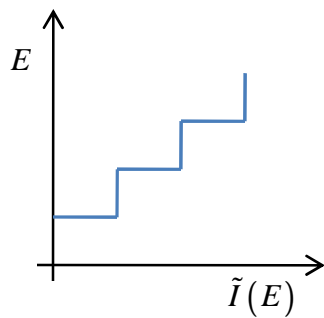
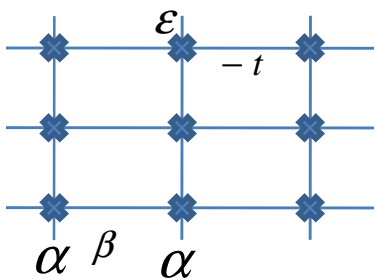
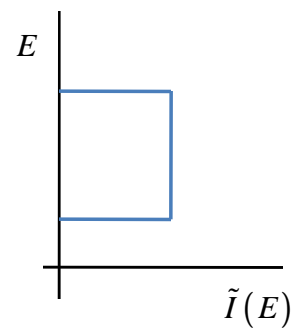
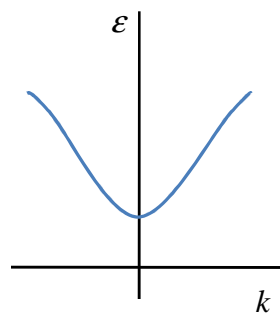
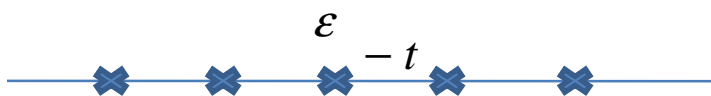
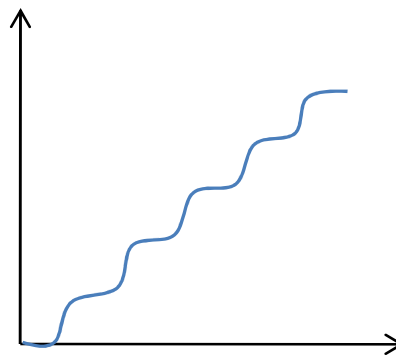
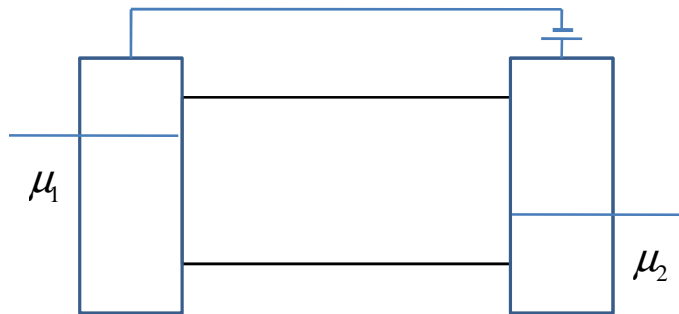


ECE 659 Quantum Transport: Atom to Transistor

Lecture 23: Transverse Modes

Supriyo Datta

Spring 2009



$$\alpha = \begin{bmatrix} 4t & -t & & \\ -t & 4t & \ddots & \\ & \ddots & \ddots & \\ & & & \ddots \end{bmatrix} \quad \beta = \begin{bmatrix} -t & & & \\ & -t & & \\ & & \ddots & \\ & & & \ddots \end{bmatrix}$$

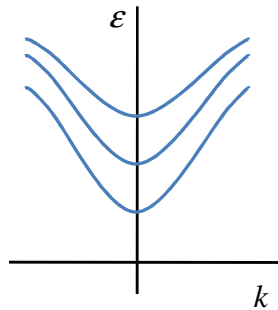
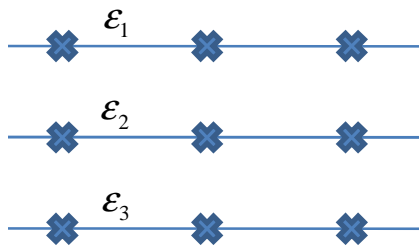
$$H = \begin{bmatrix} \alpha & \beta & & & \\ \beta^* & \alpha & \beta & & \\ & \beta^* & \alpha & \ddots & \\ & & & \ddots & \ddots \end{bmatrix}$$

$$[V, D] = \text{eig}(\alpha)$$

$$V^+ = V^{-1}$$

$$V^+ \alpha V = D$$

$$\alpha = V D V^+$$



$$E_\nu(k) = \alpha_\nu - 2t \cos ka$$

$$\alpha = \begin{bmatrix} \ddots & \ddots & & -t \\ -t & 4t & -t & \\ & -t & 4t & -t \\ -t & & \ddots & \ddots \end{bmatrix}$$

$$\alpha_\nu = 4t - 2t \cos \underbrace{ka}_{\frac{2\pi\nu}{N}}$$



$$E\psi_n = \varepsilon\psi_n - t\psi_{n-1} - t\psi_{n+1}$$

$$\psi_n = \psi_0 e^{ikna}$$

$$= \psi_0 e^{ik(n+N)a}$$

$$kNa = 2\pi\nu$$

For box boundary condition:

$$\psi_n = \psi_0 \sin kna$$

$$(E - \varepsilon) \sin kna = -t \underbrace{\left[\sin k(n-1)a + \sin k(n+1)a \right]}_{2 \sin kna \cos ka}$$

$$k_v(N+1)a = \pi v$$

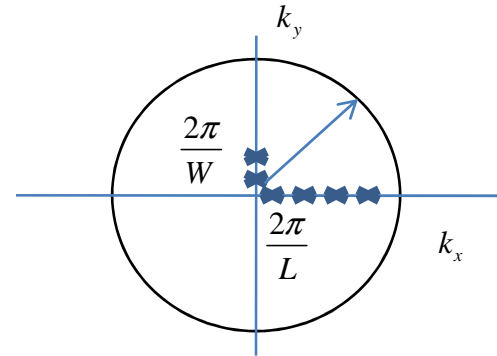
For periodic boundary:

$$k_v a = \frac{2\pi v}{N}$$

For box boundary:

$$k_v a = \frac{\pi v}{N+1}$$

$$\begin{aligned} E_v(0) &= \alpha_v - 2t \\ &= 4t - 2t \cos \underbrace{k_v a}_{\substack{2\pi v / N \\ \pi v / (N+1)}} - 2t \\ &= 2t(1 - \cos k_v a) \\ &\approx ta^2 k_v^2 \end{aligned}$$



$$N = \frac{\pi k_f^2}{2\pi} \frac{2\pi}{L} = \frac{k_f^2}{2\pi} LW$$

$$n_s = \frac{k_f^2}{2\pi} \quad 10^{12} / \text{cm}^2$$

$$\frac{2\pi}{\lambda_f}$$

$$k_f \omega = \pi v$$