10/22/08

ECE 495N, Fall'08   ME118, MWF 1130A – 1220P

HW#6: Due Wednesday Oct.29 in class.

This homework requires the use of the following formulas for the density of states and the mode density (S, area; L, length; W, width):

$$D(E) = \sum_{\vec{k}} \delta(E - \varepsilon(\vec{k}))$$

$$M(E) = \sum_{\vec{k}} \delta(E - \varepsilon(\vec{k})) \frac{\pi\hbar \left| v_x(\vec{k}) \right|}{L} \qquad \left(v_x(\vec{k}) = \frac{1}{\hbar} \frac{\partial \varepsilon}{\partial k_x} \right)$$

Assume that electrons are confined to a two-dimensional layer having an $\varepsilon(\vec{k})$ relation of the form

$$\varepsilon(\vec{k}) = \hbar^2 (k_x{}^2 + k_y{}^2) / 2m$$

1. Obtain an expression for the (a) density of states D(E) and (b) the mode density, M(E) in terms of the energy E, the area S and constants like m and $\hbar$. Assume that both L and W large enough that the summations over k_x and k_y can both be replaced with appropriate integrals.

2. How would you write the energies of the subbands if the electrons are confined to a narrow channel of width W in the y-direction?

3. Obtain an expression for the density of states D(E) and the mode density M(E), assuming that L is large enough that the summation over k_x can be replaced with an appropriate integral, but W is NOT large enough to do the same for k_y.