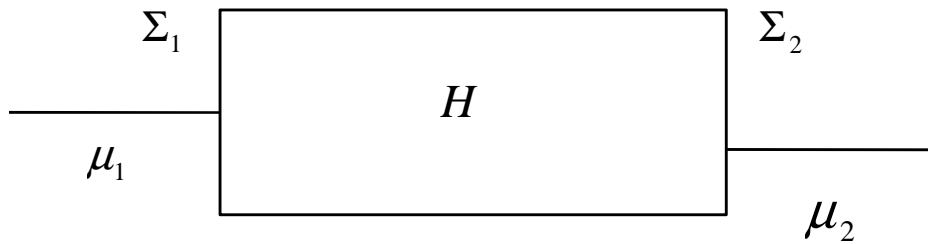


ECE 659 Quantum Transport: Atom to Transistor

Lecture 17: Non-Coherent Transport

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$$\Gamma = i[\Sigma - \Sigma^+]$$

$$G = [EI - H - \Sigma]^{-1}$$

$$G^n = G \underbrace{\Sigma^{in}}_{\Gamma_1 f_1 + \Gamma_2 f_2} G^+$$

$$A = i[G - G^+] = G\Gamma G^+ = G^+\Gamma G$$

$$G^n = 2\pi\psi\psi^+$$

$$\Sigma^{in} = 2\pi s s^+$$

$$E\psi = \underbrace{H\psi + \Sigma\psi + s}_{\Phi}$$

$$I_{op} = \frac{\Phi\psi^+ - \psi\Phi^+}{i\hbar}$$

$$i\hbar \frac{d\tilde{\psi}}{dt} = \tilde{\Phi}$$

current $\longrightarrow \frac{d}{dt}(\tilde{\psi}\tilde{\psi}^+)$

To get total current we need to take the Trace of I_{op}

$$Trace[I_{op}] = \frac{1}{2\pi} Trace \left[\frac{H \overbrace{\psi\psi^+}^{G^n} - \overbrace{\psi\psi^+}^{G^n} H}{i\hbar} \right]$$

$$= \frac{1}{i\hbar} Trace [HG^n - G^n H]$$

$$= 0$$

$$\Phi = \Sigma \psi$$

$$I_{op} = \frac{\Sigma \psi \psi^+ - \psi \psi^+ \Sigma^+}{2i\pi\hbar}$$

$$\begin{aligned} \text{Trace}[I_{op}] &= \frac{1}{ih} \text{Trace}[\Sigma G^n - G^n \Sigma^+] \\ &= \frac{1}{ih} \text{Trace}[[\Sigma - \Sigma^+] G^n] \end{aligned}$$

$$\text{Trace}[I_{op}] = \frac{-\text{Trace}[\Gamma G^n]}{h}$$

$$\Phi = s$$

$$I_{op} = \frac{s\psi^+ - \psi s^+}{ih}$$

$$\psi = Gs$$

$$I_{op} = \frac{\overbrace{ss^+}^{2\pi} G^+ - Gss^+}{ih}$$

$$\begin{aligned} \text{Trace}[I_{op}] &= \text{Trace} \frac{\Sigma^{in} G^+ - G \Sigma^{in}}{ih} \\ &= \frac{\text{Trace}[\Sigma^{in} (G^+ - G)]}{ih} \\ &= \frac{\text{Trace}[\Sigma^{in} A]}{h} \end{aligned}$$

Total current:

$$\text{Trace}[I_{op}] = \frac{\text{Trace}[\Sigma^{in} A - \Gamma G^n]}{h}$$

$$\Gamma = \Gamma_1 + \Gamma_2$$

$$\Sigma^{in} = \Gamma_1 f_1 + \Gamma_2 f_2$$

For current at each terminal we can use the subscripted expression:

$$I_m = \frac{q}{h} \text{Trace}[\Sigma_m^{in} A - \Gamma_m G^n]$$

$$I_m = \frac{q}{h} \text{Trace}[\Gamma_m (A f_m - G^n)]$$

Scatterer

