

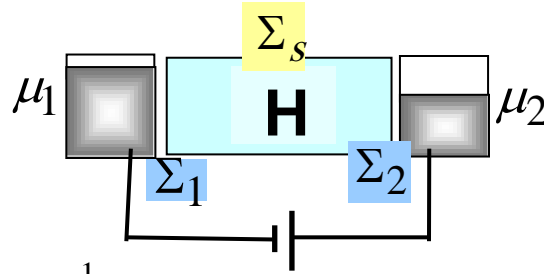
ECE 659 Spring '09 Weeks 9-11: Spin Transport
HW#6: Due Wednesday April 8, 2009 in class

NEGF equations for elastic transport

($D=0$ yields coherent transport)

$$\Gamma_1 = i[\Sigma_1 - \Sigma_1^+], \Gamma_2 = i[\Sigma_2 - \Sigma_2^+],$$

$$\Gamma_s = i[\Sigma_s - \Sigma_s^+]$$



$$1. G(E) = [EI - H - \Sigma_1 - \Sigma_2 - \Sigma_s]^{-1},$$

$$\text{where } [\Sigma_s] = D[G]$$

$$2. A(E) = i[G - G^+]$$

$$3. [G^n(E)] = [G\Gamma_1 G^+] f_1 + [G\Gamma_2 G^+] f_2 + [G\Sigma_s^{in} G^+]$$

$$\text{where } [\Sigma_s^{in}] = D[G^n]$$

$$4. i\hbar I_{op} = [HG^n - G^n H] + [\Sigma G^n - G^n \Sigma^+] + [\Sigma^{in} G^+ - G \Sigma^{in}]$$

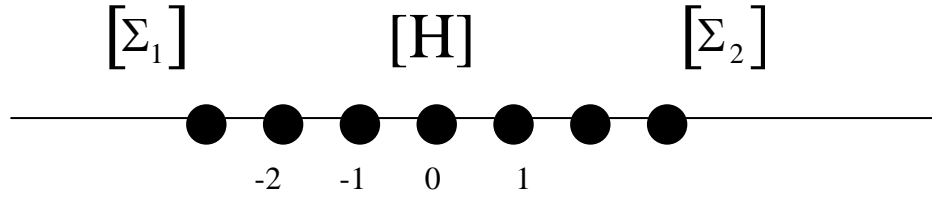
$$4a. I_{a \rightarrow b}(E) = \frac{q}{h} i[H_{ab} G_{ba}^n - G_{ab}^n H_{ba}] \quad \mathbf{a, b: Internal Points}$$

$$4b. I_i(E) = \frac{q}{h} (\text{Trace}[\Sigma_i^{in} A - \Gamma_i G^n]) \quad \mathbf{Current/energy at terminal 'i'}$$

$$4c. I_i(E) = \frac{q}{h} \sum_j \text{Trace}[\Gamma_i G \Gamma_j G^+] (f_i(E) - f_j(E)) \quad (\text{used only if } D \text{ is zero})$$

$$\text{Pauli spin matrices: } \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma_y = \begin{bmatrix} 0 & -i \\ +i & 0 \end{bmatrix}, \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Problem 1: In HW3 we analyzed a 1-D wire



modeled with a Hamiltonian having $H_{n,n} = +2t_0$, $H_{n,n+1} = -t_0 = H_{n,n-1}$ (all other elements are zero), such that the dispersion relation is given by $E = 2t_0(1 - \cos ka)$. Σ_1 is a matrix with all zeroes except for $\Sigma_1(1,1) = \Sigma$, while Σ_2 has all zeroes except for $\Sigma_2(Np, Np) = \Sigma$, where $\Sigma(E) = -t_0 e^{+ika}$. Assume $D = 0$.

We consider now the same wire but having an additional spin-orbit interaction term in the Hamiltonian for the channel: $H_{so} = \eta k_x [\sigma_y]$. The contacts do not have any spin-orbit interaction and we will assume that we have separate contacts for up and down-spin channels described by $\Sigma_{1,u}, \Sigma_{1,d}, \Sigma_{2,u}, \Sigma_{2,d}$. This will allow us to calculate the transmission coefficients T_{uu} and T_{ud} , from up to up and up to down respectively.

(a) Plot numerically the transmission coefficients T_{uu} and T_{ud} for each of three cases, namely when the up and down spins point along (i) $\pm z$, (ii) $\pm x$ and (iii) $\pm y$, as a function of the Rashba coefficient η for a device of length 100 nm. Use $E = 0.2$ eV, $a = 1$ nm, $m = 0.03$ * free electron mass, $t_0 = \hbar^2 / 2ma^2$ and $-1e-10 < \eta < 1e-10$ eV-m. **Please remember to turn in a copy of your MATLAB codes.**

(b) In part (a) you should find that the transmission oscillates as a function of η for cases (i) and (ii), but not for case (iii). Explain why. Obtain an expression for the period of the oscillation and compare with your numerical result. Hint: The Rashba interaction gives rise to an effective magnetic field that causes spins to precess.

Problem 2: (a) Prove that $[\vec{\sigma} \cdot \vec{C}][\vec{\sigma} \cdot \vec{D}]\{\psi\} = \vec{C} \cdot \vec{D}\{\psi\} + i\vec{\sigma} \cdot [\vec{C} \times \vec{D}]\{\psi\}$

where \vec{C} and \vec{D} are any two operators and $\{\psi\}$ is any two-component wavefunction.

(b) Use the result in part (a) to prove that $(\vec{B} \equiv \vec{\nabla} \times \vec{A})$

$$[\vec{\sigma} \cdot (\vec{p} + q\vec{A})][\vec{\sigma} \cdot (\vec{p} + q\vec{A})]\{\psi\} = (\vec{p} + q\vec{A}) \cdot (\vec{p} + q\vec{A})\{\psi\} + q\hbar \vec{\sigma} \cdot \vec{B}\{\psi\}$$