Fundamentals of Nanoelectronics

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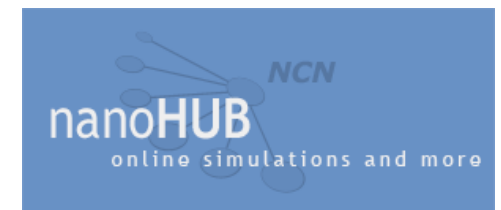
11.19.2004

Lecture 34: Current Voltage Characteristics
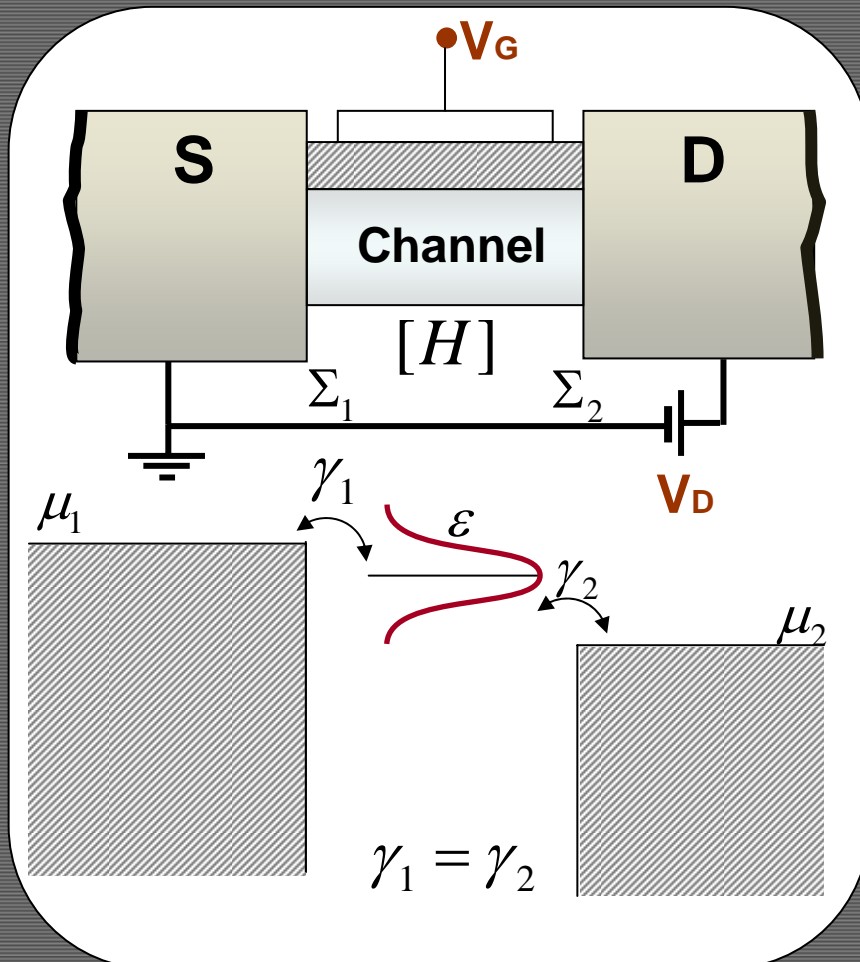Ref. Chapter 9.1

Network for Computational Nanotechnology

Model of a nanotransistor

• Figure shows a model of a nanotransistor under applied source drain voltage. Current flows if there is at least one level is available in the channel in the energy range between the two Fermi levels in the contacts.

• One important point is hat of the levels would remain sharp for all applied biases, current would get large without any upper limit. How ever as bias increases, the levels get broadened and some part of them lies outside of the energy range between the contact Fermi levels and that fraction will not contribute to the current; hence the current will saturate at some point: the more the bias, the more the broadening and current stays the same.

Model of a nanotransistor
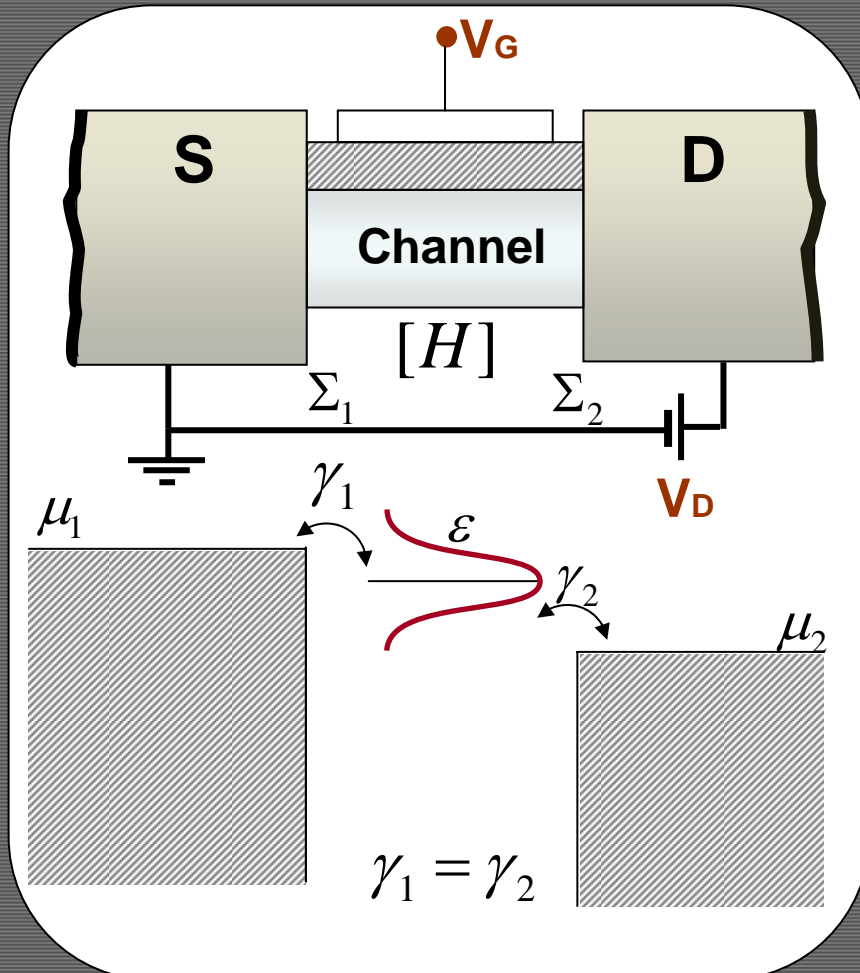
$$I_1 = q \frac{\gamma_1}{\hbar} (f_1 - N)$$

$$I_2 = q \frac{\gamma_2}{\hbar} (N - f_2)$$
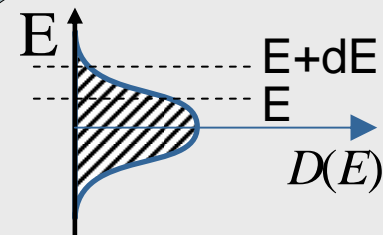
• Equating the currents at steady state gives us N:

$$N = \frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2}$$

• Substituting back N in either of the current equations we get:

$$\therefore I = I_1 = I_2 = \frac{q}{\hbar} \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} (f_1 - f_2)$$

• To include broadening, we computed DOS:

$$D(E) = \frac{\gamma/2\pi}{(E-\varepsilon)^2 + (\gamma/2)^2}, \quad (\gamma = \gamma_1 + \gamma_2)$$

- Broadening included, the new equations are:

$$N = \int dED(E) \frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2}$$

$$I = \int_{-\infty}^{\infty} dED(E) \frac{2q}{\hbar} \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} [f_1(E) - f_2(E)]$$

- Next we want to see the general version of these equations. Start with the Green's function and derive the spectral function from it:

- $G = (EI - H - \Sigma)^{-1}$ $(\Sigma = \Sigma_1 + \Sigma_2)$

- $H \Rightarrow [\varepsilon]$

- $\Sigma_1 \Rightarrow \left[\sigma_1 - \frac{i\gamma_1}{2} \right]$

- $\Sigma_2 \Rightarrow \left[\sigma_2 - \frac{i\gamma_2}{2} \right]$

$\Rightarrow \Sigma = \left[\sigma - \frac{i\gamma}{2} \right]$

$\gamma = \gamma_1 + \gamma_2$

$\sigma = \sigma_1 + \sigma_2$

$$G = \frac{1}{E - \varepsilon - \sigma + \dfrac{i\gamma}{2}}$$

$$G^+ = \frac{1}{E - \varepsilon - \sigma - \dfrac{i\gamma}{2}}$$

- $A = i(G - G^+) = i \left[\dfrac{1}{E - \varepsilon - \sigma + \dfrac{i\gamma}{2}} - \dfrac{1}{E - \varepsilon - \sigma - \dfrac{i\gamma}{2}} \right] = \dfrac{\gamma}{(E - \varepsilon - \sigma)^2 + (\gamma/2)^2}$

Density Matrix

- The general form of the electron density can be written as:

$$\rho = \int \frac{dE}{2\pi}\left(\left[G\Gamma_1 G^+\right]f_1 + \left[G\Gamma_2 G^+\right]f_2\right) \qquad \text{where} \quad \begin{cases} \Gamma_1 = i(\Sigma_1 - \Sigma_1^+) \\ \Gamma_2 = i(\Sigma_2 - \Sigma_2^+) \end{cases}$$

$$N = \int dE \left(D(E)\frac{\gamma_1 f_1}{\gamma_1 + \gamma_2} + D(E)\frac{\gamma_2 f_2}{\gamma_1 + \gamma_2} \right)$$

- For a one level device we should get what we had before:

$$G\Gamma_1 G^+ = \frac{1}{E - \varepsilon' + \dfrac{i\gamma}{2}}\gamma_1\frac{1}{E - \varepsilon' - \dfrac{i\gamma}{2}} = \frac{\gamma}{\left(E - \varepsilon'\right)^2 + \left(\gamma/2\right)^2}\frac{\gamma_1}{\gamma} = D(E)\frac{\gamma_1}{\gamma}$$

$$G\Gamma_2 G^+ = \frac{1}{E - \varepsilon' + \dfrac{i\gamma}{2}}\gamma_2\frac{1}{E - \varepsilon' - \dfrac{i\gamma}{2}} = \frac{\gamma}{\left(E - \varepsilon'\right)^2 + \left(\gamma/2\right)^2}\frac{\gamma_2}{\gamma} = D(E)\frac{\gamma_2}{\gamma}$$

$$\gamma = \gamma_1 + \gamma_2$$

• Next we wan to write the general from of the current. First rewrite the old equation for the current:

$$I = \frac{q}{2\pi\hbar} \int\limits_{-\infty}^{\infty} dE\, 2\pi D(E) \underbrace{\frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2}}[f_1(E) - f_2(E)]$$

$$\overline{T}(E): \text{ transmission}$$

• The quantity transmission is given by: $\overline{T}(E) = Trace(\Gamma_1 G \Gamma_2 G^+)$

• So for the general case current can be calculated using transmission:

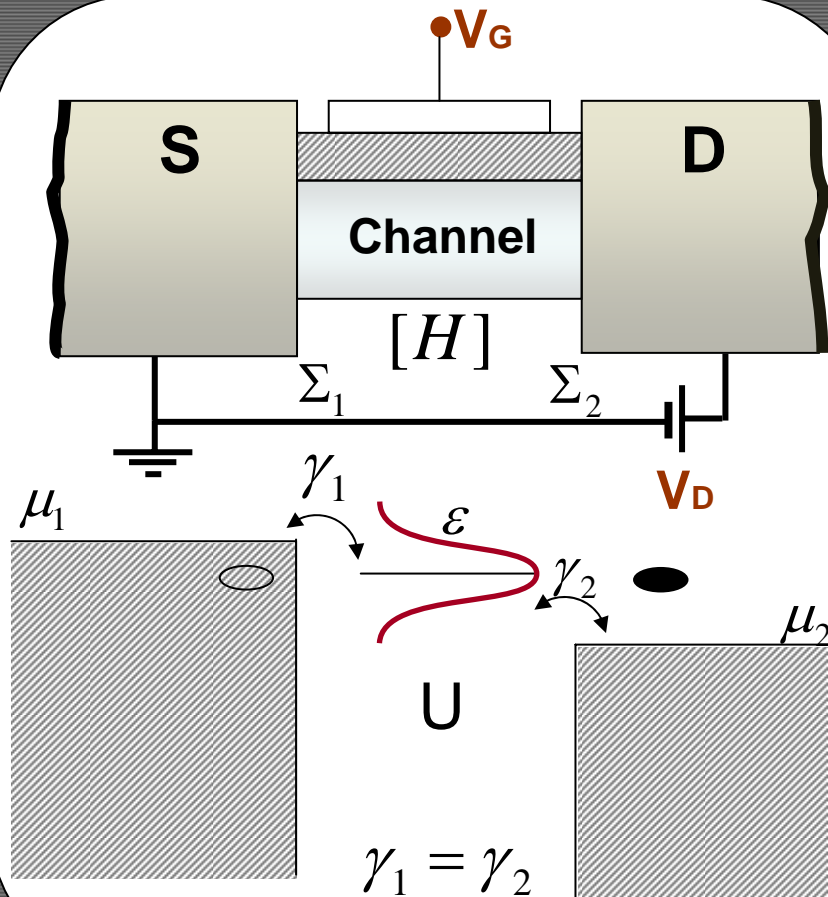$$\boxed{I = \frac{q}{h} \int dE\, \overline{T}(E)(f_1 - f_2)}$$

• Let's try again to see if the general formulation gives us the old results or the one level model:

$$\overline{T}(E) = Trace\,(\Gamma_1 G \Gamma_2 G^+) \quad = \frac{\gamma}{(E - \varepsilon')^2 + (\gamma/2)^2} \frac{\gamma_1 \gamma_2}{\gamma} = 2\pi D(E) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2}$$

Model of a nanotransistor

- In all that we've done do we need to know the number of electrons?

$$N = \int dE D(E) \frac{\gamma_1 f_1 + \gamma_2 f_2}{\gamma_1 + \gamma_2}$$

- The reason that we need N is that we have to do the calculation for current self consistently. Because the number of electron changes in the channel the potential in the channel changes due to these electrons and the levels rise up or go down. How ever here on, instead of doing the calculation self consistently, we'll assume a reasonable potential profile based on what we know about the problem and calculate the current using that U.