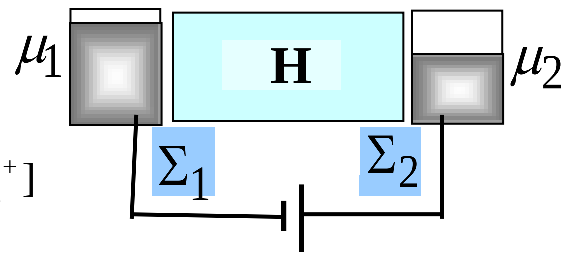
ECE 495N, Fall'08 ME118, MWF 1130A – 1220P

HW#9: Due Wednesday Dec.10 in class.

Basic equations of coherent transport



$$\Gamma_1 = i[\Sigma_1 - \Sigma_1^+], \Gamma_2 = i[\Sigma_2 - \Sigma_2^+]$$

$$G(E) = [EI - H - \Sigma_1 - \Sigma_2]^{-1},$$

$$A(E) = i[G - G^+] = G\Gamma_1 G^+ + G\Gamma_2 G^+$$ *Density of states** 2π

$$[G^n(E)] = [G\Gamma_1 G^+] f_1 + [G\Gamma_2 G^+] f_2$$ *Electron density** 2π

$$I_i(E) = \frac{q}{h}((Trace[\Gamma_i A]) f_i - Trace[\Gamma_i G^n])$$ *Current/energy*

$$I(E) = \frac{q}{h} Trace[\Gamma_1 G \Gamma_2 G^+](f_1(E) - f_2(E))$$ *2-terminal current*

Simpler version introduced earlier:

$$n(E) = D(E)\frac{\gamma_1 f_1(E) + \gamma_2 f_2(E)}{\gamma_1 + \gamma_2}$$ *Electron density*

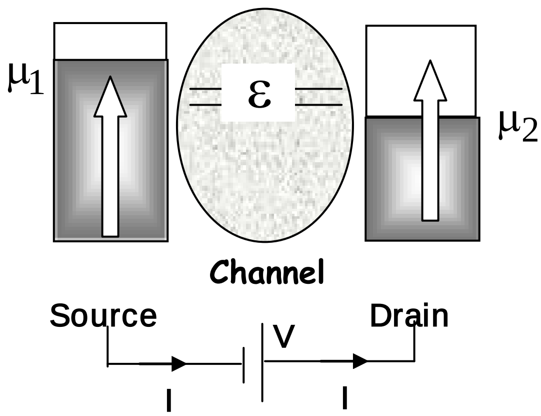
$$I_i(E) = \frac{q}{\hbar}\gamma_i\left(D(E) f_i(E) - n(E)\right)$$ *Current/energy*

$$I(E) = \frac{q}{\hbar}\frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2} D(E)(f_1(E) - f_2(E))$$ *2-terminal current*

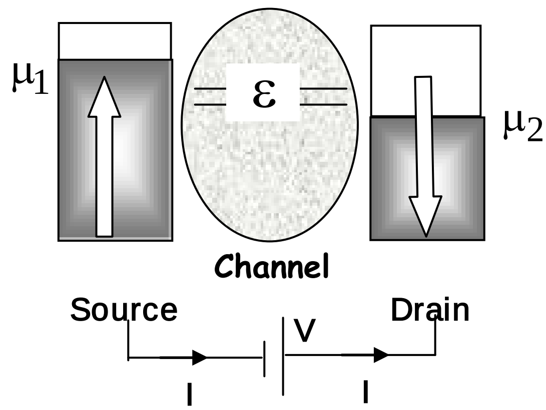
In general H has to be replaced with H+U, and D(E) with D(E-U) where U has to be calculated self-consistently from an appropriate "Poisson"-like equation, but you can ignore this aspect in the following problems.

Problem 1 (Spin valve): A spin valve device consists of an ordinary channel with magnetic contacts that couple differently to the two spin levels which are assumed to have the same energy ε so that the Hamiltonian is given by $[H]=\begin{bmatrix}\varepsilon & 0\\ 0 & \varepsilon\end{bmatrix}$. The spin valve device has very different conductances in the parallel and anti-parallel configurations and this difference is exploited to "read" information stored in magnetic disks. Our objective is to model this change in the conductance.

Parallel Configuration



Anti-Parallel Configuration



In the parallel configuration (both contacts magnetized in the ***same*** direction) we can write

$$[\Sigma_1]=-\frac{i}{2}\begin{bmatrix}\alpha & 0\\ 0 & \beta\end{bmatrix},[\Sigma_2]=-\frac{i}{2}\begin{bmatrix}\alpha & 0\\ 0 & \beta\end{bmatrix}$$

while in the anti-parallel configuration (contacts magnetized in ***opposite*** directions) we can write

$$[\Sigma_1]=-\frac{i}{2}\begin{bmatrix}\alpha & 0\\ 0 & \beta\end{bmatrix},[\Sigma_2]=-\frac{i}{2}\begin{bmatrix}\beta & 0\\ 0 & \alpha\end{bmatrix}$$

where α and β are real numbers with say, $\alpha > \beta$.

(a) Calculate the transmission function $\overline{T}_P$ (E) for the parallel configuration.
(b) Calculate the transmission function $\overline{T}_A$ (E) for the anti-parallel configuration.

(c) Obtain an expression for the magnetoresistance ratio defined as

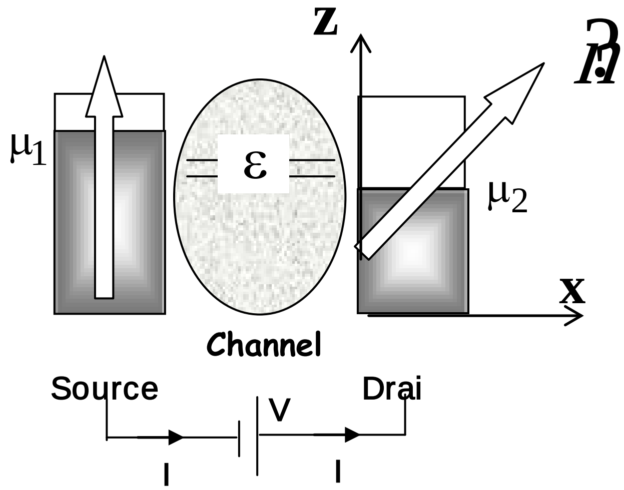
$$MR=\frac{\overline{T}_{P0}-\overline{T}_{A0}}{\overline{T}_{P0}} \quad \text{where} \quad \overline{T}_{P0}\equiv\int_{-\infty}^{\infty}dE\ \overline{T}_P(E)\ ,\ \overline{T}_{A0}\equiv\int_{-\infty}^{\infty}dE\ \overline{T}_A(E)$$

(d) Calculate the total density of states, D(E) in the channel for the parallel configuration. Find $\int_{-\infty}^{\infty} dE\ D(E)$. **(e)** Calculate the total density of states, D(E) in the channel for the antiparallel configuration. Find $\int_{-\infty}^{\infty} dE\ D(E)$.

Problem 2

(Generalized spin-valve) :

Consider the same device as in Problem 1, except that contact 2 is magnetized at an angle θ to z in the z-x plane.

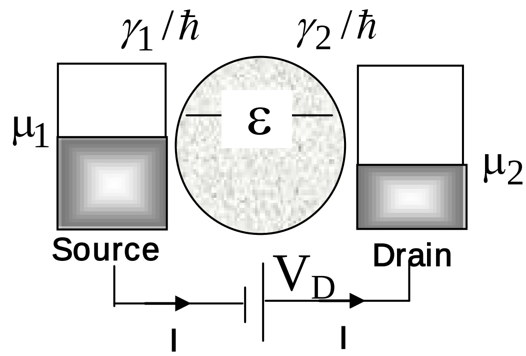


As before we can write $[H]=\begin{bmatrix}\varepsilon & 0\\ 0 & \varepsilon\end{bmatrix}$ and $[\Gamma_1]=\begin{bmatrix}\alpha & 0\\ 0 & \beta\end{bmatrix}$, $\alpha>\beta$. The problem is to write $[\Gamma_2]$ for the drain contact. ***If we use up- and down-spins along $\hat{n}$ as our basis,*** $[\Gamma_2]=\begin{bmatrix}\alpha & 0\\ 0 & \beta\end{bmatrix}$. We have to transform this into the regular basis using up- and down-spins along $\hat{z}$ before we can calculate the current using the equations for coherent transport.

(a) Noting that up-spin and down-spin along $\hat{n}$ are given by $\begin{bmatrix}\cos(\theta/2)\\ \sin(\theta/2)\end{bmatrix}$ and $\begin{bmatrix}-\sin(\theta/2)\\ \cos(\theta/2)\end{bmatrix}$, obtain Γ_2 in the regular basis using up- and down-spins along $\hat{z}$, by performing the required basis transformation.

(b) Plot the transmission versus energy (-0.1 eV < E < +0.1 eV), assuming $\varepsilon=0, \alpha=0.25eV, \beta=0.25\alpha$ for $\theta=0,\pi/2,\pi,3\pi/2$. Please turn in your MATLAB codes for this part.

Problem 3: Consider a device having one energy level ε (whose broadening you may neglect) with equal escape rates $\gamma_1/\hbar = \gamma_2/\hbar = 10^{12}$/sec . and an applied voltage $V_D = (\mu_1 - \mu_2)/q$ such that $\varepsilon - \mu_1 = k_B T$ and $\varepsilon - \mu_2 = 2 k_B T$.



Calculate the (a) current, (b) power delivered by the battery, (c) heat removed from contact 1 and (d) heat dissipated in contact 2. Assume $k_B T$ = 0.025 eV.

(e) What value of $\varepsilon - \mu_1$ will maximize the amount of heat removed from contact 1, assuming that $\varepsilon - \mu_2 >> k_B T$.

Hint: Every electron absorbs an energy $\varepsilon - \mu_1$ from contact 1, dissipates $\varepsilon - \mu_2$ into contact 2 and takes the balance $\mu_1 - \mu_2$ from the battery.