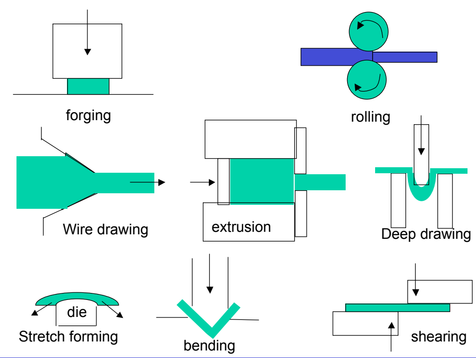
Some Processing Examples Using Stress

- **Plastic Deformation to Fracture: Engineering is in between**
  - **Stress-strain and microscopic factors can be used to engineer materials.**

- **Processing and Avoiding Failures**
  - **How can we use elastic, plastic, and fracture information?**
  - **How does temperature affect engineering considerations?**

Typical Forming Operations



Recall: Complex State of Stress and Strain in 3-D linear elastic, isotropic Solid

- Linear superposition of Hooke's Law and Poisson effect ($\varepsilon_{width} = -\nu\varepsilon_{axial}$) yield the 3 normal strain components:

X-direction: $\varepsilon_1 = \sigma_1/E - \nu(\sigma_2+\sigma_3)/E$

Y-direction: $\varepsilon_2 = \sigma_2/E - \nu(\sigma_1+\sigma_3)/E$

Z-direction: $\varepsilon_3 = \sigma_3/E - \nu(\sigma_2+\sigma_1)/E$

In x-direction, the total linear strain is:

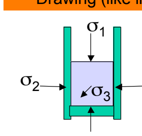
$$\varepsilon_1 = \frac{1}{E}\{\sigma_1 - \nu(\sigma_2+\sigma_3)\} \quad or \quad \frac{1}{E}\{(1+\nu)\sigma_1 - \nu(\sigma_1+\sigma_2+\sigma_3)\}$$

For uniaxial tension test, $\sigma_1 = \sigma_2 = 0$, so $\varepsilon_3 = \sigma_3/E$ and $\varepsilon_1 = \varepsilon_2 = -\nu\varepsilon_3$,

and volume strain is $\Delta V/V = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = (1-2\nu)\sigma_3/E$, as before.

Processing Example: Stress, Strain, and Poisson Effect

Drawing (like in soda cans), Dyes, Pressing, Stamping, etc.



For pressing, etc., applied stress is σ_1. Z-direction is free surface.

Why is $\sigma_3 = 0$? **What is σ_2?**

Wall Effect: NO contraction **What is strain?**

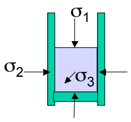
Direction 2: Equilibrium requires that $F_{wall} = F_2$, so $\varepsilon_2 = 0$

From general strain eq.: $\varepsilon_2 = 0 = \sigma_2/E - \nu(\sigma_1+\sigma_3)/E$. So $\sigma_2 = \nu\sigma_1$.

Direction 1: $\varepsilon_1 = \sigma_1/E - \nu(\sigma_2+\sigma_3)/E$. So $\varepsilon_1 = \sigma_1(1-\nu^2)/E$.

In general, one CANNOT IGNORE POISSON EFFECT for Stresses and Strains.

Processing Example: Stress, Strain, and Poisson Effect



So σ_3 = 0, ε_2 = 0, and $\sigma_2 = \nu\sigma_1$ by Poisson Effect.

Strain in x-direction: $\varepsilon_1 = \sigma_1(1-\nu^2)/E$

Consider using Cu: **E = 110 GPa, YS= 69 MPa, TS= 200 MPa and ν= 0.34.**

For applied stress $\sigma_1 < \sigma_{YS}$, the combined strains for linear-elastic behavior is OK.

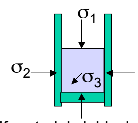
Let **σ_1 =–69 MPa**, so ε_1 = –69 MPa (1– 0.34^2)/110 GPa = –0.00056

Notice this is less than if Poisson Effect was ignored, i.e. $\varepsilon_1 = \sigma_1/E$ = –0.00063
Therefore, constraint due to wall decreases yield strain.

What is the stress due to wall: **$\sigma_2 = \nu\sigma_1$** = 0.34(–69 MPa) = **–23.5 MPa**.

What is the strain along dir. 3? **$\varepsilon_3 = -\nu(1+\nu)\sigma_1/E$** σ_3 is zero, not strain!
From **Poisson Effect** Show ε_3 = 0.00029

Processing Example: Stress, Strain, and Poisson Effect



So σ_3 = 0, due to wall ε_2 = 0, and Applied Stress is σ_1

$\sigma_2 = \nu\sigma_1$ and $\varepsilon_1 = \sigma_1(1-\nu^2)/E$ by Poisson Effect.
E = 110 GPa, YS= 69 MPa, UTS = 200 MPa and ν= 0.34.

• If material yields, i.e. $\sigma_1 > \sigma_{YS}$, there is plastic deformation and volume is conserved (until necking), so $\Delta V/V = 0 = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$, and ν=1/2.

Is that correct? $0 = \varepsilon_1 + 0 + \varepsilon_3 = (1-\nu^2)\,\sigma_1/E - \nu(1+\nu)\sigma_1/E$
True only acts like ν=1/2, as expected.

During uniform plastic deformation $\varepsilon_1 = -\varepsilon_3$ and $\varepsilon_2 = 0$.

What if $\sigma_1 = -\sigma_{UTS}$ = –200 MPa? Equations no longer valid! Why?
But you have certainly *yielded* and $\varepsilon_1 = -\varepsilon_3$, so Cu decreases in x-dir. and extends in z-dir. (one-to-one), and any higher stress can cause *fracture*.
Still, it is clear roughly, $\sigma_2 = \nu\sigma_1$ = 0.34 (-200 MPa) = 68 MPa (*no yielding* in y-dir.)

Thermal Expansion and Thermal Stress

Temperature changes always generate residual stresses that can lead to fracture, e.g., ceramic cooled quickly.

Thermal shock-induced fracture has been studied for more than 2200 years!

It is reported that Hannibal's military engineers used thermal shock to fracture rocks that blocked the path of the Carthaginian army (from Spain) while crossing the Alps in 218 B.C.

Hannibal got within 150 km of Rome. He finally attacked Rome in 211only after the Carthaginian government stopped sending reinforcements. He lost.

Linear and Volume Thermal Expansion

Thermal Strain: $\varepsilon_T = \Delta L/L_0 = (L_f - L_0)/L_0 = \alpha_T\,\Delta T = \alpha_T\,(T_f - T_0)$
units = inverse temperature

$$\alpha_L(T) = \frac{1}{L}\frac{\Delta L}{\Delta T}$$

see Table 19.1:
e.g., Al: 23.6 x 10^{-6}/C; Al_2O_3: 7.6 x 10^{-6}/C; SiO_2: 0.4 x 10^{-6}/C

Volume Thermal Strain: $\varepsilon_T = dV/V = \alpha^V_T\,dT$
$\alpha^V_T = (1/V)(\Delta V/\Delta T) \approx 3\,\alpha_T$ for cubic crystals since L^3~V.

- *In Invar systems, like $Fe_{63}Ni_{36}$, α_T~0 near R.T.* (Invar = volume *Invar*iant)
- *α-Uranium has 3 different α_T, with 1 being negative!*
- *Rubber has α_T < 0 due to entropy of polymer chains* (lots of wiggle room).
- *α_T may be discontinuous at allotropic boundaries* (e.g., FCC → BCC).

Linear Thermal Expansion: General Trend

$$\alpha_L(T) = \frac{1}{L}\frac{\Delta L}{\Delta T}$$

- **Thermal Expansion** depends upon the bond strengths between atoms and the *asymmetry in U vs r*.

- **Recall General Trend from Chapter 2**

  Increasing bond strength (higher T_{melt}) → Decreasing atomic spacing → Higher elastic stiffness → Lower α_T

- For ceramics, α_T decreases with bond strength and %covalent bond.

α_T is highest → lowest

Organic solids (lowest bond strength) → Metals → Ceramics (highest bond strength)

Thermal Stress and Strains

- **For linear superposition:** $\varepsilon_{Total} = \varepsilon_{mechanical} + \varepsilon_{Thermal}$

For normal strains, e.g.

X-direction: $\varepsilon_1 = \sigma_1/E - \nu(\sigma_2+\sigma_3)/E + \alpha_T \Delta T$

Y-direction: $\varepsilon_2 = \sigma_2/E - \nu(\sigma_1+\sigma_3)/E + \alpha_T \Delta T$

Z-direction: $\varepsilon_3 = \sigma_3/E - \nu(\sigma_2+\sigma_1)/E + \alpha_T \Delta T$

- If free thermal expansion is prevented by geometric constraint, e.g., $\varepsilon_{Total} = \varepsilon_{mechanical} + \varepsilon_{Thermal} = 0$ along a particular dimension, then a sufficient ΔT will cause large stresses to develop!

- This is how Hannibal's engineers thermally shocked the rocks. How large is large?

Thermal Stress from Constraints

Consider a Rod in 1-D
e.g., constrained beams, gas pipelines, sensors…

- **Total Strain:** $\varepsilon_{Total} = \varepsilon_{applied\text{-}stress} + \varepsilon_{Thermal}$

  $\varepsilon_{applied\text{-}stress} = \sigma_{applied}/E$ and $\varepsilon_{Thermal} = \sigma_{Thermal}/E$

- **Constrained rod:** $\varepsilon_{Total} = \varepsilon_{applied\text{-}stress} + \varepsilon_{Thermal} = 0$ **Why?**

  So, $\varepsilon_{applied\text{-}stress} = -\varepsilon_{Thermal} = -\alpha_T \Delta T$ and $\sigma_{applied} = -\alpha_T \Delta T E$

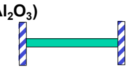
- **Unconstrained rod:** $\varepsilon_{Total} = \varepsilon_{thermal}$ **Why?**

  So, $\varepsilon_{Thermal} = \alpha_T \Delta T$ and $\sigma_{Thermal} = \alpha_T \Delta T E$

Why are pipes usually buried?

Example Thermal Stress with Constraints: Alumina

Consider a Rod in 1-D **10-cm rod of Alumina (Al_2O_3)**
99.8% dense decreases from 200 C → 0 C.
E = 385 GPa, UTS = 205 MPa and $\alpha_T = 6.7 \times 10^{-6}$/C



(a) What is change in length if unconstrained?
Expect contraction since cooled. $\varepsilon_{Thermal} = \Delta L/L_0 = \alpha_T (T_f - T_0)$.

$\Delta L = \alpha_T (T_f - T_0) L_0 = (6.7 \times 10^{-6}/C)\ (0–200)C\ (10\ cm)$
$= –0.0134$ cm (–0.134%) contraction!

(b) If rod is constrained (thermal stresses), what is σ_{wall}?
$\varepsilon_{Total} = \varepsilon_{applied\text{-}stress} + \varepsilon_{Thermal} = 0.$ Or, $\varepsilon_{wall} = -\varepsilon_{Thermal}$

Therefore, $\sigma_{wall} = -\sigma_{Thermal} = -\alpha_T \Delta T E = (6.7 \times 10^{-6}/C)(200C)(385\ GPa)$

σ_{wall} = 516 MPa **Does rod break?**
It is tensile, i.e. alumina wants to contract.

Example Thermal Stress with Constraints: Alumina

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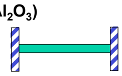


(c) What is maximum temperature change so as not to fracture rod?

Fracture will occur for $\sigma > \sigma_{UTS}$. To not fracture, it must be no larger.

That is, $\sigma_{Thermal} = \alpha_T \Delta T\ E = \sigma_{UTS}$.

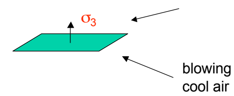
Solving ΔT = (205 MPa)/[(6.7 x 10^{-6}/C)(385 GPa)] = 79.5 C

Recap: ΔT = 79.5 C produces 205 MPa of stress, and 200 C produces 516 MPa!

In general, there is a ΔT capable of fracturing the rod.
ΔT is smaller the smaller the UTS or α_T or the larger the E. **(as in rocks)**

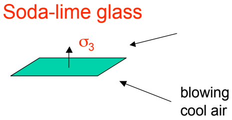
Thermal Stress with Constraints: Safety Glass

Consider a 2-D Sheet of Glass being rolled out of furnace



- Surfaces of sheet are cooled (**$T_f < T_0$**) rapidly by *blowing air* as it is rolled out.
- Free surface has no applied stress: **$\sigma_3 = 0$**.
- Surface layer will reach T_f quickly, but total strains in x-y plane are zero because material does not have time to adjust temperature, $\varepsilon_{total\text{-}1} = \varepsilon_{total\text{-}2} = 0$.
- Thermal Strains Required: $\varepsilon_{total\text{-}1} = 0 = \sigma_1/E - \nu(\sigma_2+\sigma_3)/E + \alpha_T \Delta T$
- Thermal Stresses Created: $(\sigma_1 - \nu\sigma_2)/E = -\alpha_T (T_f - T_0)$ (and $\sigma_1 = \sigma_2$)
- So, for this planar stress case, $\sigma_1 = \sigma_2 = -\alpha_T (T_f - T_0)E/(1-\nu)$

Example: 2-D Sheet of Soda-lime glass



For Soda-lime glass: α_T = 9.6 x 10^{-6}/C, E = 69 GPa, ν=0.23, UTS = 69 MPa

For a 40^0 C decrease in surface temperature

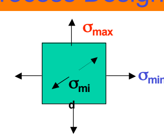
$\sigma_1 = \sigma_2 = -\alpha_T (T_f - T_0)E/(1-\nu)$

= $-(-40)$ (9.6 x 10^{-6}/C) (69 GPa)/(0.77) = 34.4 MPa (tension)

Where does stored energy go?

Why is no YS reported for Soda-lime glass?

Process Design: Use CRSS and Max. Shear on 45^0 planes



Consider 3-D axial stress state
Recall that normal stresses produce shear!

Plastic deformation (yielding) due to shear stresses with maximum on 45^0 planes.

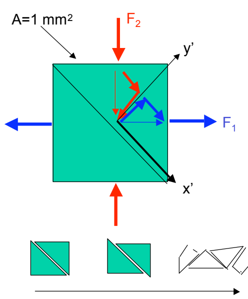
Can develop an approximate estimate — the **Minimum Shear Stress Criterion** from knowing just normal stresses.

$$\sigma_{YS} = (\sigma_{max} - \sigma_{min}) > \tau_{CRSS}$$

Now we can predict in engineering application yielding, just from σ-ε info.

Pure Shear From Only Axial Stresses. How?

Consider 2-D stress state: F_1 = +10 N (tensile)= – F_2 (compressive)



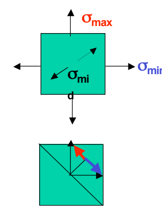
- **Stresses:** σ_2 = F_2/A= –10 MPa = –σ_1
- **Normal stress on 45⁰ Plane** (A_{45}=√2A): $F_{2,y'}$ + $F_{1,y'}$ = 0
- **Shear Stresses on 45⁰ Plane** (A_{45}=√2A): $F_{2,x'}$ = 10 N/√2 and $F_{1,x'}$ = 10 N/√2

τ_x = ($F_{2,x'}$ + $F_{1,x'}$)/A_{45} = 10 MPa.

PURE SHEAR!

Increasing deformation cause *strain and thinning*

Yielding Reach at CRSS: Shear on 45⁰ planes a maximum



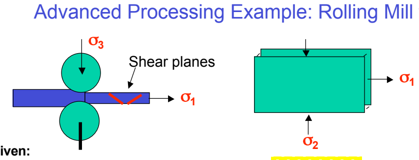
Consider 3-D axial stress state

- **Shear stresses maximum on 45⁰ planes.**
- Maximum shear stress τ_{max} = (σ_{max} - σ_{min})/2
- Critical Resolved Shear Stress: τ_{CRSS} = σ_{YS}/2
- Initial Yielding when τ_{max} = τ_{CRSS}

OR σ_{YS}/2 = (σ_{max} - σ_{min})/2
which is approximately correct and called the **Minimum Shear Stress Criterion**

Now we can predict in engineering application yielding, just from σ-ε info.

Advanced Processing Example: Rolling Mill for Cu



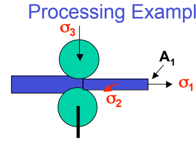
Given:
- Applied tensile force in the plane of the sheet is F = 0.22 MN (x-direction).
- Sheet is lubricated so that NO shear forces act.
- 0.5 m wide (y-direction), 0.6 cm thick (z-direction)

Knowns: σ_3 must be *compressive* to get *decrease* in z-dimension.
- YS= 145 MPa for Cu sheet.
- For rolling, the plastic deformation occurs by plane-strain such that there is NO increase in the width of the sheet (y-direction) and thus ε_2 **=0**.

This is like shearing a deck of playing cards: the width strain is zero.
- We must have *yielding*, so ν = 1/2 and volume is maintained.

DESIGN NEED: What applied stress is required to make sheet yield, i.e. σ_3 ?

Processing Example: Rolling Mill for Cu (Pressing to Yield)



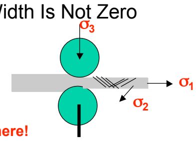
- NO shear forces act. *YS=145 MPa for Cu*
- Note that s_2 ≠ 0 due to Poisson Effect.
- A_1 = (0.5 m) (6 x 10⁻³ m) = **3 x 10⁻³ m²**.
- With F_1 = 0.22 MN,
  σ_1 = F/A = (0.22 MN)/(3 x 10⁻³ m²)= **73.3 MPa**

- **How are σ_2 and σ_3 related?** By Poisson Effect, as we want to just yield.
- **Due to plane-strain:** $\varepsilon_2 = 0 = [\sigma_2 - \nu(\sigma_1 + \sigma_3)]/E$ or $\sigma_2 = \nu(\sigma_1 + \sigma_3)$
- **Just at yielding ν = 1/2, so:** $\sigma_2 = (\sigma_1 + \sigma_3)/2$ = (73.3 MPa + σ_3)/2
  Note: σ_1 (max) > σ_2 > σ_3 (min)
- **Yielding at** ***Maximum Shear Stress Criterion***: (σ_{max} - σ_{min})/2 > σ_{YS}/2
  or (σ_1 - σ_3)/2 > σ_{YS}/2 → σ_3 = **–(145 - 73.3) MPa = –71.7 MPa**

Engineered deformation is obtained by applying (compressive) rolling pressure of –71.7 MPa to initiate yielding to make sheet correct thickness.

In Rolling, *Internal* Stress Normal to Width Is Not Zero

Why is $\sigma_2 = (\sigma_1 + \sigma_3)/2$ and NOT $\sigma_2 = 0$?



- **Of course, σ_2 = 0 at the free surface, but only there!**
  Rolling creates internal 3-D stress state.

- **But for ε_2 =0** (plane-strain), $\sigma_2 = (\sigma_1 + \sigma_3)/2$ **internally due to Poisson Effect.**
  This depends on the width of sheet compared to other dimension, of course.
  Mostly the slip systems in x-z plane are initiated, hence ε_2 =0 (plane-strain).

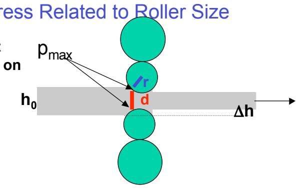
- **So, σ_2 = 0 at surface and rapidly becomes $(\sigma_1 + \sigma_3)/2$ inside.**

- At yielding, **$\Delta V = 0 = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$** and $\nu = 1/2$**!** **With ε_2 =0, $\varepsilon_1 = -\varepsilon_3$**.
  Only for $\sigma_2 = (\sigma_1 + \sigma_3)/2$ and $\nu = 1/2$ is $\varepsilon_1 = -\varepsilon_3$ in strain equations.

- **For $\varepsilon_2 = 0$, $\sigma_2 = (\sigma_1 + \sigma_3)/2$ = (73.3 - 71.7)/2 MPa = 0.8 MPa ~ 0 MPa**.
  But this is not true in general, only for almost pure shear.

Rolling Maximum Stress Related to Roller Size

- **Δh is reduction in height of sheet**
- **d is height at maximum pressure on the sheet, which is $d \sim h_0 - \Delta h$**
- **r is radius of roller.**



- **Can show approximately:** $p_{max} = \sigma_{YS}\,(1 + \sqrt{2}\,(r/d)^{0.5}\,(\Delta h/d)^{0.5})$

- **For roller radius about r ~ d, p_{max} is not vastly bigger than YS.**
  SMALL rollers better than BIG roller.

- **For *aluminum foil* primary rollers have the diameter of a pencil, with 18 secondary rollers!**

Why the extra rollers? **Consider smallness of roller and deformations.**