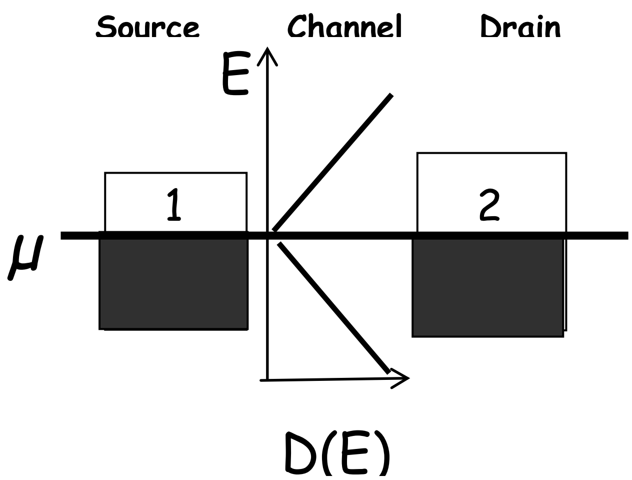
9/10/08

ECE 495N, Fall'08 ME118, MWF 1130A – 1220P

HW#3: Due Friday Sept.26 in class.

Problem 1: A sheet of graphene (Width: W, Length: L) has a density of states given by $D(E) = LW\,|E|/(\hbar v_f)^2$ where $v_f \approx 10^6\, m/s$. The equilibrium electrochemical potential μ = 0 eV, $k_B T_1 = k_B T_2 = 0.025\, eV$ and assume escape rates $\gamma_1 = \gamma_2 = (\hbar v_f / L)\dfrac{\lambda}{L+\lambda} \approx \hbar v_f / L$, for ballistic transport L << λ.



(a) Plot $(N\text{-}N_0)/LW$ versus U for -0.5 eV < U < +0.5 eV (Note: $N_0 = N(U=0)$)

using the equation
$$N = \int_{-\infty}^{+\infty} dE\, D(E-U)\, \frac{\gamma_1 f_1(E) + \gamma_2 f_2(E)}{\gamma_1 + \gamma_2} \qquad (1)$$

assuming $f_1 \approx f_2$ (close to equilibrium),

and (b) G versus U, using the equation
$$G = \frac{q^2}{\hbar} \int_{-\infty}^{+\infty} dE D(E-U) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \left(\frac{-\partial f_1}{\partial E} \right) \qquad (2)$$

Compare your plots with the expressions

(a) $(N - N_0)/LW = -0.5(U/\hbar v_f)^2, U > 0$

$(N - N_0)/LW = 0.5(U/\hbar v_f)^2, U < 0$ (3)

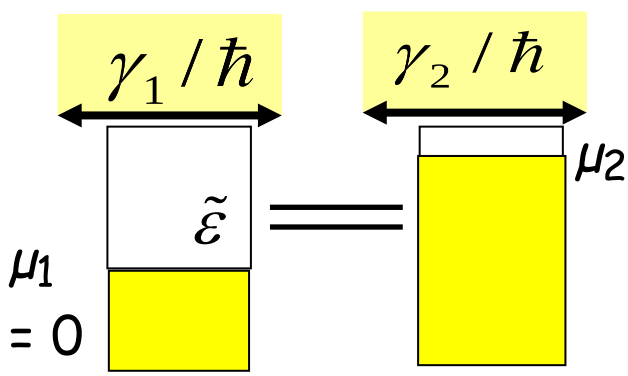
and (b) $G/W = \dfrac{q^2}{2\hbar} |U| / \hbar v_f$ (4)

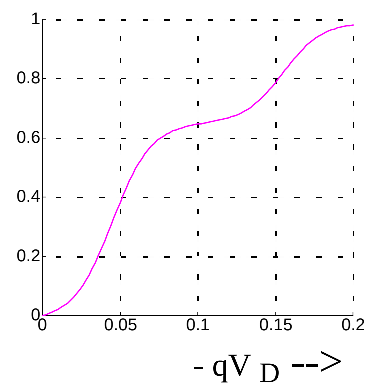
Show how you could obtain the approximate expressions (3) and (4) from Equations (1) and (2).

For Problems 2,3 you can use the MATLAB code at the end of the text as a guide, but the code you turn in should be your own work, not copied from the text.

Problem 2: Exercise E.2.1, Page 49 from *S.Datta, Quantum Transport: Atom to Transistor, Cambridge (2005) ISBN 0-521-63145-9.*

Problem 3: A box has two degenerate energy levels having energy $\tilde{\varepsilon}$ = 0.05 eV, before including any self-consistent field due to electron-electron interactions. Write a MATLAB code to calculate the current for a negative voltage on the drain (contact 2), assuming that the the energy level remains fixed with respect to the source and neglecting any broadening of the energy levels. Assuming $\gamma_1 = \gamma_2 = 0.005\, eV$, U_0 = 0.1 eV and $k_B T = 0.005\, eV$ you should obtain a plot like this for the current normalized to $I(maximum) = \frac{q}{\hbar}\frac{2\gamma_1\gamma_2}{\gamma_1+\gamma_2}$.





Show that the current value at the plateau for intermediate voltages (around 0.1V) is given by $I(plateau) = \frac{q}{\hbar}\frac{2\gamma_1\gamma_2}{\gamma_1+2\gamma_2}$ (Hint: at this voltage electrons are not available in either contact to take the channel to the 11 state; so it resides in the 00, 01 and 10 states).

Problem 4: A channel has two energy levels ε_1 and ε_2 corresponding to four levels 00, 01, 10 and 11 in the multi-electron picture. Apply the law of equilibrium in the multi-electron picture to obtain the equilibrium occupation probabilities assuming zero interaction energy (U_0 = 0) for the four levels and show that

$$P_{00} = (1 - f_1)(1 - f_2) \quad , \; P_{01} = (1 - f_1)\, f_2, \quad P_{10} = f_1(1 - f_2) \text{ and } P_{11} = f_1\, f_2$$

where f_1 and f_2 are the equilibrium Fermi functions corresponding to the two energy levels.

Problem 5: A channel has four degenerate energy levels all having the same energy $\varepsilon = 0$ eV with an interaction energy that can be written as $U_{ee} = U_0 N(N-1)/2$, where $U_0 = 0.1$ eV. The figure below shows the change in the ***equilibrium*** number of electrons, N inside the channel as the electrochemical potential μ is changed. What are the values of μ at which the transitions in N take place (labeled μ1, μ2, μ3 and μ4 in the figure) ?