ECE 659 Quantum Transport: Atom to Transistor

Lecture 9: Landauer-Buttiker Formalism

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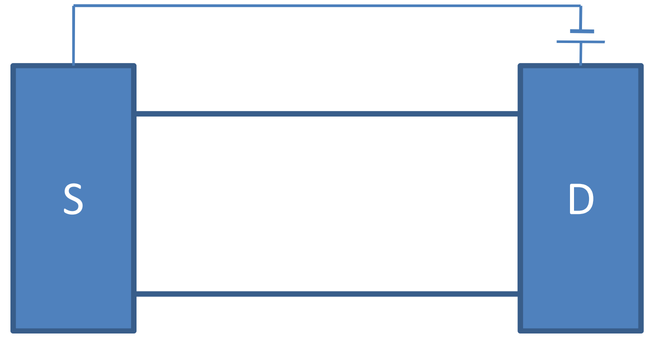
$$I = q\int dE\left(-\frac{df}{dE}\right)\frac{Dv_z}{2L}\underbrace{\frac{\lambda}{\lambda+L}(f_1-f_2)}_{f^+-f^-}$$

$$I = q\int dE\left(-\frac{df}{dE}\right)\frac{Dv_z}{2L}\underbrace{\frac{\lambda}{\lambda+L}(\mu_1-\mu_2)}_{\mu^+-\mu^-}$$

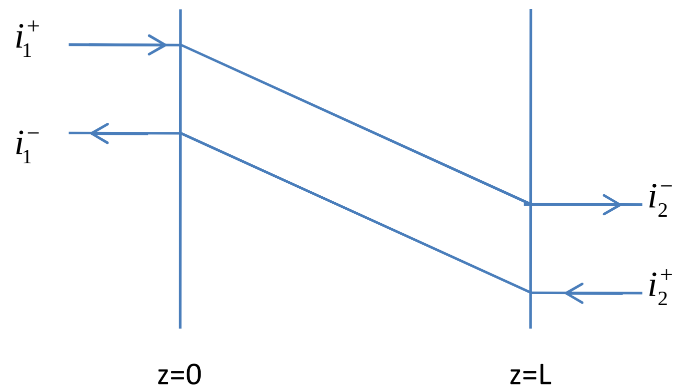


$$i^+ = \frac{q}{h}M\mu^+$$

$$i^- = \frac{q}{h}M\mu^-$$

$$i_1^- = Ri_1^+ + Ti_2^+ \qquad T = \frac{\lambda}{L+\lambda}, R = \frac{L}{L+\lambda}$$

$$i_2^- = Ti_1^+ + Ri_2^+$$



$$\{i^-\} = [S]\{i^+\} \Rightarrow S = \begin{bmatrix} R & T \\ T & R \end{bmatrix}$$

$$\{i\} = \{i^{+}\} - \{i^{-}\}$$
$$= [I - S]\{i^{+}\}$$
$$\{i^{+}\} = M\{\mu^{+}\}$$
$$\{i\} = [I - S]M\{\mu^{+}\}$$

$$[I - S] = \begin{bmatrix} T & -T \\ -T & T \end{bmatrix}$$

$$\{i\} = MT\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\{\mu^{+}\}$$

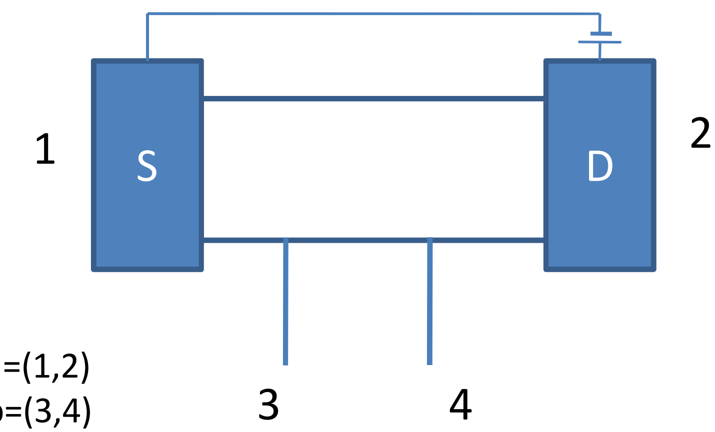
$$i_1 = MT(\mu_1 - \mu_2)$$
$$i_2 = -MT(\mu_1 - \mu_2)$$

$$Y = [I - S][M]$$

If we have defined voltages in terms
Of average electrochemical potentials $\frac{\mu^{+} + \mu^{-}}{2}$

$$Y = 2[I - S][I + S]^{-1}[M]$$

4 terminal measurements:



a=(1,2)
b=(3,4)

$$i_a^{-} = Ai_a^{+} + Ci_b^{+}$$
$$i_b^{-} = Di_a^{+} + Bi_b^{+}$$

Each A, B, C, D are 2x2 matrices

Open circuit condition: $i_b^{+} = i_b^{-}$

$$[I-B]i_b^+ = Di_a^+$$

$$i_b^+ = [I-B]^{-1} Di_a^+$$

$$i_a^- = \underbrace{\left[A + C[I-B]^{-1} D\right]}_{\equiv P} i_a^+$$

$$i_a^- = Pi_a^+$$

$$\{i\} = \{i_a^+\} - \{i_a^-\} = [I-P]\{i_a^+\}$$

The voltage probes can be thought of as scatterers

To find voltages in b terminals, we need to express i in terms of i_b

$$\{i_a^-\} = D^{-1}[I-B]\{i_b^+\}$$

$$\{i\} = [I-P]D^{-1}[I-B]\{i_b^+\}$$

$$\{i\} = [I-P]D^{-1}[I-B][M_B]\{\mu_b^+\}$$

$$Y = [I-P]D^{-1}[I-B][M_B]$$