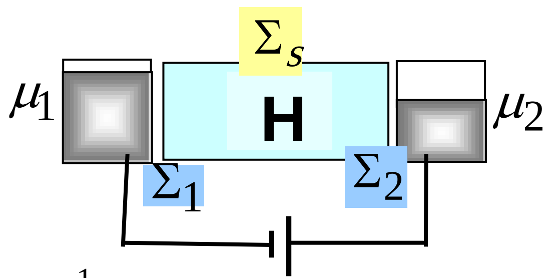
ECE 659 Spring '09 Weeks 9-11: Spin Transport

HW#6: Due Wednesday April 8, 2009 in class

NEGF equations for elastic transport

(D=0 yields coherent transport)



$\Gamma_1 = i[\Sigma_1 - \Sigma_1^+], \Gamma_2 = i[\Sigma_2 - \Sigma_2^+],$

$\Gamma_s = i[\Sigma_s - \Sigma_s^+]$

1. $G(E) = [EI - H - \Sigma_1 - \Sigma_2 - \Sigma_s]^{-1},$

   $where \quad [\Sigma_s] = D[G]$

2. $A(E) = i[G - G^+]$

3. $[G^n(E)] = \quad [G\Gamma_1 G^+]\ f_1 + [G\Gamma_2 G^+]\ f_2 + [G\Sigma_s^{in} G^+]$

   $where \quad \left[\Sigma_s^{in}\right] = D\left[G^n\right]$

4. $i\hbar I_{op} = [HG^n - G^n H] + [\Sigma G^n - G^n \Sigma^+] + [\Sigma^{in} G^+ - G\Sigma^{in}]$

4a. $I_{a\to b}(E) = \dfrac{q}{h}\, i[H_{ab} G^n_{ba} - G^n_{ab} H_{ba}]$ *a, b: Internal Points*

4b. $I_i(E) = \dfrac{q}{h}(Trace[\Sigma_i^{in} A - \Gamma_i G^n])$ *Current/energy at terminal 'i'*

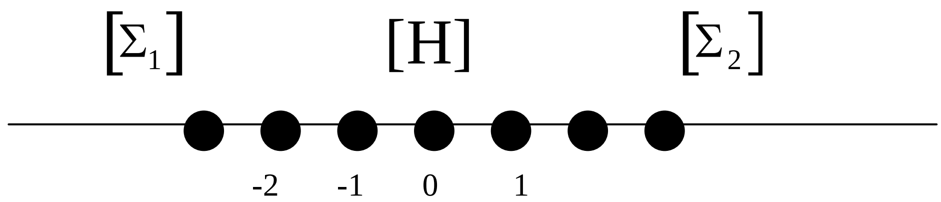
4c. $I_i(E) = \dfrac{q}{h} \sum_j Trace[\Gamma_i G \Gamma_j G^+](f_i(E) - f_j(E))$ ***(**used only if D is zero**)***

****

Pauli spin matrices: $\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},\ \sigma_y = \begin{bmatrix} 0 & -i \\ +i & 0 \end{bmatrix},\ \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

****

Problem 1: In HW3 we analyzed a 1-D wire



modeled with a Hamiltonian having $H_{n,n} = +2t_0$, $H_{n,n+1} = -t_0 = H_{n,n-1}$ (all other elements are zero), such that the dispersion relation is given by $E = 2t_0(1-\cos ka)$. Σ_1 is a matrix with all zeroes except for $\Sigma_1(1,1) = \Sigma$, while Σ_2 has all zeroes except for $\Sigma_2(Np,Np) = \Sigma$, where $\Sigma(E) = -t_0\, e^{+ika}$. Assume D = 0.

We consider now the same wire but having a additional spin-orbit interaction term in the Hamiltonian for the channel : $H_{so} = \eta\, k_x[\sigma_y]$. The contacts do not have any spin-orbit interaction and we will assume that we have separate contacts for up and down-spin channels described by $\Sigma_{1,u}, \Sigma_{1,d}, \Sigma_{2,u}, \Sigma_{2,d}$. This will allow us to calculate the transmission coefficients T_{uu} and T_{ud}, from up to up and up to down respectively.

(a) Plot numerically the transmission coefficients T_{uu} and T_{ud} for each of three cases, namely when the up and down spins point along (i) ± z, (ii) ± x and (iii) ± y, as a function of the Rashba coefficient η for a device of length 100 nm. Use E = 0.2 eV, a = 1 nm, m = 0.03 * free electron mass, $t_0 = \hbar^2/2ma^2$ and $-1e-10 < \eta < 1e-10\ eV-m$. ***Please remember to turn in a copy of your MATLAB codes.***

(b) In part (a) you should find that the transmission oscillates as a function of η for cases (i) and (ii), but not for case (iii). Explain why. Obtain an expression for the period of the oscillation and compare with your numerical result. Hint: The Rashba interaction gives rise to an effective magnetic field that causes spins to precess.

Problem 2: (a) Prove that $[\vec{\sigma}.\vec{C}][\vec{\sigma}.\vec{D}]\{\psi\} = \vec{C}.\vec{D}\{\psi\} + i\vec{\sigma}.[\vec{C}x\vec{D}]\{\psi\}$

where $\vec{C}$ and $\vec{D}$ are any two operators and $\{\psi\}$ is any two-component wavefunction.

(b) Use the result in part (a) to prove that ($\vec{B} \equiv \vec{\nabla}x\vec{A}$)

$$[\vec{\sigma}.(\vec{p}+q\vec{A})][\vec{\sigma}.(\vec{p}+q\vec{A})]\{\psi\} = (\vec{p}+q\vec{A}).(\vec{p}+q\vec{A})\{\psi\} + q\hbar\vec{\sigma}.\vec{B}\{\psi\}$$