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ECE 495N, Fall'08 ME118, MWF 1130A – 1220P

HW#4: Due Wednesday Oct.15 in class.

Note: Problems 4 and 5 are carried over from HW#3

Problem 1: Consider the (2x2) matrix $A = \begin{bmatrix} \cos\theta & \sin\theta\, e^{-i\varphi} \\ \sin\theta\, e^{+i\varphi} & -\cos\theta \end{bmatrix}$

Show that the following

$$V_1 \equiv \begin{Bmatrix} \cos(\theta/2)\, e^{-i\varphi/2} \\ \sin(\theta/2)\, e^{+i\varphi/2} \end{Bmatrix} \text{ and } \qquad V_2 \equiv \begin{Bmatrix} -\sin(\theta/2)\, e^{-i\varphi/2} \\ \cos(\theta/2)\, e^{+i\varphi/2} \end{Bmatrix}$$

are eigenvectors of [A]. What are the corresponding eigenvalues?

Are they orthogonal (that is, is $V_1^+ V_2 = 0$) ?

Note: the superscript '+' denotes Hermitian conjugate.

Problem 2: Define a (2x2) matrix $[V] \equiv [\{V_1\}\ \{V_2\}]$

Now calculate the matrix [B] = $[V^+]$ [A] [V].

(A and $[V] \equiv [\{V_1\}\ \{V_2\}]$ are given in Problem 1).

Assuming $\theta = \dfrac{\pi}{2}, \varphi = 0$, use MATLAB to find [V] and check that [B] = $[V^+]$ [A] [V].

(HINT: use "[V, B]=eig(A)" command)

Problem 3: Using the 2s and the three 2p levels as basis functions write down the Hamiltonian matrix for a hydrogen atom in an electric field F directed along the x-axis. Find the eigenvalues and eigenvectors. You may find Section 4.4.2 (page 99) of the text useful.

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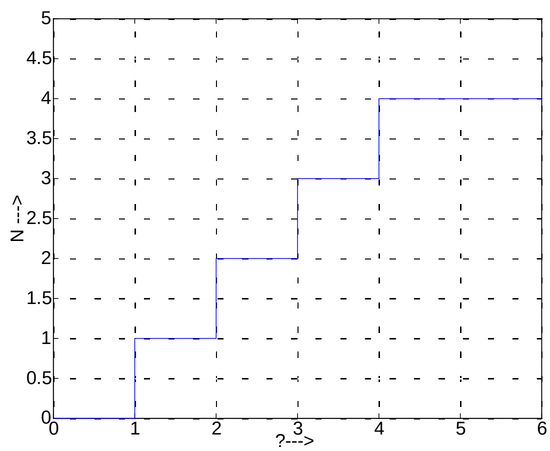
Problem 4: A channel has two energy levels ε_1 and ε_2 corresponding to four levels 00, 01, 10 and 11 in the multi-electron picture. Apply the law of equilibrium in the multi-electron picture to obtain the equilibrium occupation probabilities assuming zero interaction energy (U_0 = 0) for the four levels and show that

$$P_{00} = (1 - f_1)(1 - f_2) \quad , \; P_{01} = (1 - f_1)\, f_2, \quad P_{10} = f_1(1 - f_2) \text{ and } P_{11} = f_1 f_2$$

where f_1 and f_2 are the equilibrium Fermi functions corresponding to the two energy levels.

Problem 5: A channel has four degenerate energy levels all having the same energy $\varepsilon = 0$ eV with an interaction energy that can be written as $U_{ee} = U_0 N(N-1)/2$, where $U_0 = 0.1$ eV. The figure below shows the change in the ***equilibrium*** number of electrons, N inside the channel as the electrochemical potential μ is changed. What are the values of μ at which the transitions in N take place (labeled μ1, μ2, μ3 and μ4 in the figure) ?



μ1 μ2 μ3 μ4

μ --->