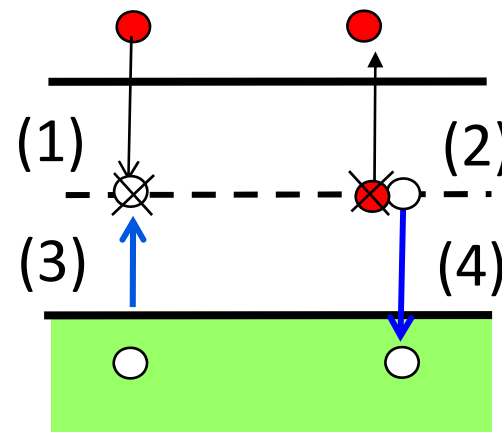
(2)+(4): one hole created in valence band and one electron created in conduction band

# SRH Recombination

Physical picture



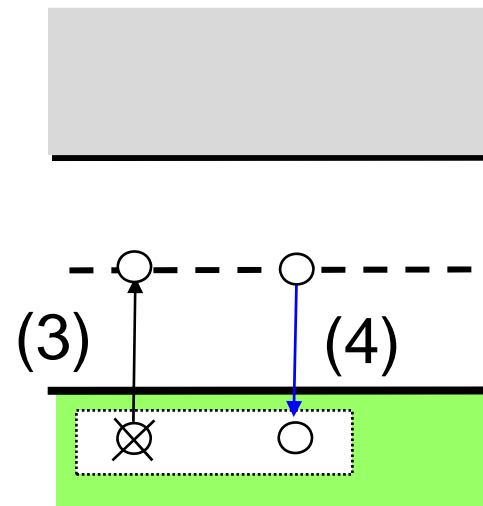
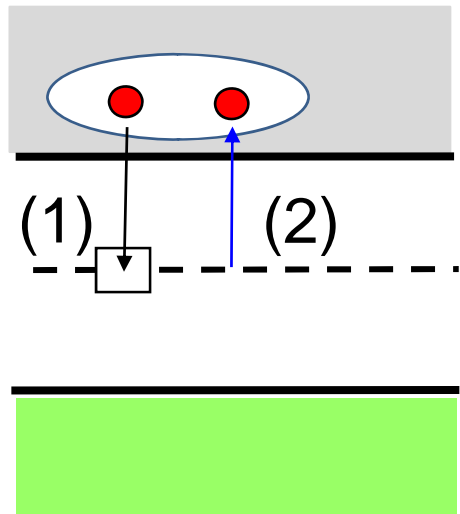
Equivalent picture



(1)+(3): **one electron reduced from C-band & one-hole reduced from valence-band**

(2)+(4): **one hole created in valence band & one electron created in conduction band**

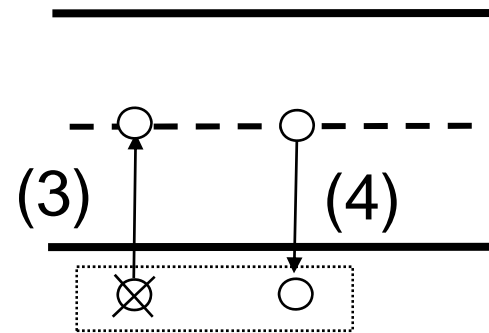
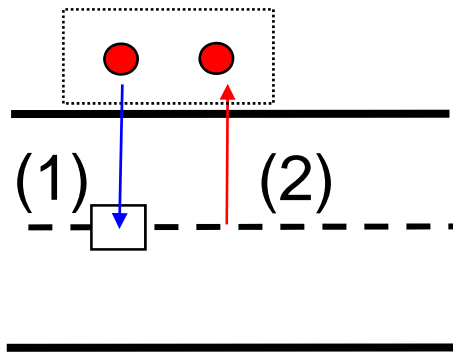
# Changes in electron and hole Densities



$$\left. \frac{\partial n}{\partial t} \right|_{1,2} = -c_n n p_T + e_n n_T (1 - f_c)$$

$$\left. \frac{\partial p}{\partial t} \right|_{3,4} = -c_p p n_T + e_p p_T f_v$$

## Detailed Balance in *Equilibrium*



$$\left. \frac{\partial n}{\partial t} \right|_{1,2} = -c_n n_0 p_{T0} + e_n n_{T0}$$

$$0 = -c_n n_0 p_{T0} + e_n n_{T0}$$

$$e_n = c_n \frac{n_0 p_{T0}}{n_{T0}} \equiv c_n n_1$$

$$0 = -c_n (n_0 p_{T0} - n_{T0} n_1)$$

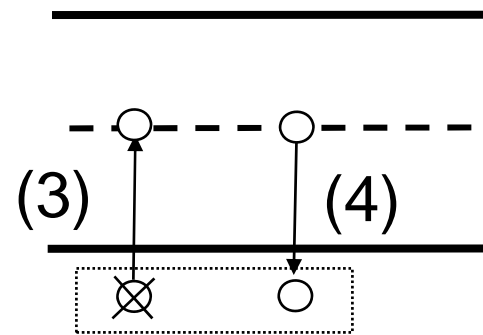
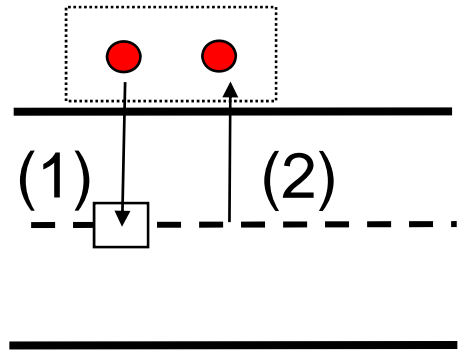
$$\left. \frac{\partial p}{\partial t} \right|_{3,4} = -c_p p n_T + e_p p_T$$

$$0 = -c_p p_0 n_{T0} + p_{T0} e_p$$

$$e_p \equiv \frac{c_p p_0 n_{T0}}{p_{T0}} = c_p p_1$$

$$0 = -c_p (p_0 n_{T0} - p_{T0} p_1)$$

## Expressions for ( $n_1$ ) and ( $p_1$ )



$$n_1 = \frac{n_0 p_{T0}}{n_{T0}}$$

$$p_1 = \frac{p_0 n_{T0}}{p_{T0}}$$

$$n_1 p_1 = \frac{n_0 p_{T0}}{n_{T0}} \times \frac{p_0 n_{T0}}{p_{T0}} = n_0 p_0 = n_i^2$$

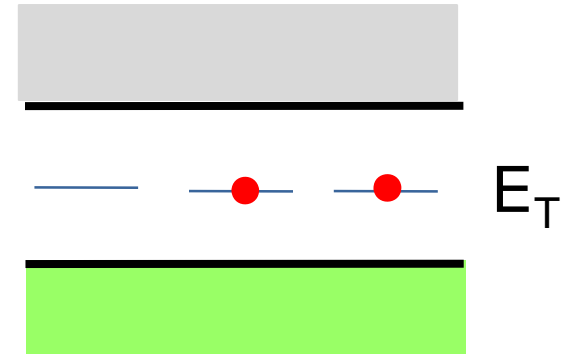
## Expressions for ( $n_1$ ) and ( $p_1$ )

$$n_{T0} = N_T (1 - f_{00}) = \frac{N_T}{1 + g_D e^{\beta(E_T - E_F)}}$$

$$n_1 = \frac{n_0 p_{T0}}{n_{T0}} = n_0 \frac{(N_T f_{00})}{N_T (1 - f_{00})}$$

$$n_1 = n_i e^{\beta(E_F - E_i)} \left[ 1 + g_D e^{\beta(E_T - E_F)} - 1 \right]$$

$$= n_i g_D e^{\beta(E_T - E_i)}$$



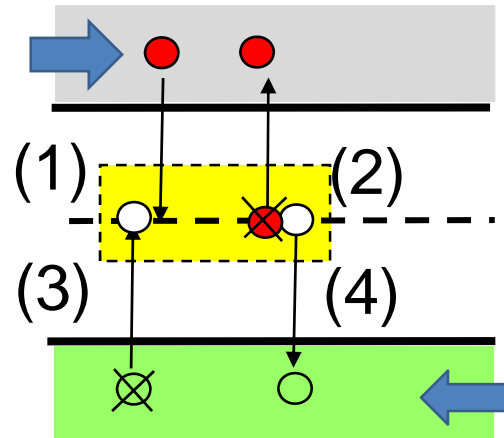
$$p_1 n_1 = n_i^2$$

$$p_1 = n_i^2 / n_1$$

$$= n_i g_D^{-1} e^{\beta(E_i - E_T)}$$



# **Dynamics** of Trap Population

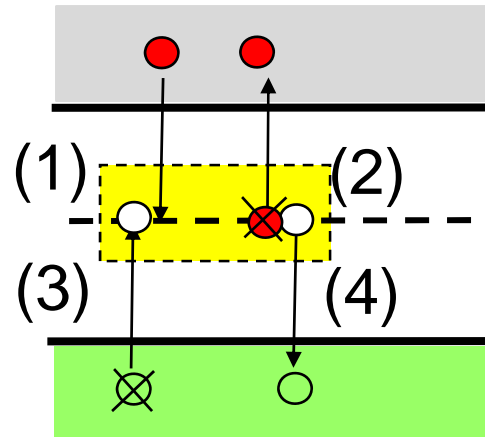


$$\frac{\partial n_T}{\partial t} = -\left. \frac{\partial n}{\partial t} \right|_{1,2} + \left. \frac{\partial p}{\partial t} \right|_{3,4}$$

$$= c_n n p_T - e_n n_T - c_p p n_T + e_p p_T$$

$$= c_n (n p_T - n_T n_1) - c_p (p n_T - p_T p_1)$$

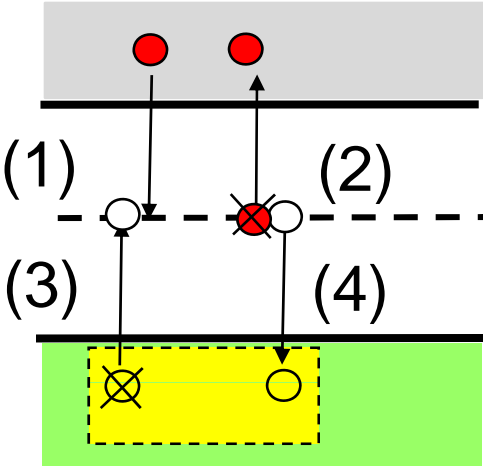
## Steady-state Trap Population



$$\frac{\partial n_T}{\partial t} = 0 = c_n (np_T - n_T n_1) - c_p (p n_T - p_T p_1)$$

$$n_T = \frac{c_n N_T n + c_p N_T p_1}{c_n (n + n_1) + c_p (p + p_1)} = c_n (np_T - n_T n_1)$$

# Net Rate of Recombination-Generation



$$\begin{aligned}
 R &= -\frac{dp}{dt} = c_p (p n_T - p_T p_1) \\
 &= \frac{np - n_i^2}{\left(\frac{1}{c_p N_T}\right)(n + n_1) + \left(\frac{1}{c_n N_T}\right)(p + p_1)}
 \end{aligned}$$

$\tau_n$    $\tau_p$

# Outline

- 1) Derivation of SRH formula
- 2) **Application of SRH formula for special cases**
- 3) Direct and Auger recombination
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# Case 1: Low-level Injection in p-type

$$R = \frac{np - n_i^2}{\tau_p (n + n_1) + \tau_n (p + p_1)}$$

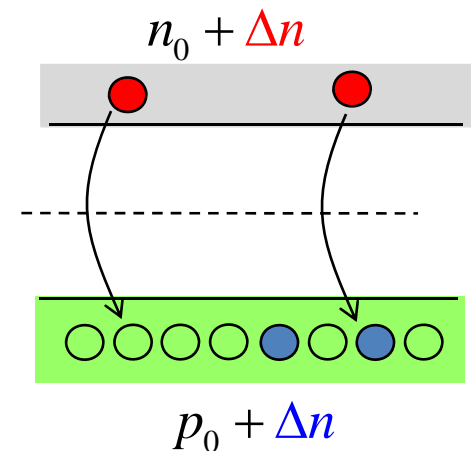
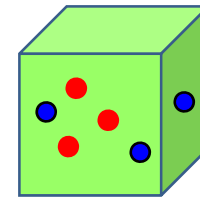
$$= \frac{(n_0 + \Delta n)(p_0 + \Delta n) - n_i^2}{\tau_p (n_0 + \Delta n + n_1) + \tau_n (p_0 + \Delta p + p_1)}$$

$$= \frac{\Delta n (n_0 + p_0) + \cancel{\Delta n^2}}{\tau_p (\cancel{n_0} + \Delta n + n_1) + \tau_n (p_0 + \Delta p + p_1)}$$

$$= \frac{\Delta n (p_0)}{\tau_n (p_0)} = \frac{\Delta n}{\tau_n}$$

$$\Delta n^2 \approx 0$$

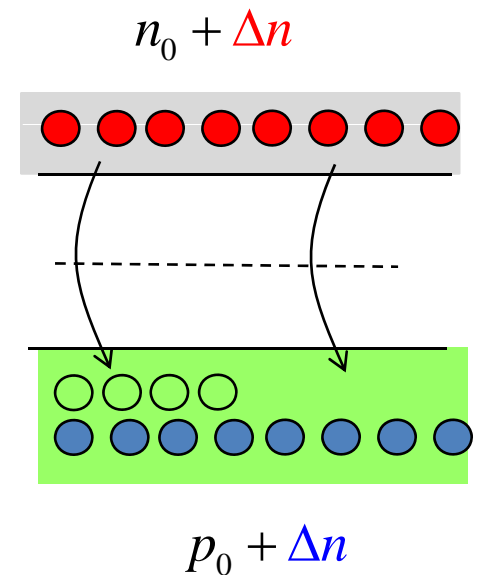
$$p_0 \gg \Delta n \gg n_0$$



## Case 2: High-level Injection

$$\begin{aligned}
 R &= \frac{np - n_i^2}{\tau_p (n + n_1) + \tau_n (p + p_1)} \\
 &= \frac{(n_0 + \Delta n)(p_0 + \Delta p) - n_i^2}{\tau_p (n_0 + \Delta n + n_1) + \tau_n (p_0 + \Delta p + p_1)} \\
 &= \frac{\cancel{\Delta n}(n_0 + p_0) + \Delta n^2}{\tau_p (\cancel{n_0} + \Delta n + n_1) + \tau_n (\cancel{p_0} + \Delta n + p_1)} \\
 &= \frac{\Delta n^2}{(\tau_n + \tau_p) \Delta n} = \frac{\Delta n}{(\tau_n + \tau_p)}
 \end{aligned}$$

e.g. organic solar cells



$$\Delta n \gg p_0 \gg n_0$$

## High/Low Level Injection ...

$$R_{high} = \frac{\Delta n}{(\tau_n + \tau_p)}$$

$$\Delta n \gg p_0 \gg n_0$$

$$R_{low} = \frac{\Delta n}{\tau_p}$$

$$p_0 \gg \Delta n \gg n_0$$

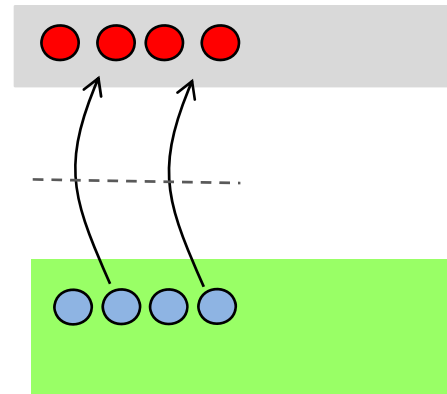
which one is larger and why?

## Case 3: Generation in Depletion Region

$$R = \frac{\cancel{np} - n_i^2}{\tau_p (\cancel{n} + n_1) + \tau_n (\cancel{p} + p_1)}$$

$$= \frac{-n_i^2}{\tau_p (n_1) + \tau_n (p_1)}$$

$$n \ll n_1 \quad p \ll p_1$$





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# Band-to-band Recombination

$$R = B \left( np - n_i^2 \right)$$

Direct recombination at low-level injection

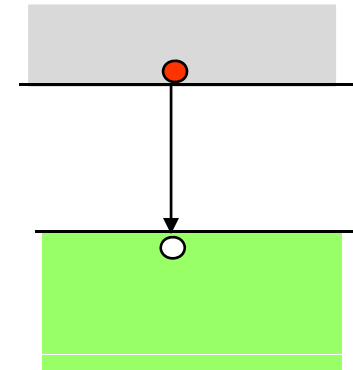
$$n_0 \ll (\Delta n = \Delta p) \ll p_0$$

$$R = B \left[ (n_0 + \Delta n)(p_0 + \Delta p) - n_i^2 \right] \approx B p_0 \times \Delta n$$

Direct generation in depletion region

$$n, p \sim 0$$

$$R = B \left( np - n_i^2 \right) \approx -B n_i^2$$



# Auger Recombination

2 electron & 1 hole

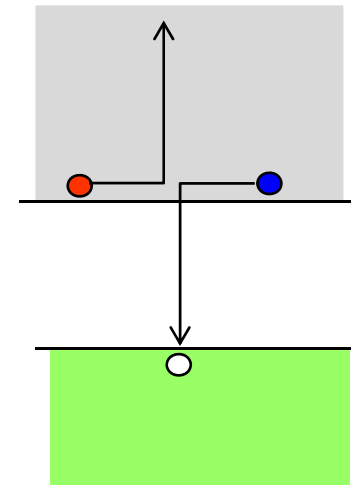
$$R = c_n \left( n^2 p - n_i^2 n \right) + c_p \left( np^2 - n_i^2 p \right)$$

$$c_n, c_p \sim 10^{-29} \text{ cm}^6/\text{sec}$$

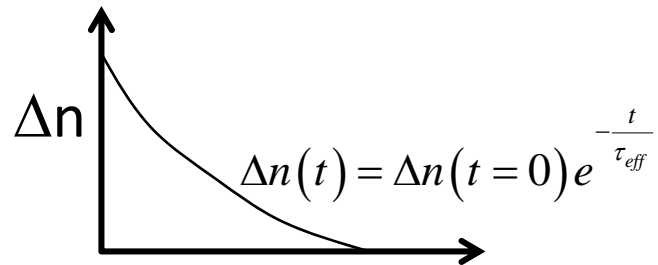
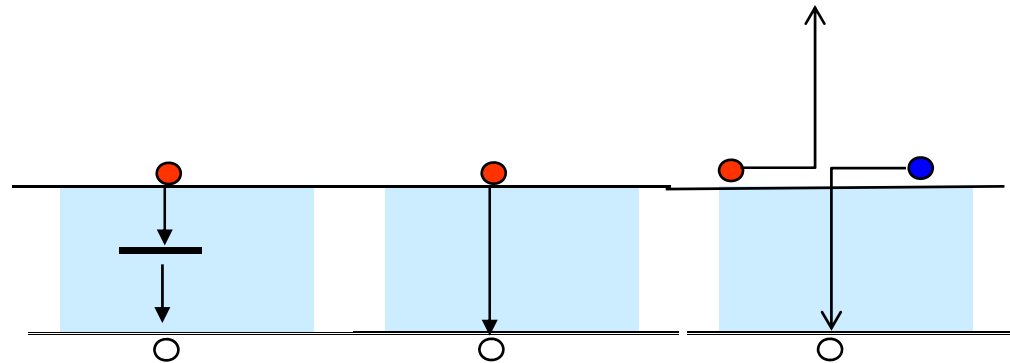
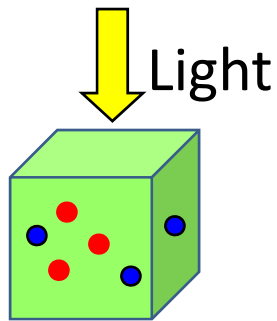
Auger recombination at low-level injection

$$n_0 \ll (\Delta n = \Delta p) \ll (p_0 = N_A)$$

$$R \approx c_p N_A^2 \Delta n = \frac{\Delta n}{\tau_{auger}} \quad \tau_{auger} = \frac{1}{c_p N_A^2}$$



# Effective Carrier Lifetime



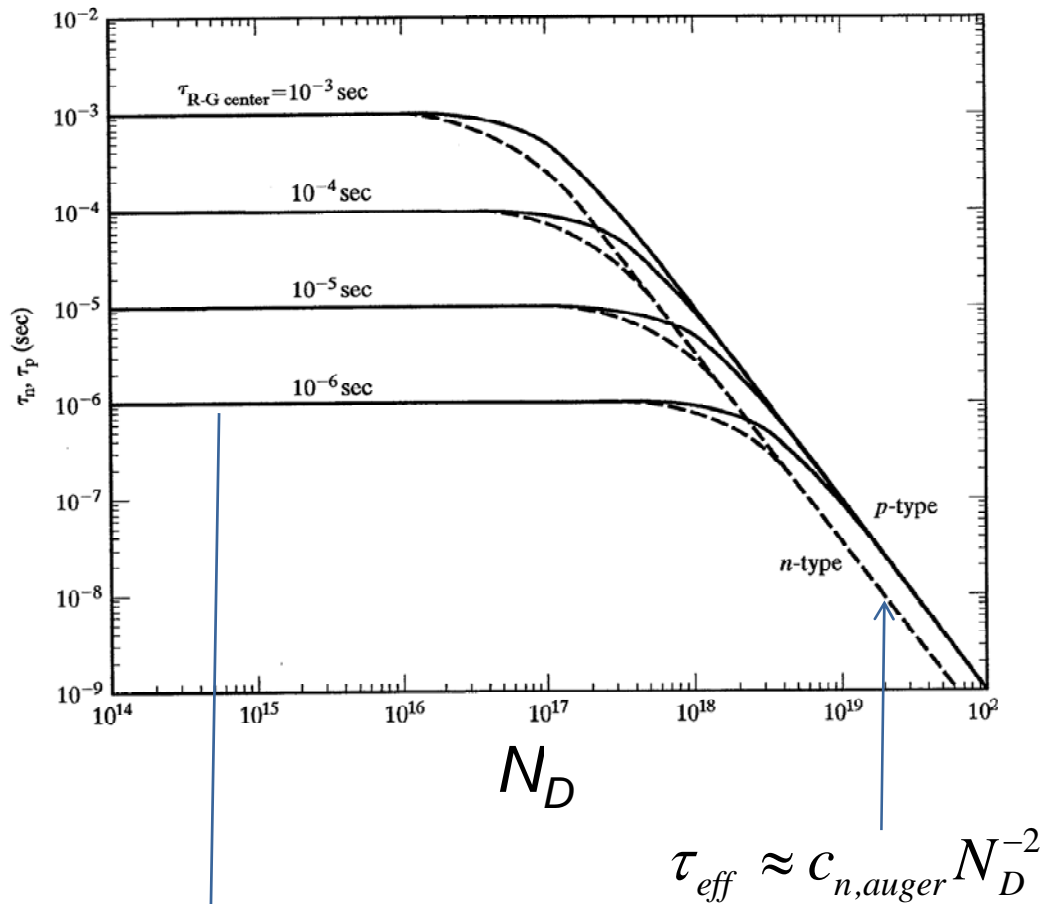
$$\tau_{eff} = \left( c_n N_T + B N_D + c_{n, auger} N_D^2 \right)^{-1}$$

$$R = R_{SRH} + R_{direct} + R_{Auger}$$

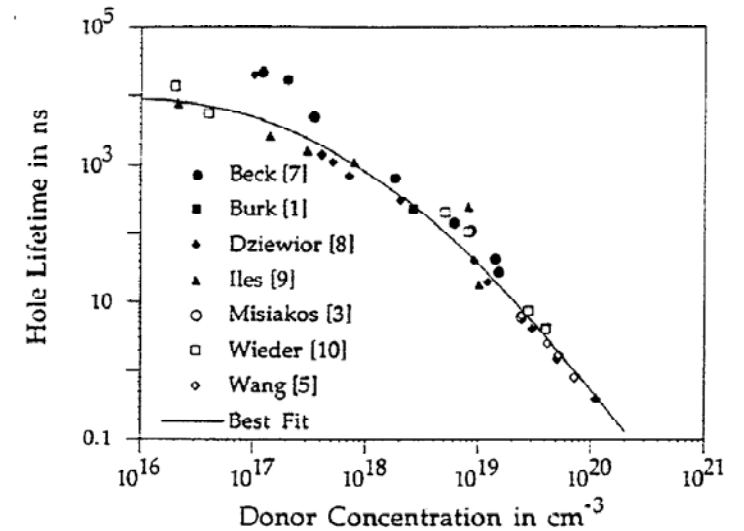
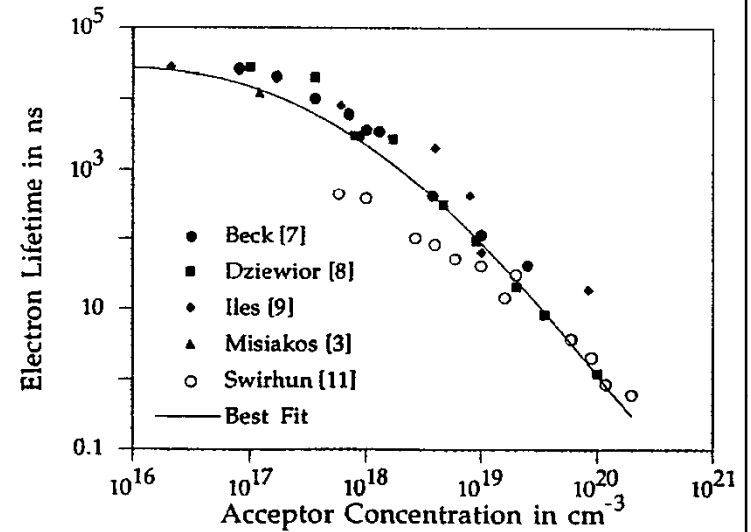
$$= \Delta n \left( \frac{1}{\tau_{SRH}} + \frac{1}{\tau_{direct}} + \frac{1}{\tau_{Auger}} \right)$$

$$= \Delta n \left( c_n N_T + B N_D + c_{n, auger} N_D^2 \right)$$

# Effective Carrier Lifetime with all Processes



$$\tau_{eff} = \left( c_n N_T + B N_D + c_{n, auger} N_D^2 \right)^{-1}$$



# Conclusion

SRH is an important recombination mechanism in important semiconductors like Si and Ge.

SRH formula is complicated, therefore simplification for special cases are often desired.

Direct band-to-band and Auger recombination can also be described with simple phenomenological formula.

These expressions for recombination events have been widely validated by measurements.