ECE 659 Quantum Transport: Atom to Transistor

Lecture 12: Cyclotron Frequency

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$$\frac{d\vec{x}}{dt} = \frac{1}{\hbar}\nabla_k E$$

$$\hbar\frac{d\vec{k}}{dt} = \nabla E$$

$$\hbar\vec{k} = m\vec{v}$$

$$E = U(x) + \frac{(\hbar\vec{k} - q\vec{A})\cdot(\hbar\vec{k} - q\vec{A})}{2m}$$

$$\frac{d\vec{x}}{dt} = \frac{\hbar\vec{k} - q\vec{A}}{m} = \frac{\hbar\vec{k}'}{m}$$

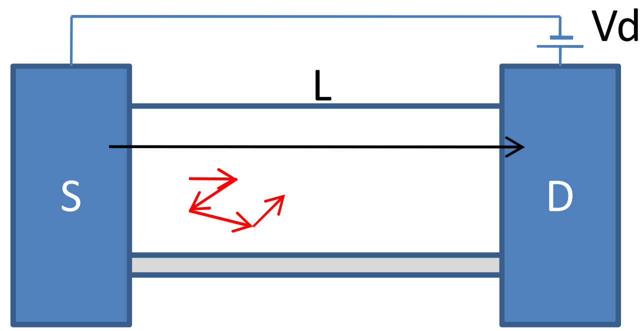
$$\hbar\frac{d\vec{k}'}{dt} = q\left(\vec{v}\times\vec{B}\right)$$

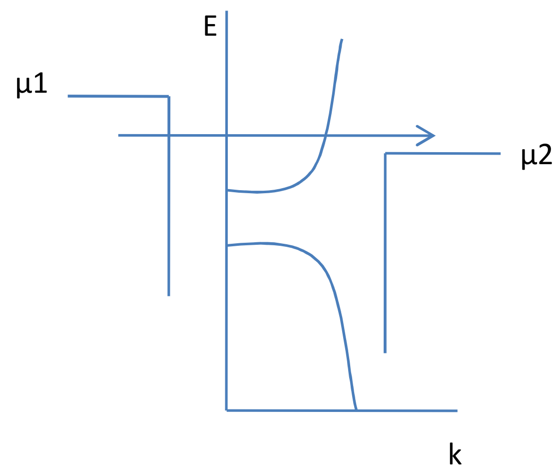
We keep U as constant and drop it for convenience

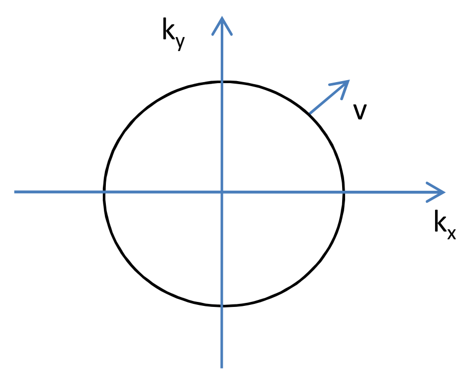
$$E = \sum_i \frac{(\hbar k_i - qA_i)^2}{2m} \qquad i = x, y, z$$

$$\frac{dx_i}{dt} = \frac{1}{\hbar}\frac{\partial E}{\partial k_i}$$

$$\hbar\frac{dk_i}{dt} = -\frac{\partial E}{\partial x_i}$$







We change i to j in the E expression to avoid confusion

$$\frac{dx_i}{dt} = \frac{1}{\hbar}\frac{\partial E}{\partial k_i} = \frac{\hbar k_i - qA_i}{m}$$

More formally

$$\frac{1}{\hbar}\frac{\partial E}{\partial k_i} = \frac{1}{\hbar}\sum_i \frac{\hbar k_i - qA_i}{m}\hbar\delta_{ij}$$
$$= \frac{\hbar k_i - qA_i}{m}$$

Vector notation then follows naturally

Some Tensor notations:

Kronecker delta

$$\delta_{ij} = \begin{cases} 1 & if\ i = j \\ 0 & if\ i \neq j \end{cases}$$

Levi-Civita tensor

$$\varepsilon_{ijk} = \begin{cases} +1 & if\ i \neq j \neq k, \text{in cyclic order} \\ -1 & if\ i \neq j \neq k, \text{in anti-cyclic order} \\ 0 & if\ i = j \text{ or } j = k \text{ or } i = k \text{ or } i = j = k \end{cases}$$

$$\hbar\frac{dk_i}{dt} = -\sum_j \underbrace{\frac{\hbar k_j - qA_j}{m}}_{v_j} \cdot -q\frac{\partial A_j}{\partial x_i}$$

$$\hbar\frac{dk_i}{dt} = q\sum_j v_j \frac{\partial A_j}{\partial x_i} \qquad -A$$

$$\frac{d}{dx}(qA_i) = q\sum_j \frac{\partial A_i}{\partial x_j}\frac{dx_j}{dt} \qquad -B$$

Subtracting B from A

$$\hbar\frac{dk_i}{dt} = q\sum_j v_j\left[\frac{\partial A_j}{\partial x_i} - \frac{\partial A_i}{\partial x_j}\right]$$
$$= q\sum_j v_j\left[\nabla_i A_j - \nabla_j A_i\right]$$

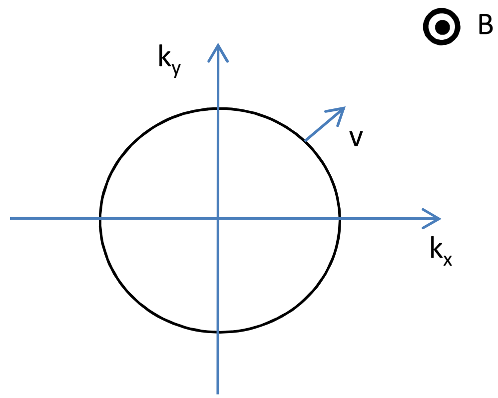
Now: $\left(\vec{A}\times\vec{B}\right)_k = \sum_{i,j}\varepsilon_{ijk}A_iB_j = A_xB_y - A_yB_x$

so $$\hbar\frac{dk_i'}{dt} = q\sum_{j,k} v_j\varepsilon_{ijk}\left(\nabla\times\vec{A}\right)_k$$
$$= q\left(\vec{v}\times\vec{B}\right)$$

$$\frac{d\vec{x}}{dt} = \frac{1}{\hbar}\nabla_k E = \frac{\hbar\vec{k} - q\vec{A}}{m} = \frac{\hbar\vec{k}'}{m}$$

$$\hbar\frac{d\vec{k}'}{dt} = q\left(\vec{\xi} + \vec{v}\times\vec{B}\right)$$

0



$$\frac{\hbar d\left(k'\right)}{v} = qBdt$$

Integrating over time T

$$\frac{\hbar 2\pi k}{\frac{\hbar k}{m}} = qBT$$

$$T = \frac{2\pi}{\frac{qB}{m}} \rightarrow \omega_c$$

More generally:

$$\frac{\hbar k d\theta}{\frac{1}{\hbar}\frac{dE}{dk}} = qBdt$$

Integrate over T

$$\Longrightarrow \hbar^2\frac{d}{dE}\left(A(k)\right) = qBT$$

Area in k-space

$$\omega_c\tau = \frac{qB\tau}{m}$$

$$= \mu B$$