Fundamentals of Nanoelectronics

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Lecture 3: Quantum of Conductance

Reference: Chapters 1.3, 111

Network for Computational Nanotechnology

NCN nanoHUB online simulations and more

Review of Current Expression

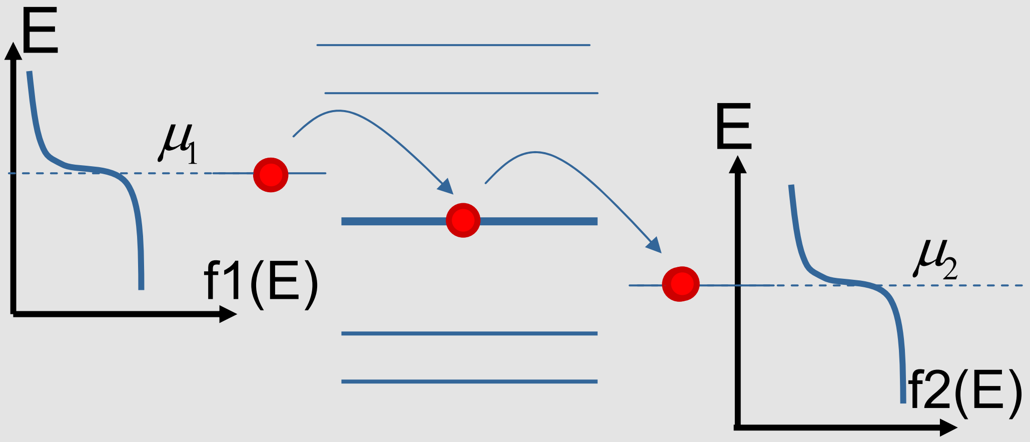
00:12

- Last time we learned how to calculate current through some really small channel namely one that has only one electronic energy level. Remember that you can think of any material in the channel as one that provides energy levels for conduction (transport of electrons). But are all of these levels important for conduction? (See next slide)

$$\left.\begin{aligned} I_1 &= q\frac{\gamma_1}{\hbar}(f_1 - N) \\ I_2 &= q\frac{\gamma_2}{\hbar}(N - f_2) \end{aligned}\right\}$$

At steady state:

$$\therefore I = I_1 = I_2 = \frac{q}{\hbar}\frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2}(f_1 - f_2)$$



Conducting Levels

00:36

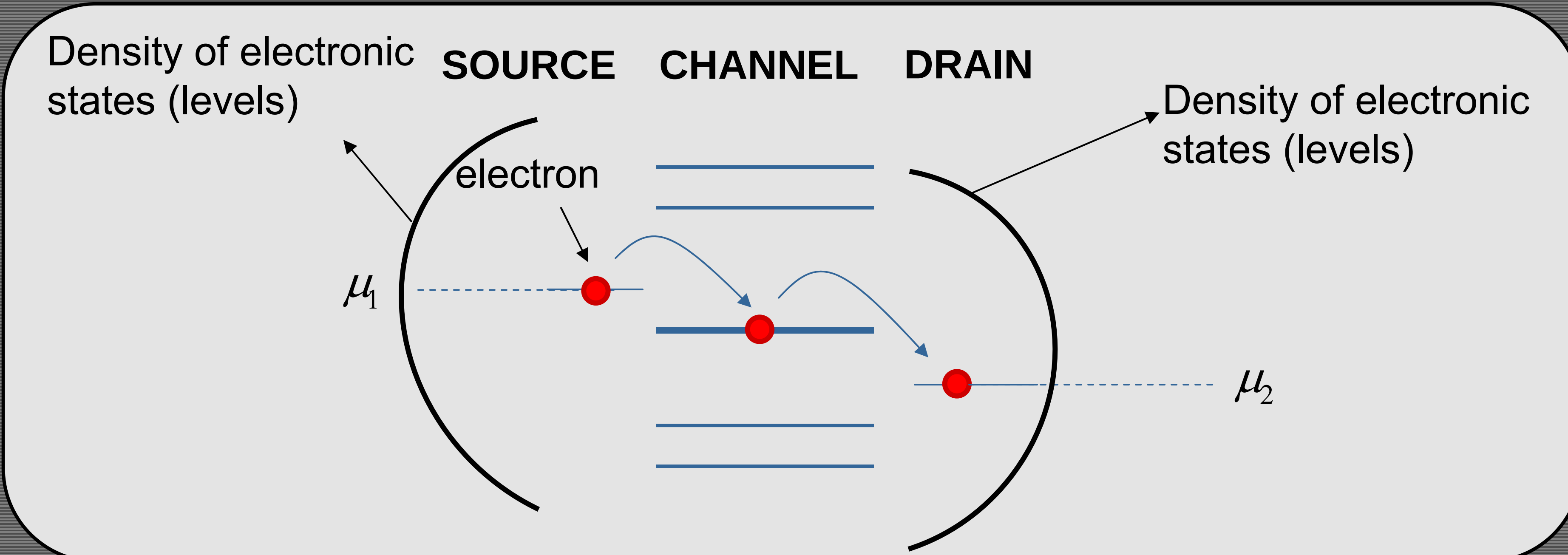
- As it turns out conduction only cares about those levels that are in close vicinity of chemical potential say within a few $k_B T$ of it. (Chemical potential or Fermi level is a hypothetical border that at 0 K divides the full levels from the empty ones. All levels above it are empty and every level below it is full). But what happens when we apply a voltage between the two contacts of the device?

- Suppose one keeps the source channel at 0 potential and applies a positive voltage to the drain contact. Then the electronic levels in the drain will shift down. This in turn will result in the lowering of Fermi level. How about an illustration of this?...

What makes electrons flow?

02:11

• In a channel electrons flow because of the difference of agenda of the Fermi functions in the two contacts. One keeps filling up the level(s) while the other empties them. The result of this process is a net flow of current.



• Note: The two contacts are connected such that the system makes a loop. Of course without that there would be no current. When source looses an electron, another one enters the source from left and the process continues.

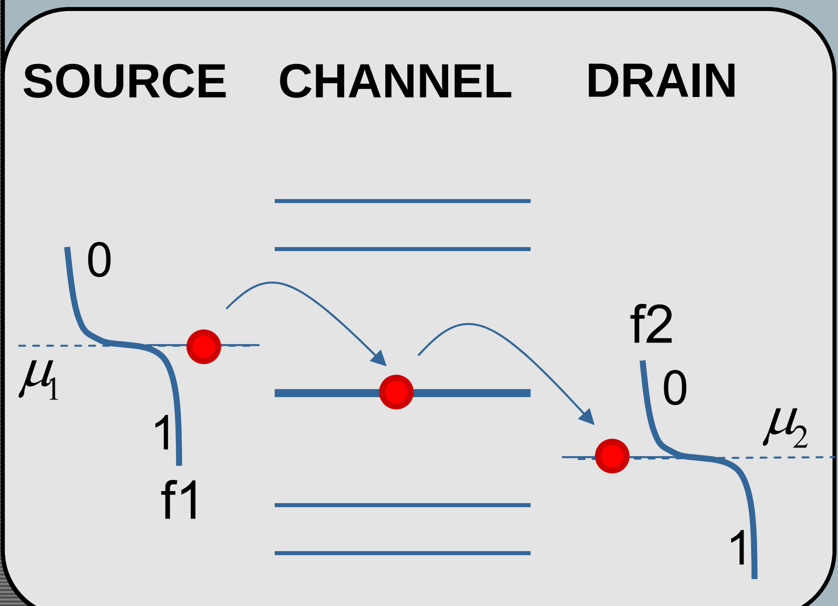
• Last time we calculated this current…

Why do some levels matter for conduction while others don't?

04:19

$$I = \frac{q}{\hbar}\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}\left(f_1 - f_2\right)$$

- Where γ_1 and γ_2 are two parameters that tell us how is easily an electron can escape into the source or the drain. Consider the dimension: if you divide them by $\hbar$ then you get the units of per second which is the rate of escape.
- The last factor in the equation in fact shows that current flows if there is a difference between the Fermi functions in the contacts. If the two are equal then the current will be 0.



- From the shape of the two Fermi functions you can see that the levels deep down and levels way above do not contribute to the current because at those levels f1 and f2 have the same value of 1 (deep down) and 0 (way above).
- The above expression for current is valid when the applied voltage is small. If the coupling is stronger than a certain amount, then we have to take into account for the effect called "broadening" which comes next …

Quantum of Conductance/ Towards Broadening 1

08:46

• To understand broadening let us simplify the expression of current a bit. Suppose the level in the channel is located in a position where f1=1 and f2=0. Then
f1-f2=1. Assume more over that $\gamma_1 = \gamma_2$
Current becomes:

$$I = \frac{q}{\hbar}\frac{\gamma_1}{2}$$

• What this tells us is that the more the coupling the more the current which is intuitive. Higher voltage => higher current.

• How ever it has been shown by experiment that there is a maximum current that is obtainable. This means that there is a maximum conductance:

• This maximum conductance is called: "Quantum of Conductance" and its value is:

$$G_{max} = 2q^2/h$$

Correspondingly there is a minimum resistance: $R_{min} = h/2q^2$

• This is significant in a sense that even with the best conceivable contacts the resistance will not be 0 and it has a certain minimum. Notice that this is not quite in agreement with Ohm's law which state states that $R = \rho L / A$. Here as L gets really small the resistance tends to 0. What is now known is that for the Ohm's law to account for really small devices we have to add a constant L0 to L such that resistance won't become 0.

• Next let's calculate minimum R …

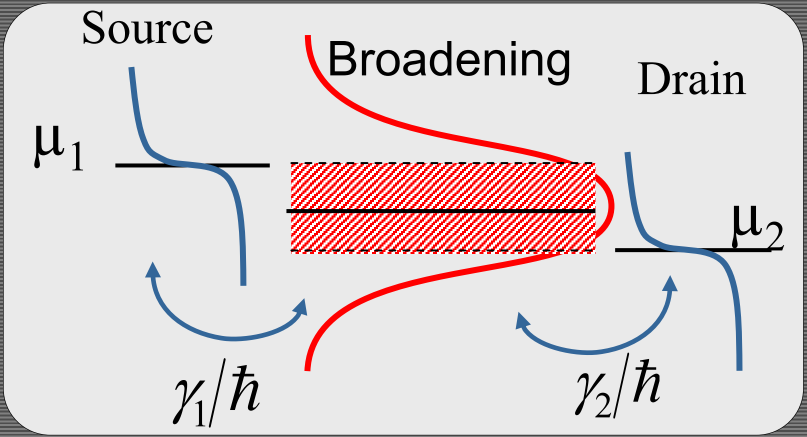
Quantum of Conductance/ Towards Broadening 2

13:00

$$\frac{h}{2q^2} = \frac{6.6\times10^{-34}\,J-s}{(1.6\times10^{-19})^2\,coul^2} = 12.9K\Omega$$

• Now we need to know where this quantum of conductance is coming from and modify our expression of current to actually reflect this fact:

Broadening of a level



• When a strong voltage is applied to the contacts the sharp discrete level that we had before broadens out and becomes a continuum of states sum of which is still one level and can accommodate only one electron. This is not trivial and will be discussed more later on. But can you see why we have a maximum conductance now?

• As the level broadens out some of it will lie outside of the region between the two chemical potentials and this does not contribute to the current because outside of this region the two contact Fermi functions are equal and their subtraction gives 0 current.

• So how does the current get modified?

Approximate Calculation of Maximum Conductance

- Since only part of the level contributes to the current, the expression for the current should only include the fraction of it which is active in transporting electrons. This fraction (or ratio) is equal to qVd which is the width of the region between the two chemical potentials divided by the total broadening (2 γ_1 in this case). Current becomes:

$$I = \frac{q}{\hbar}\frac{\gamma_1}{2} \times \frac{qV_D}{2\gamma_1} = \frac{q^2}{4\hbar}V_D$$

- The more the applied voltage, the more the coupling, the more the broadening and lesser the part of the level that is in between chemical potentials. So more coupling will not result in a better conductance.

- Let's now discuss the physics of the phenomena:
- We have a sea of electronic states in a contact and one sharp level in the channel. When these two are coupled together, the channel level looses some of its weight to the contact which is significant for the channel but which will not influence anything in the contact analogous to the fact if you add a drop of water to a sea of water nothing happens in the sea. At the same time the contact levels also loose some of their weight to the channel in the vicinity of the original level which changes the physics of the channel because there was no level in the close proximity of the sharp level before and there is some now. More rigorously…

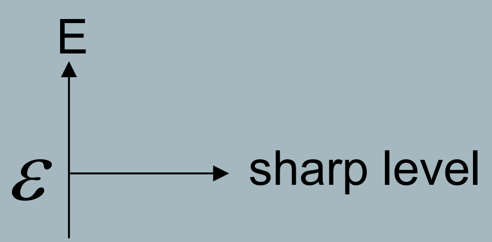
31:56

Towards the Precise Expression of Broadening

• Each electronic level has a wave function Ψ associated with it. With no coupling this wave function has the following description:

$$\psi \propto e^{-i\varepsilon t/\hbar}$$

Which is an expression in the time domain. To get the corresponding description of physics in the energy domain we Fourier transform this which gives us an impulse function



• Back to Ψ: the physical interpretation of this is that its module squared gives us the probability of finding the electron at a point. But $|\Psi|^2$ is one for the above expression of Ψ. This … means that nothing changes with time. After coupling, one would expect the electron to escape into the channel. Now there is a lifetime associated with the electron. The wavefunction gets modified to show this lifetime:

$$\psi \propto e^{-i\varepsilon t/\hbar} e^{-t/2\tau}$$

The factor of 2 is in there so that

$$|\psi|^2 = e^{-t/\tau}$$

Note that this is the measurable quantity in the experiments and not Ψ. We have set the expression of wavefunction in a way to give us the lifetime of τ for the probability of finding the electron in the channel.

• The Fourier transform of this in energy domain gives us the precise expression of broadening which is called a Lorentzian…

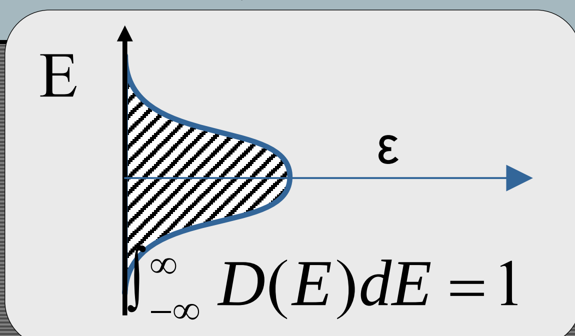
Lorentzian Distribution & Modified Expression of Current

- For the wavefunction:

$$\psi \propto e^{-i\varepsilon t/\hbar} e^{-t/2\tau}$$

- The expression for the density of states (after Fourier transforming the wavefunction) is:

$$D(E) = \frac{\gamma/2\pi}{(E-\varepsilon)^2 + (\gamma/2)^2}, \gamma = \gamma_1 + \gamma_2 = \hbar/2\tau$$



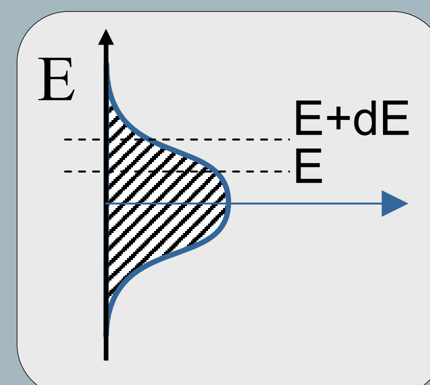
$$\int_{-\infty}^{\infty} D(E)dE = 1$$

- D(E) is a Lorentzian
- Lorentzian characteristics: peak value of $2/\pi\gamma$; which depends on γ.
- The area under the curve is 1 because it can accommodate exactly one electron.

- What we want to do now is to evaluate the current based on this expression of broadening. Previously we had only one level.

$$\therefore I = \frac{q}{\hbar} \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} (f_1 - f_2)$$

- Now we have a distribution of levels which signals that we need to integrate…



$$I = \int_{-\infty}^{\infty} dE \cdot D(E) \frac{2q}{\hbar} \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} [f_1(E) - f_2(E)]$$