ECE 495N, Fall'08 ME118, MWF 1130A – 1220P

HW#7: Due Friday Nov.21 in class.

Problem 1: Below is some recent experimental data on a sheet of graphene reported by one of the leading groups in the field (http://arxiv.org/abs/0805.1830). It plots the "conductivity" σ obtained from the conductance G, using the relation $G = \sigma W / L$, W and L being the width and length respectively.

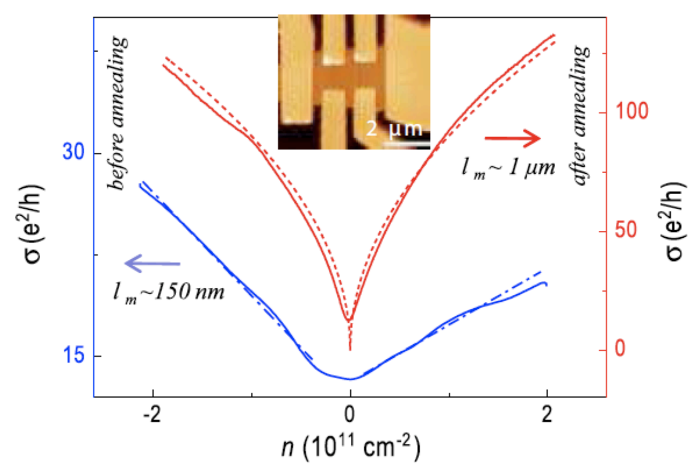


FIG. 1: (color online) Conductance of the suspended graphene sample S1 before (blue line) and after (red line) annealing as a function of carrier density. Data are shown for $T = 40$ K to suppress universal conductance fluctuations. Note the change from near-linear to sub-linear behavior before and after annealing, respectively. The dotted red line is the expectation for ballistic transport (see text). Inset: atomic force microscope image of the suspended device (S1).

We wish to try to understand this data based on the expressions developed in this course (you may want to review Problem 1 of HW#3):

$$N - N_0 = \int_{-\infty}^{+\infty} dE [D(E-U) - D(E)] \, f_{eq}(E) \qquad (1)$$

$$G = \frac{q^2}{h} \int_{-\infty}^{+\infty} dE M(E-U) \frac{\lambda}{\lambda + L} \left(\frac{-\partial f_{eq}}{\partial E} \right) \qquad (2)$$

where λ is the mean free path and $f_{eq}(E) = \dfrac{1}{1 + \exp(E/kT)}$.

A. Starting from the approximate dispersion relation $\varepsilon(\vec{k}) \approx \hbar v k$, use the relations

$$D(E) = \sum_{\vec{k}} \delta(E - \varepsilon(\vec{k}))$$

$$M(E) = \sum_{\vec{k}} \delta(E - \varepsilon(\vec{k})) \frac{\pi\hbar |v_x(\vec{k})|}{L}$$

to show that $D(E) = \frac{2LW}{\pi} \frac{|E|}{(\hbar v)^2}$ *and* $M(E) = \frac{4W}{\pi} \frac{|E|}{\hbar v}$

including a factor of 2 for the two spins and another factor of 2 for the two valleys.

B. Now use Eqs.(1) and (2) and assume very low temperatures so that $-\frac{\partial f_{eq}(E)}{\partial E} \approx \delta(E)$ to show that

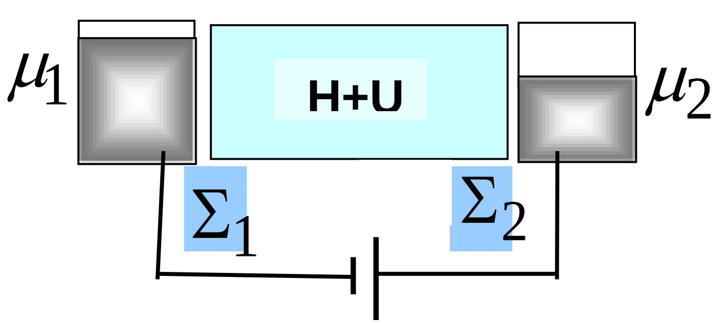
$$n_s \equiv (N - N_0) / LW = \frac{1}{\pi} \left| \frac{U}{\hbar v} \right|^2$$

$$\sigma \equiv \frac{GL}{W} = \frac{q^2}{h} \frac{4}{\pi} \left| \frac{U}{\hbar v} \right| \frac{\lambda L}{\lambda + L}$$

and $$\sigma = \frac{q^2}{h} \frac{4}{\pi} \frac{\lambda L}{\lambda + L} \sqrt{\pi n_s}$$

Plot σ versus n_s, assuming L = 4 µm using (a) λ= 2 µm and with (b) λ= 300 nm. Note that these values of the mean free path are twice those indicated in the paper: we believe the difference arises from a difference in our definition of mean free path.

Problem 2:
Coherent transport



Basic equations of coherent transport

$$\Gamma_1 = i[\Sigma_1 - \Sigma_1^+], \Gamma_2 = i[\Sigma_2 - \Sigma_2^+]$$

$$G(E) = [EI - H - \Sigma_1 - \Sigma_2]^{-1},$$

$$A(E) = i[G - G^+] = G\Gamma_1 G^+ + G\Gamma_2 G^+ \qquad \textit{Density of states}$$

$$[G^n(E)] = [G\Gamma_1 G^+] f_1 + [G\Gamma_2 G^+] f_2 \qquad \textit{Electron density}$$

$$I_i(E) = \frac{q}{h}((Trace[\Gamma_i A]) f_i - Trace[\Gamma_i G^n]) \qquad \textit{Current/energy}$$

$$I(E) = \frac{q}{h} Trace[\Gamma_1 G \Gamma_2 G^+](f_1(E) - f_2(E)) \qquad \textit{2-terminal current}$$

In general H has to be replaced with H+U, where U has to be calculated self-consistently from an appropriate "Poisson"-like equation, but you can ignore this aspect in the following problem.

Consider a channel described by a (2x2) Hamiltonian matrix: $H = \begin{bmatrix} \varepsilon_1 & t \\ t & \varepsilon_2 \end{bmatrix}$ whose connection to two contacts are described by $\Sigma_1 = -\frac{i}{2}\begin{bmatrix} \gamma_1 & 0 \\ 0 & 0 \end{bmatrix}$ and $\Sigma_2 = -\frac{i}{2}\begin{bmatrix} 0 & 0 \\ 0 & \gamma_2 \end{bmatrix}$ respectively. Assume ε_1 = +0.5 eV, ε_2 = -0.5 eV, t = 1 eV, γ_1 = γ_2 = 0.5 eV, $f_1(E)$=1, $f_2(E)$=0.

(a) Plot the current/energy over the energy range -2 eV < E < +2 eV. The plot should have two peaks: Compare the location of these peaks with the eigenvalues of [H].

(b) Plot the local density of states A(1,1) and A(2,2) over the same energy range. It should have two peaks too, but of unequal height. Compare the ratio of the peaks for A(1,1) and A(2,2) with $|u_1/u_2|^2$ and $|v_1/v_2|^2$ where $\begin{pmatrix} u_1 \\ v_1 \end{pmatrix}$ and $\begin{pmatrix} u_2 \\ v_2 \end{pmatrix}$ are the normalized eigenvectors