G2: Graphene Fundamentals

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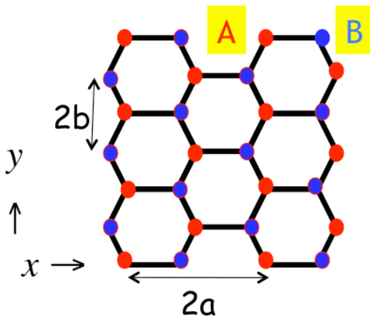
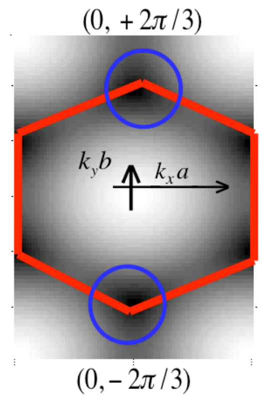
G2.1. Graphene Bandstructure



$$\exp[+ik_x x + ik_y y]$$



$$h_0(\vec{k}) = -t\,(1 + 2\exp(+ik_x a)\cos(k_y b))$$

$$\begin{bmatrix} 0 & h_0{}^* \\ h_0 & 0 \end{bmatrix} \begin{Bmatrix} \psi_A \\ \psi_B \end{Bmatrix} = E \begin{Bmatrix} \psi_A \\ \psi_B \end{Bmatrix} \qquad E = \pm \left| h_0(\vec{k}) \right|$$

G2.2. Graphene Bandstructure (Approximate)

$$\frac{\partial h_0}{\partial k_x} = -2iat\,\exp(+ik_x a)\cos(k_y b) \rightarrow iat$$

$$\frac{\partial h_0}{\partial k_y} = 2bt\,\exp(+ik_x a)\sin(k_y b) \rightarrow \pm at$$

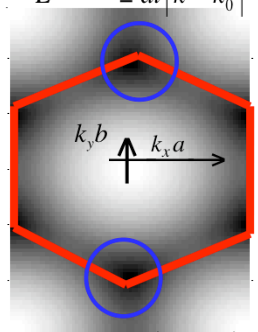
$$K(0,+2\pi/3): \quad h_0 \approx at\,(ik_x + k_y)$$

$$K'(0,-2\pi/3): \quad h_0 \approx at\,(ik_x - k_y)$$

$$h_0(\vec{k}) = -t\,(1 + 2\exp(+ik_x a)\cos(k_y b))$$

$$\begin{bmatrix} 0 & h_0^* \\ h_0 & 0 \end{bmatrix} \begin{Bmatrix} \psi_A \\ \psi_B \end{Bmatrix} = E \begin{Bmatrix} \psi_A \\ \psi_B \end{Bmatrix} \qquad E = \pm \left| h_0(\vec{k}) \right|$$

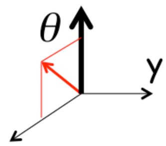
$$E = \pm at \left| \vec{k} - \vec{k}_0 \right|$$



$$E = \pm at \left| \vec{k} + \vec{k}_0 \right|$$

G2.3. Spinor <--> Vector

$$\begin{Bmatrix} up : \cos\theta/2 \\ dn : \sin\theta/2 \end{Bmatrix}$$

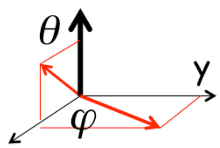


$$\begin{Bmatrix} z : \cos\theta \\ x : \sin\theta \end{Bmatrix}$$

$\theta = \pi :$ "*Orthogonal*"

$\theta = \pi/2 :$ "*Orthogonal*"

$$\begin{Bmatrix} up : \cos\theta/2 \; e^{-i\varphi/2} \\ dn : \sin\theta/2 \; e^{+i\varphi/2} \end{Bmatrix}$$



$$\begin{Bmatrix} x : \sin\theta\cos\varphi \\ y : \sin\theta\sin\varphi \\ z : \cos\theta \end{Bmatrix}$$

G2.4. Spin matrices

$$= \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{\sigma_x} \underbrace{\sin\theta\cos\varphi}_{\hat{x}.\hat{n}} + \underbrace{\begin{bmatrix} 0 & -i \\ +i & 0 \end{bmatrix}}_{\sigma_y} \underbrace{\sin\theta\sin\varphi}_{\hat{y}.\hat{n}} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}_{\sigma_z} \underbrace{\cos\theta}_{\hat{z}.\hat{n}}$$

$$= \begin{bmatrix} \cos\theta & \sin\theta\, e^{-i\varphi} \\ \sin\theta\, e^{+i\varphi} & -\cos\theta \end{bmatrix} \equiv \vec{\sigma}\,.\hat{n}$$

$$\left[\vec{\sigma}\,.\hat{n}\right]^2 = I$$

Dirac Equation

$$E\{\psi\} = \underbrace{\begin{bmatrix} mc^2 I & c\,\vec{\sigma}.\vec{p} \\ c\,\vec{\sigma}.\vec{p} & -mc^2 I \end{bmatrix}}_{H} \{\psi\}$$

Setting $p_z = 0$

$$E\{\psi\} = \begin{bmatrix} mc^2 & 0 & 0 & c(p_x - ip_y) \\ 0 & mc^2 & c(p_x + ip_y) & 0 \\ 0 & c(p_x - ip_y) & -mc^2 & 0 \\ c(p_x + ip_y) & 0 & 0 & -mc^2 \end{bmatrix} \{\psi\}$$