2009 NCN@Purdue-Intel Summer School

Notes on Percolation and Reliability Theory

Lecture 3
Electrical Conduction in Percolative Systems

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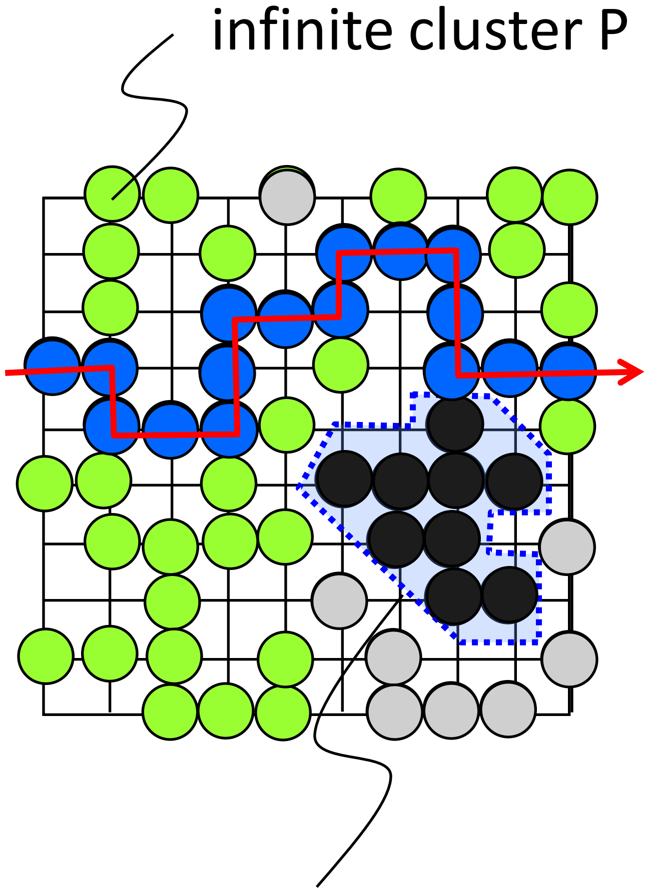
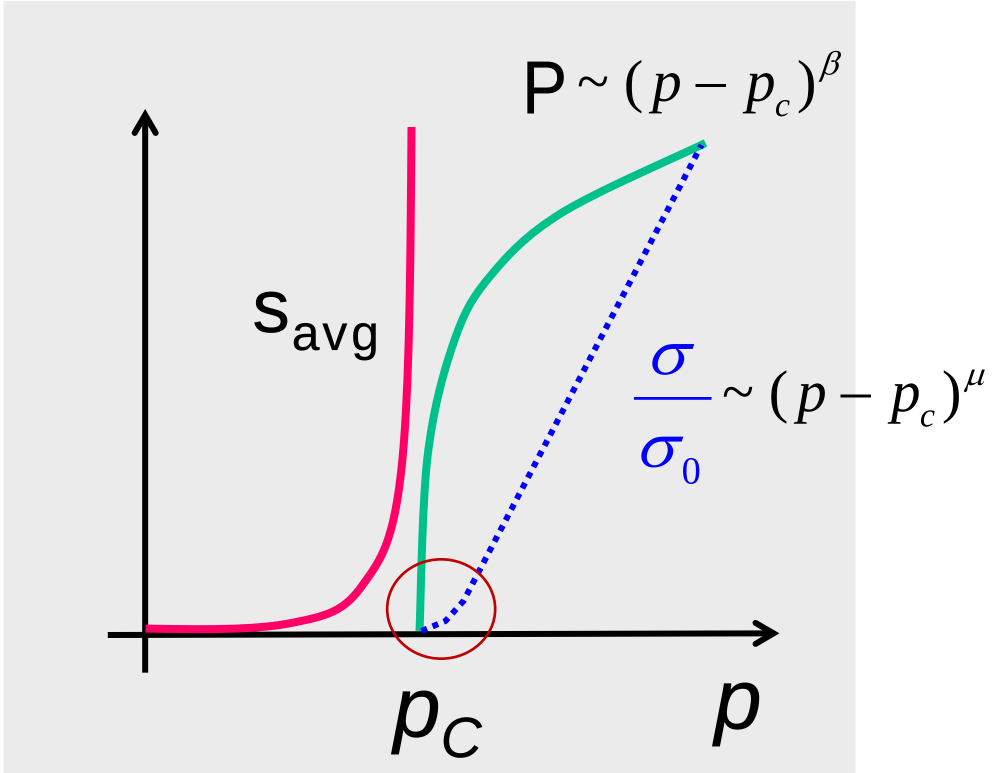
outline of lecture 3

1) **Basic concepts of percolative conduction**
2) Non-ohmic conduction: cell-based percolation
3) Non-ohmic conduction: renormalization
4) Finite width transition: physics of striping
5) Conclusion

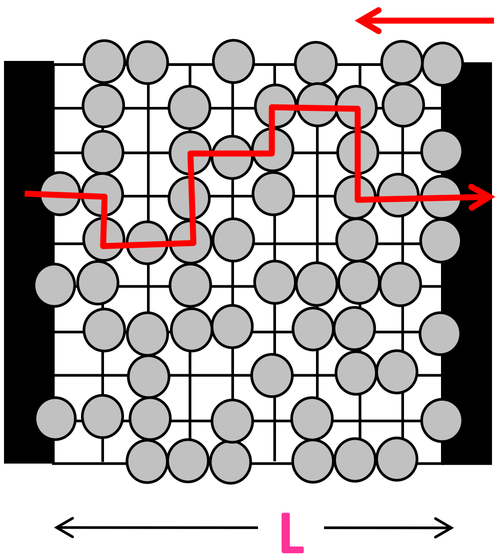
basics: cluster-size and conduction





More details in the lab-session …

finite sizes and end of Ohm's law

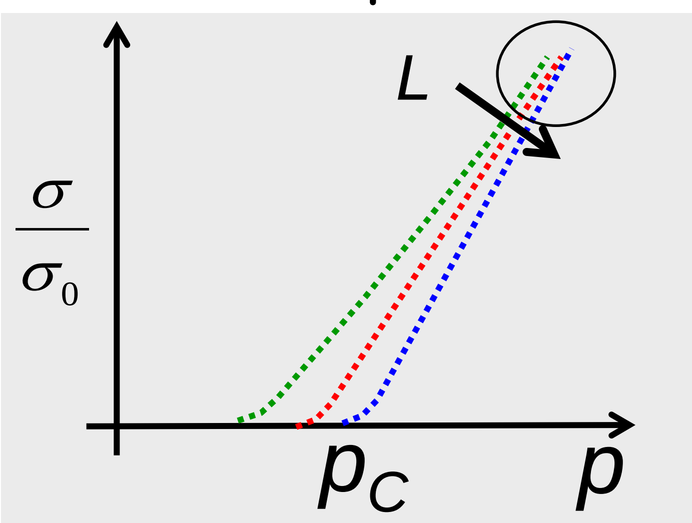


Ohm's law says …

$$\frac{\sigma(p \gg p_c)}{\sigma_0} \propto \frac{1}{L^0}$$

$$G \sim \sigma_0 \frac{W}{L}$$

Threshold depends on L



but close to percolation ….

$$\frac{\sigma(p \sim p_c)}{\sigma_0} \propto \frac{1}{L^{\frac{\mu}{\nu}-1}} = \frac{1}{L^{0.93}}$$

$$G \sim \sigma \frac{W}{L} \sim \frac{1}{L^{\frac{\mu}{\nu}}}$$

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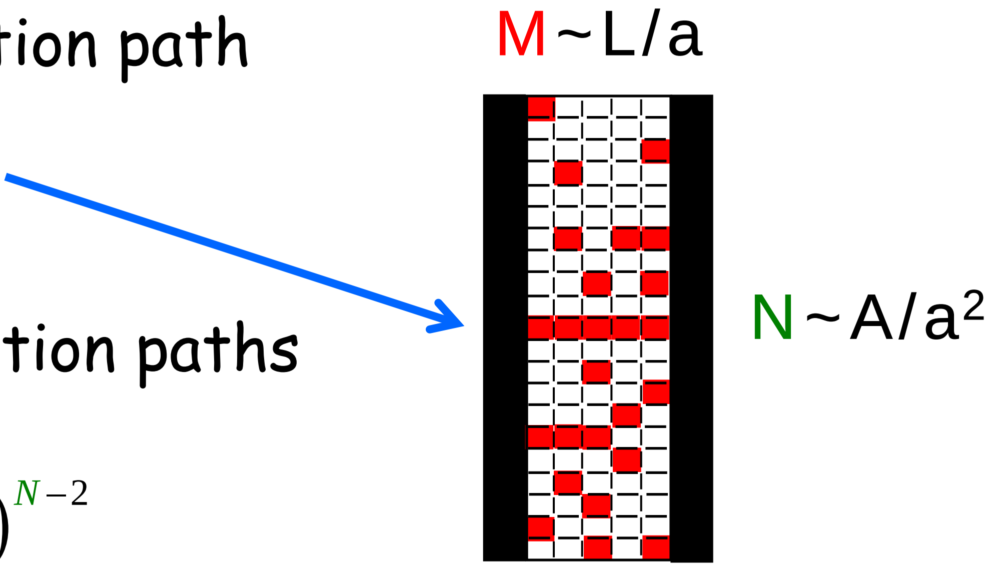
non-ohmic scaling by cell percolation

Prob. of a filled row $\mathbf{P} = p^{M}$

Probability of 1-percolation path

$$P_1 = N \times \mathbf{P} \times (1-\mathbf{P})^{N-1}$$



Probability of 2-percolation paths

$$P_2 = \frac{N(N-1)}{2} \times \mathbf{P}^2 \times (1-\mathbf{P})^{N-2}$$

$$P_n = \frac{x^n}{n!} exp(-x) \qquad x \equiv \mathrm{P}N = p^M N$$

Homework

finite size "percolation threshold"

Prob. of a filled row $\mathrm{P} = p^{M}$

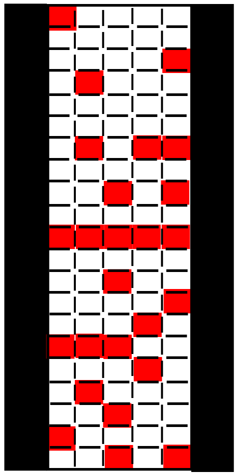
Probability of not conducting

$$1 - P_0 = (1 - \mathrm{P})^{N}$$

Probability of conducting

$$P_0 = 1 - (1 - p^{M})^{N} \sim 1 - (1 - p^{L/a})^{A/a^2}$$

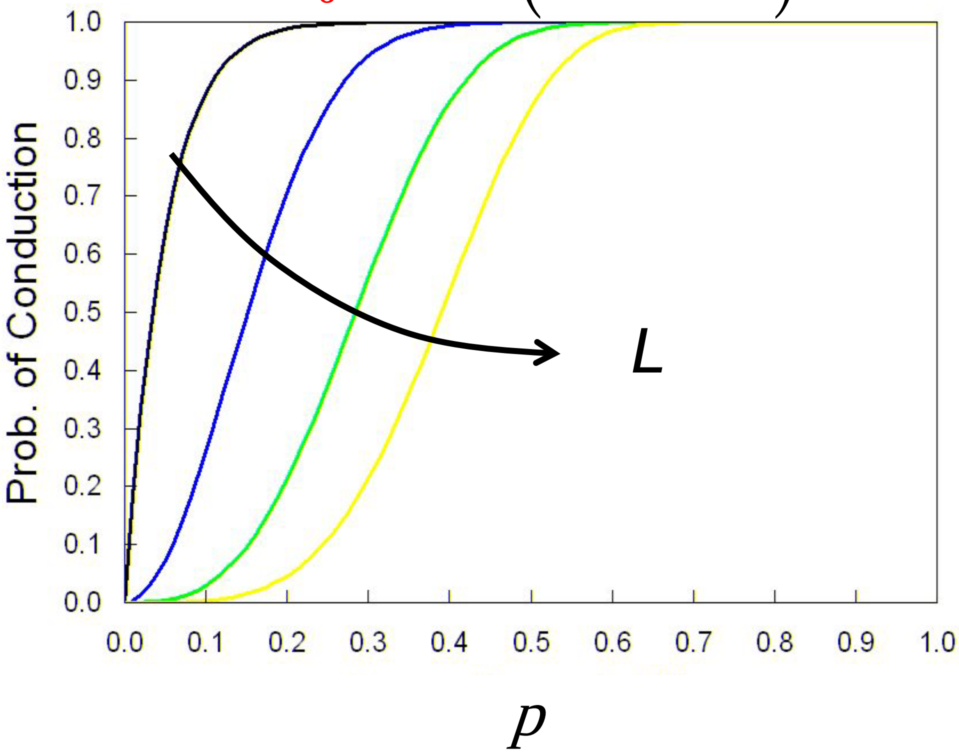
M ~ L/a

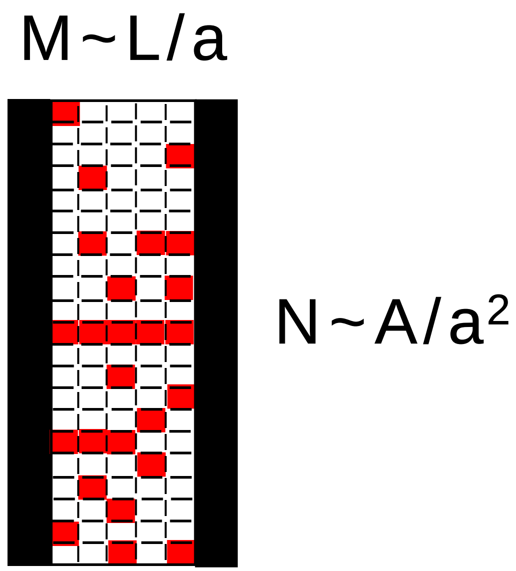


N ~ A/a²

finite size “percolation threshold”

$$P_0 = 1 - \left(1 - p^{L/a}\right)^{A/a^2}$$





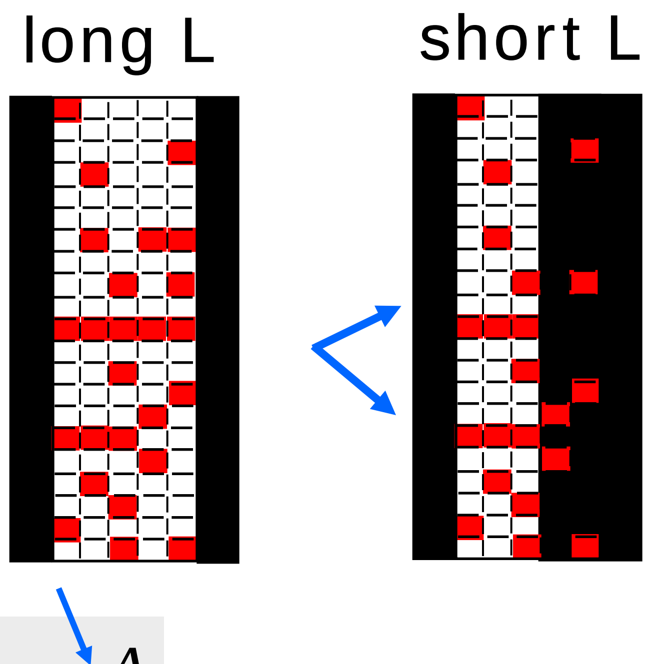
Simple cell-percolation model
anticipates L-dependent threshold

conductance of the random resistor

Average number of percolation paths

$$\langle n \rangle = P_1 + 2P_2 + 3P_3 + ...$$

$$= \sum_{i>0} i \bullet P_i = N\mathrm{P} = N \times p^{M}$$



Average paths/area

$$\frac{\langle n \rangle}{N} = p^{M} \sim p^{L/a}$$

$$G \sim \frac{\sigma_0^*}{L} \times N \times \frac{\langle n \rangle}{N} = \frac{\sigma_0^*}{L} \frac{A}{a^2} p^{L/a} \propto \sigma_0{}^* p^{L/a} \frac{A}{L}$$

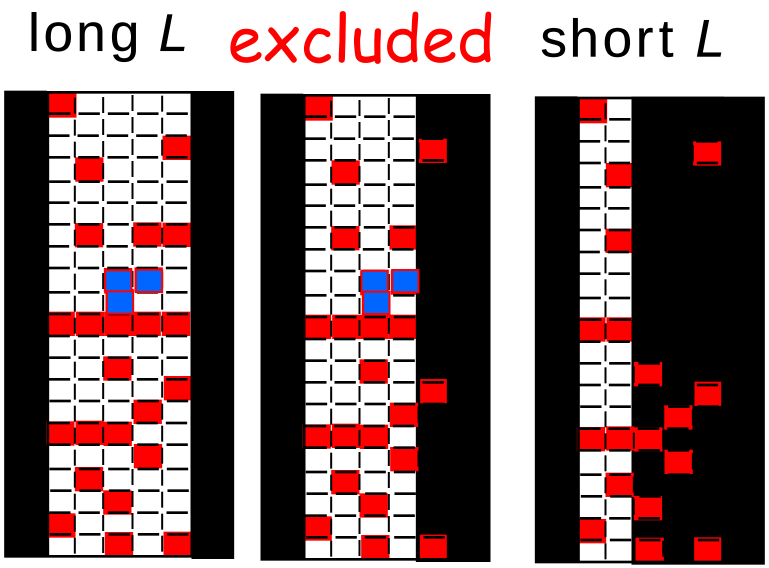
- With p close to 1 (>>pc), we return to 1/L dependence
- Nonlinearity in G arises from cluster-size distribution

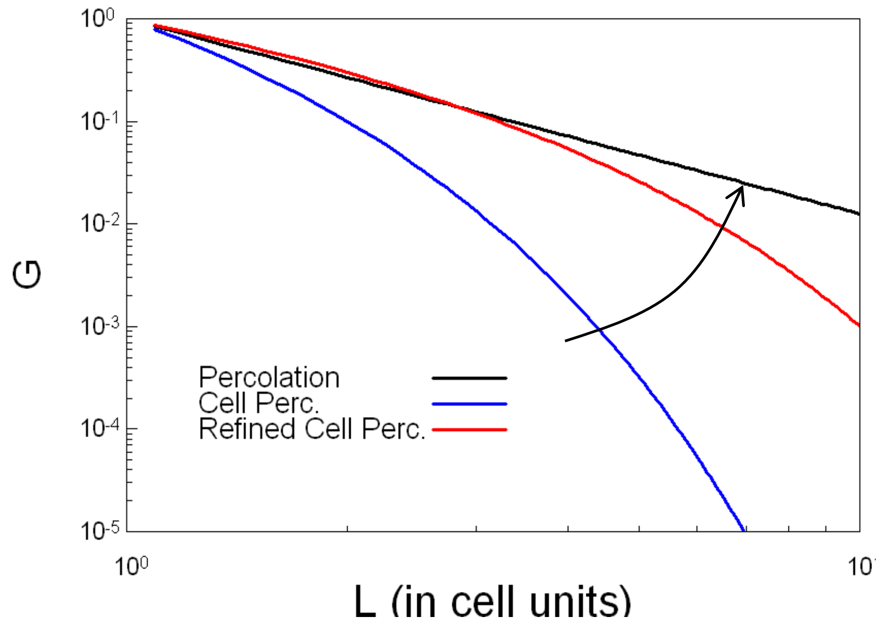
length dependence for small vs. large systems

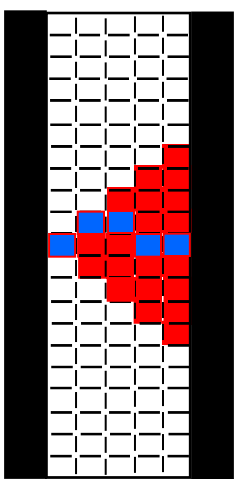
$$G \sim \sigma_{row} p^{L} \frac{W}{L}$$ (very small systems)

$$G \sim \sigma_{row} \frac{W}{L^{1.93}}$$ (very large systems)

.... crooked paths for long conductors







$$G \propto \frac{3^{L-1} p^{L}}{L}$$

The number of nonlinear paths increase with L

Inclusion of these paths improves the long-L limit

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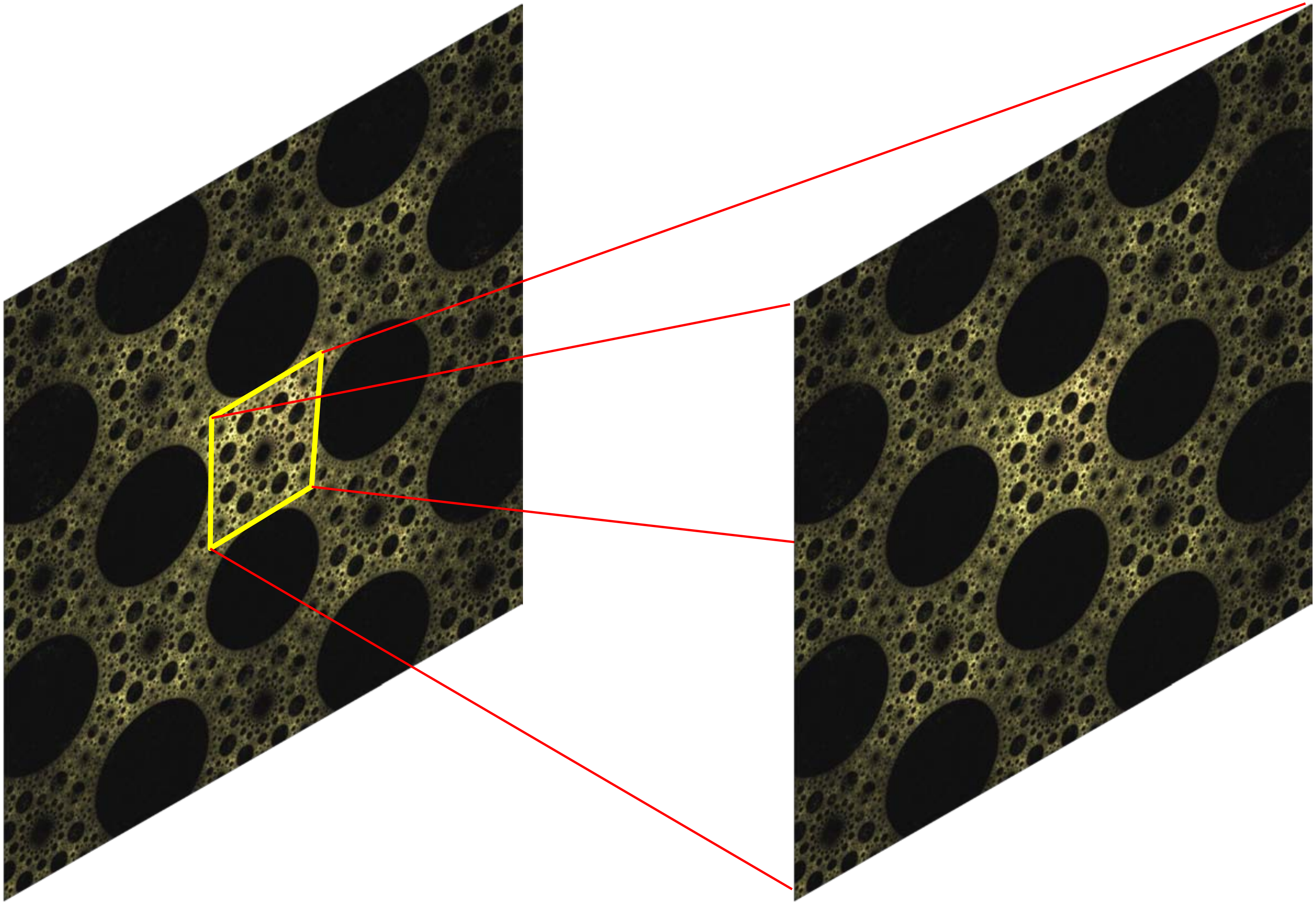
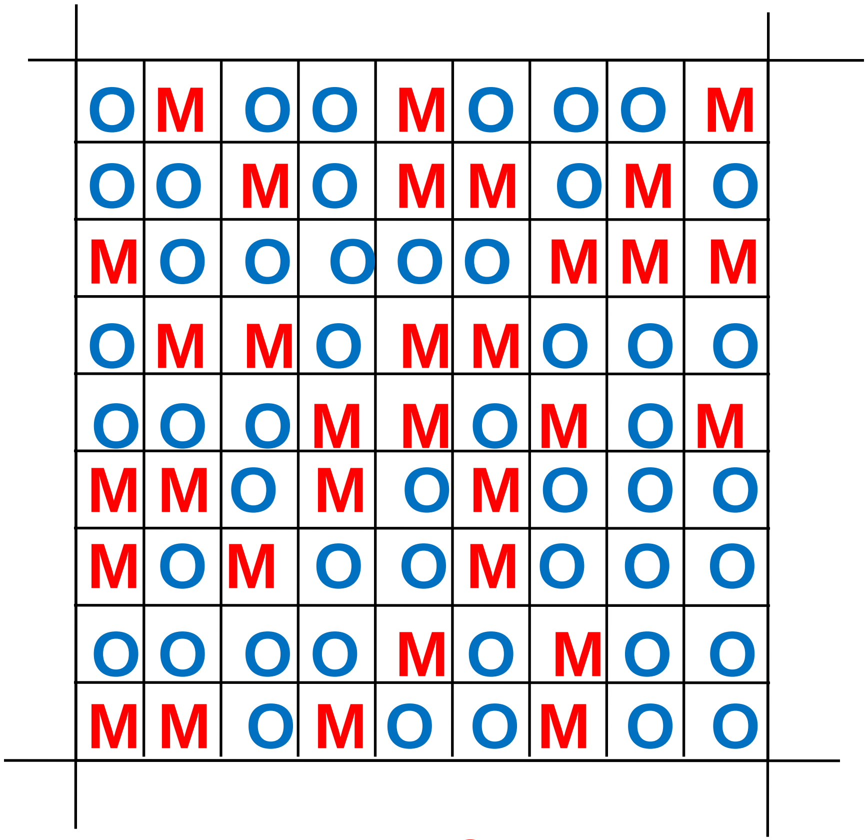
self-similarity

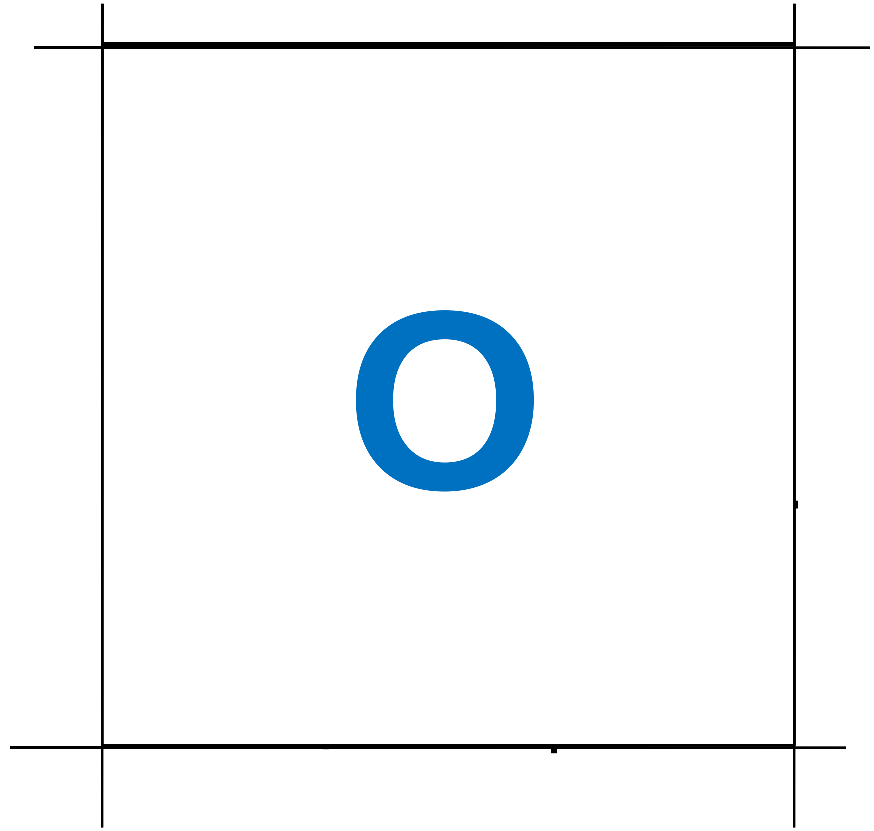


Invariant under magnification or scaling

renormalization of Ohama v. McCain

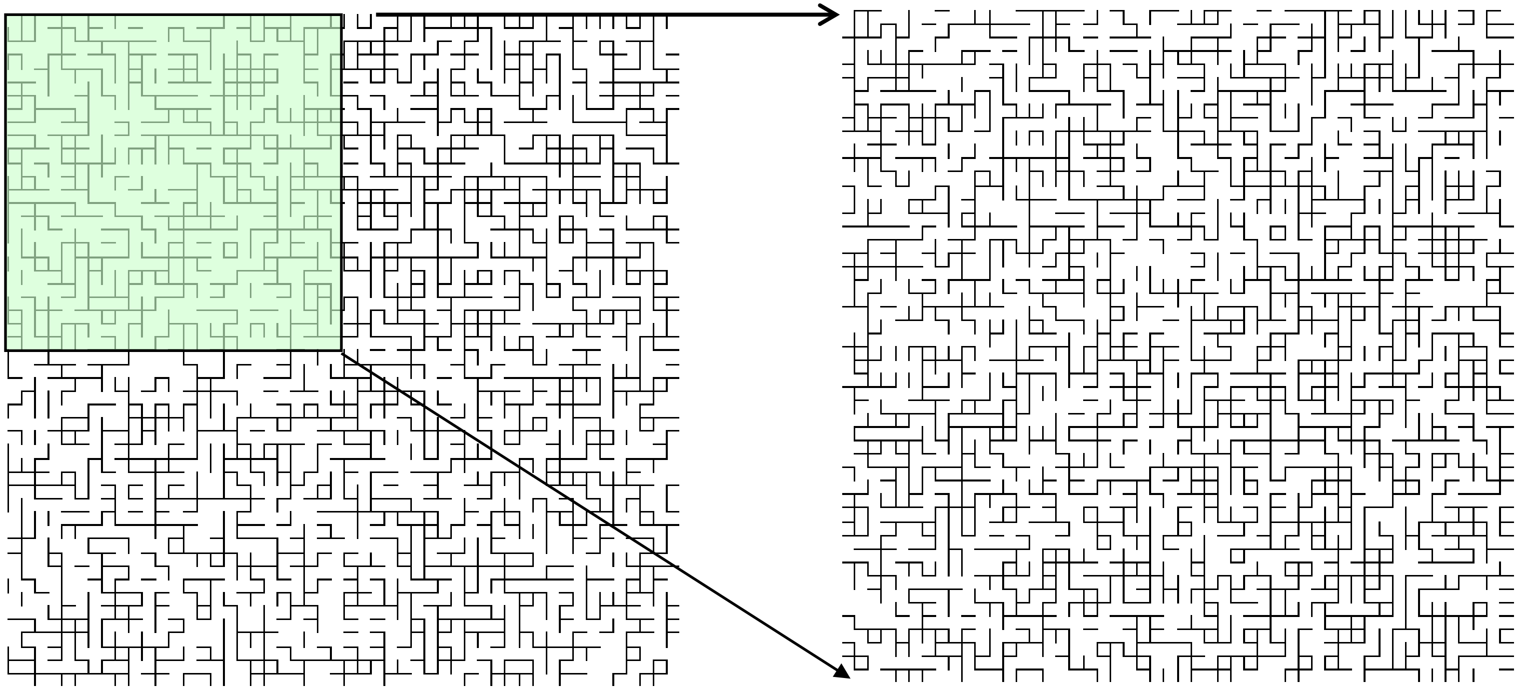


44 vs. 37



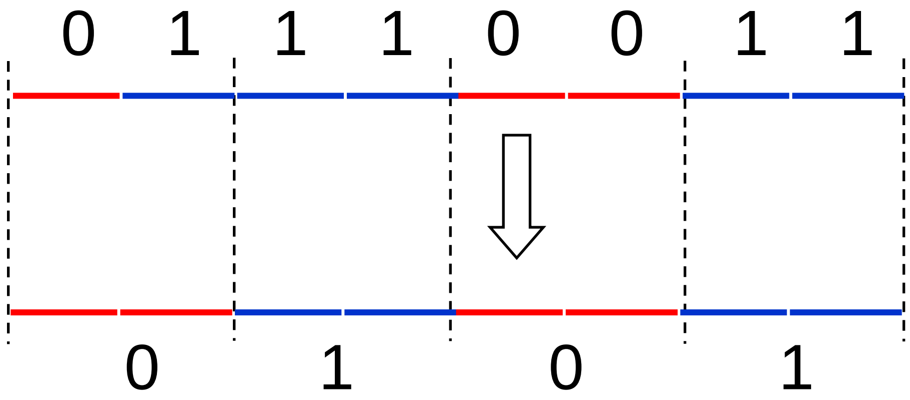
At percolation threshold, the self-similar pattern will not change on rescaling …

renormalization and self-similarity .....



- At $p=p_c$, the islands sizes are self-similar
- The probability of connection at smaller scale must be preserved for scale invariance to work.

1D percolation threshold ....



$$p$$

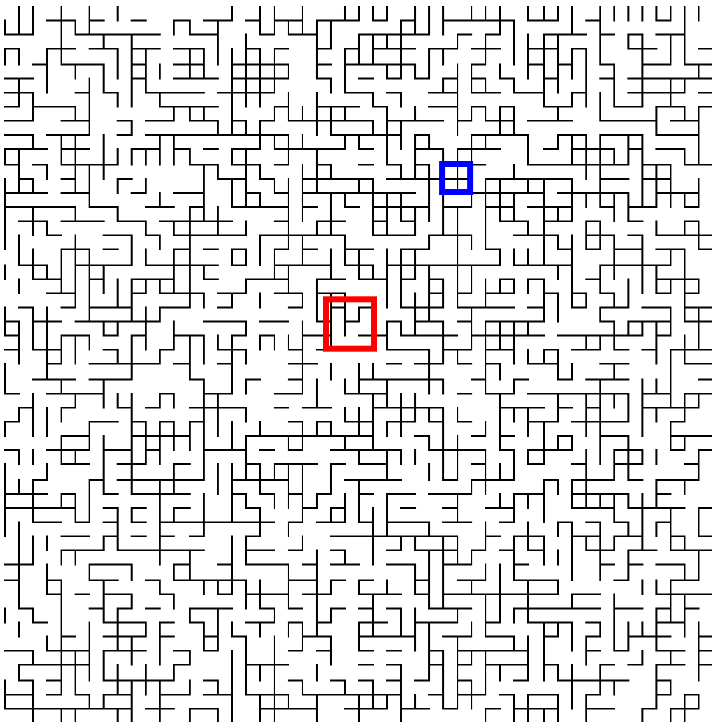
$$p' = p^2$$

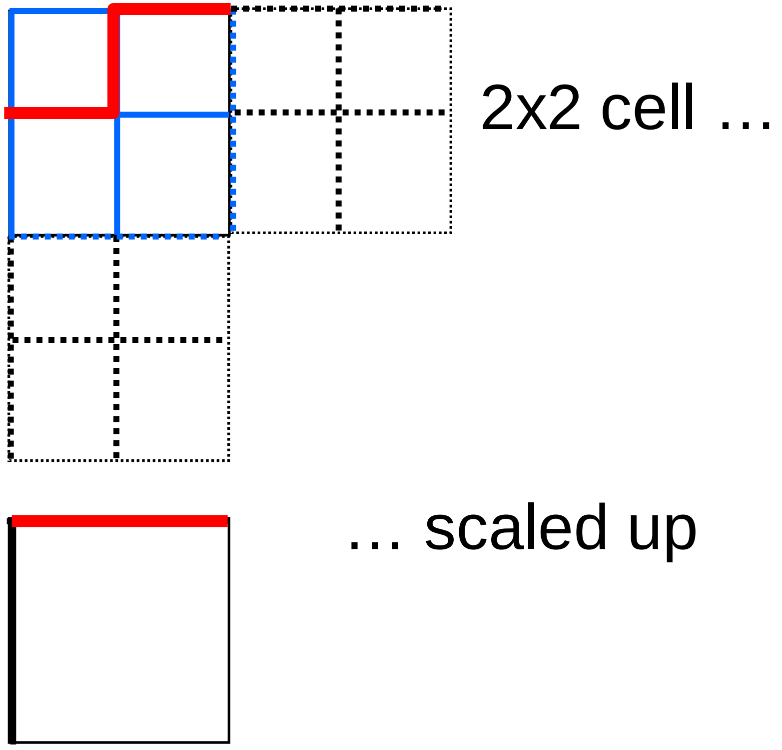
At the threshold, scaling does not change p ....

$$p_c = p_c^{\;2} \quad \Rightarrow p_c = 0,1$$

Either all are connected (p_c=1) or all are broken (p_c=0)

now 2D threshold ....

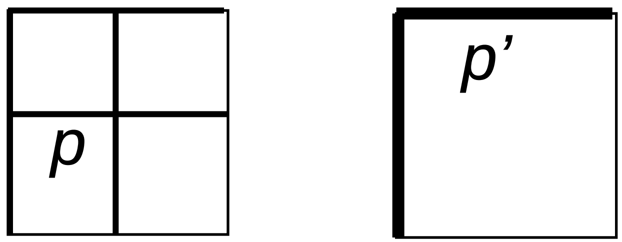


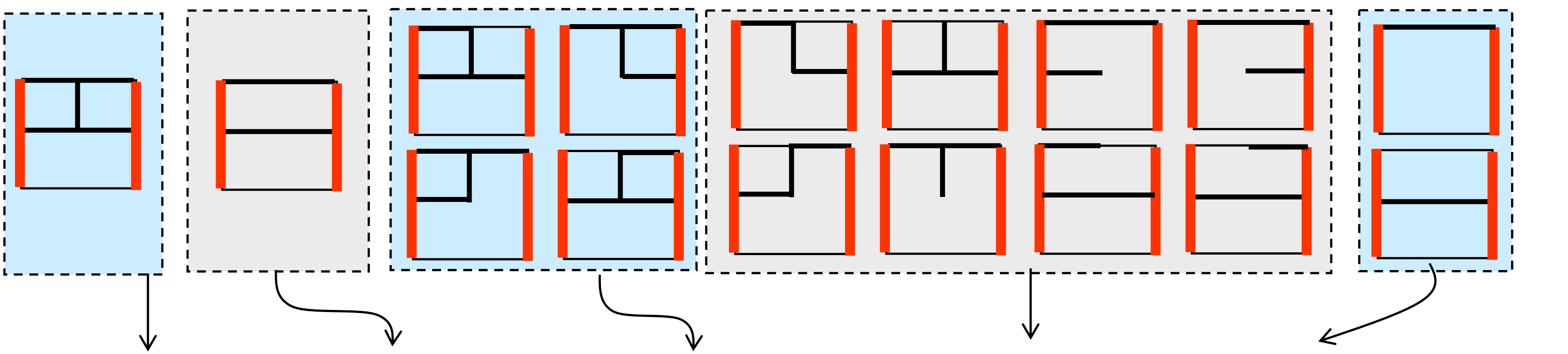


Rule: ability to connect from left to right

The probability of connection at smaller scale must be preserved for scale invariance to work.

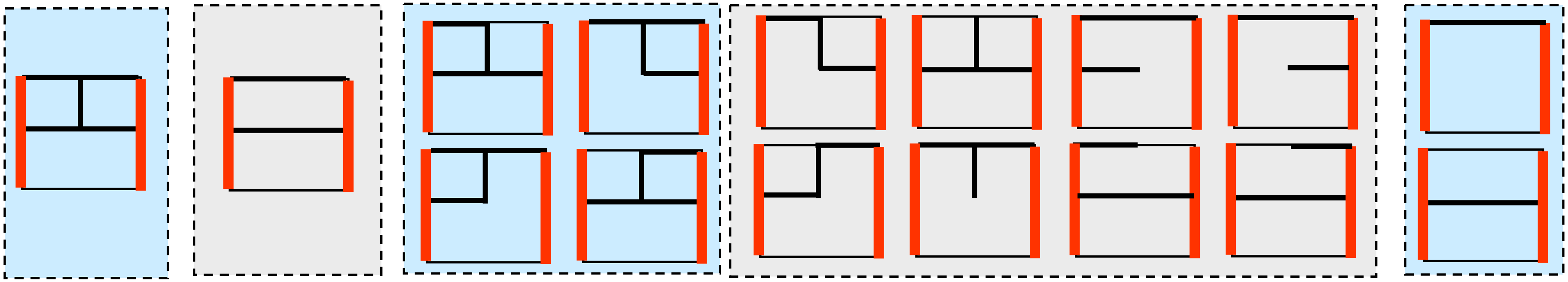
percolation threshold ....





$$p' = p^5 + p^4 \times (1 - p) + 4p^4 \times (1 - p) + 8p^3 \times (1 - p)^2 + 2p^2 \times (1 - p)^3$$

percolation threshold ….

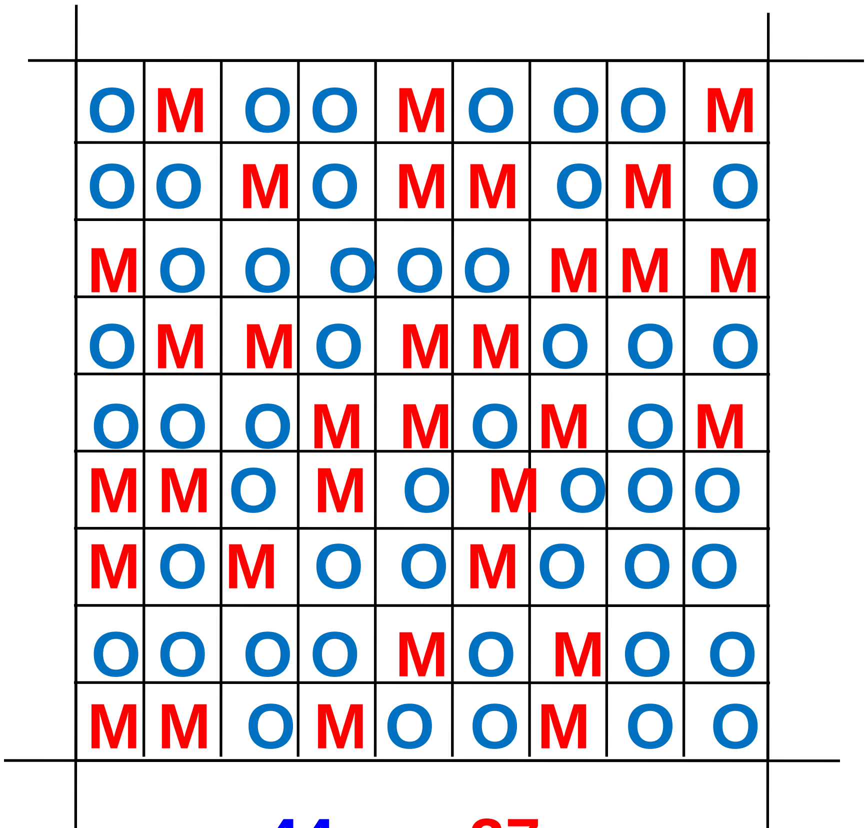


$$p' = p^5 + 5p^4(1-p) + 8p^3(1-p)^2 + 2p^2(1-p)^3$$

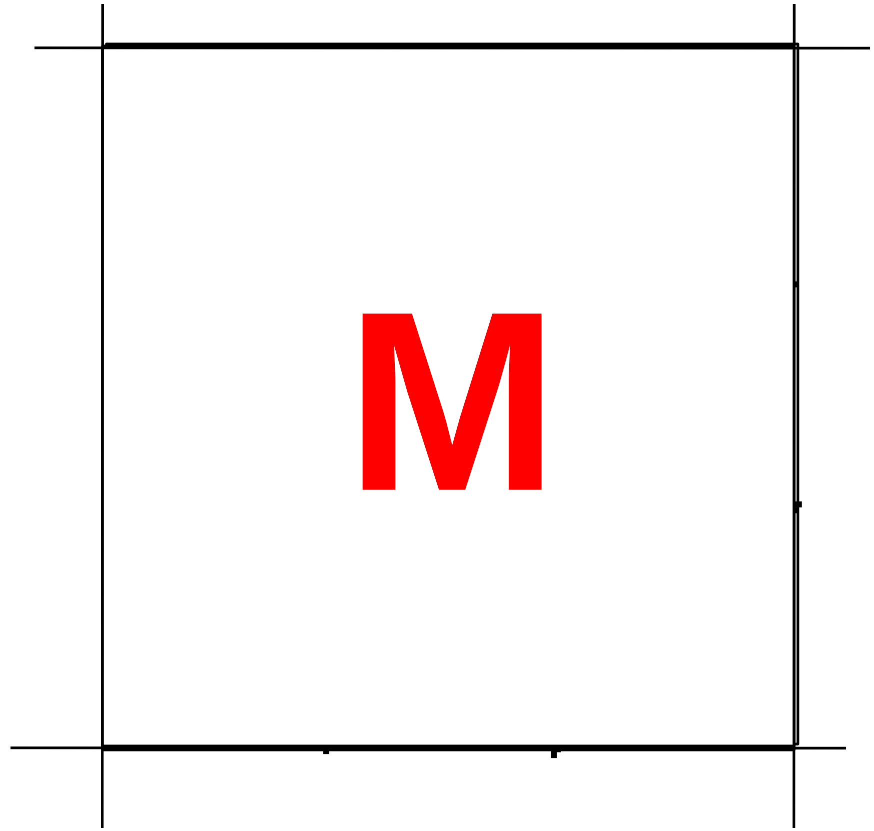
At percolation threshold, $p' = p$ so that $p = \frac{1}{2}, 0, 1$

From lecture 2, recall that bon percolation threshold was indeed 0.5 …

New Rule: 1 M =1.75 O

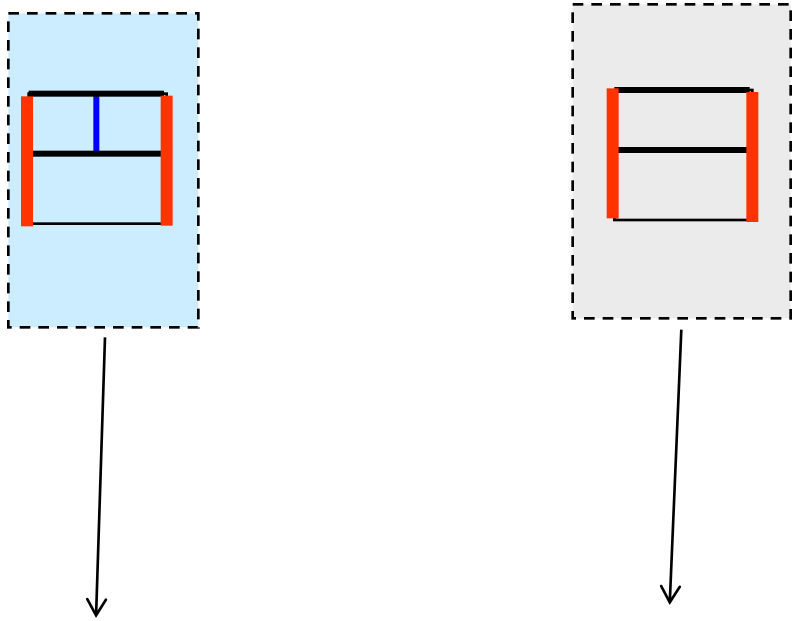


44 vs. 37



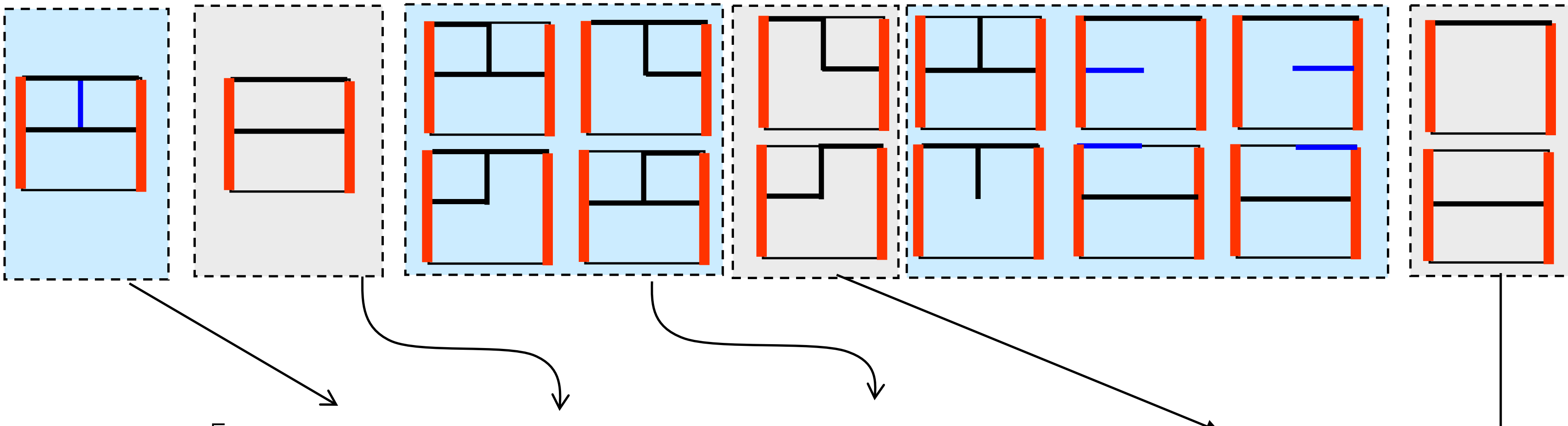
If weight for different bonds are different,
the scaling will obviously proceed differently …

conductivity ....



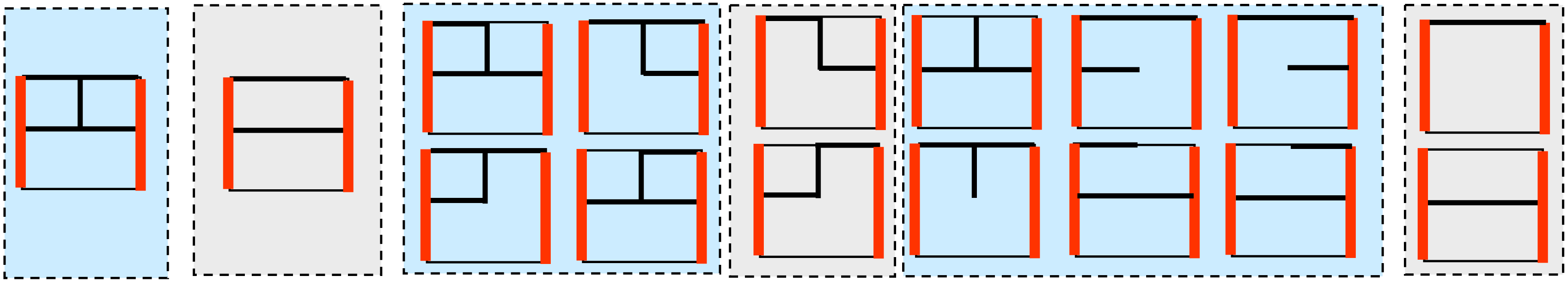
$$\left[\; p^{5}\left(\frac{1}{2}+\frac{1}{2}\right)^{-1} \qquad p^{4}\times(1-p)\left(\frac{1}{2}+\frac{1}{2}\right)^{-1} \;\right]$$

conductivity ....



$$\frac{p'}{\sigma_{s_2}} = \frac{1}{\sigma_{s_1}}\left[p^5\left(\frac{1}{2}+\frac{1}{2}\right)^{-1} + p^4 \times (1-p)\left(\frac{1}{2}+\frac{1}{2}\right)^{-1} + 4p^4(1-p)\left(\frac{3}{5}\right)^{-1} + 2p^3(1-p)^2\left(\frac{1}{3}\right)^{-1} + 6p^3(1-p)^2\left(\frac{1}{2}\right)^{-1} + 2p^2 \times (1-p)^3\left(\frac{1}{2}\right)^{-1} \right]$$

conductivity ....



$$\frac{p'}{\sigma_{s_2}} = \frac{1}{\sigma_{s_1}}\left[p^5 + p^4(1-p) + \frac{5}{3}\times 4p^4(1-p) + 6p^3(1-p)^2 + 12p^3(1-p)^2 + 4p^2(1-p)^3 \right]$$

If $p' = p$ then $\left[\frac{\sigma_{s_1}}{\sigma_{s_2}}\right] = 1.917 = \left(\frac{s_2}{s_1}\right)^{\frac{\mu}{\nu}-1} = 2^{\frac{\mu}{\nu}-1} \Rightarrow \frac{\mu}{\nu} = 1.93$

$$\sigma \sim \frac{1}{L^{0.93}} \quad \mathrm{G} = \sigma \frac{W}{L} \sim \frac{1}{L^{1.93}}$$

summary

	Ballistic	Ohmic	Percolation
σ	$\sim L$	$qn\mu \frac{1}{L^0}$	$\sim \frac{1}{L^{0.93}}$
$G \equiv \sigma \frac{W}{L}$	$\frac{2q^2}{h} \frac{1}{L^0}$	$qn\mu \frac{W}{L}$	$\sim \frac{1}{L^{1.93}}$

The danger of using classical SPICE model …

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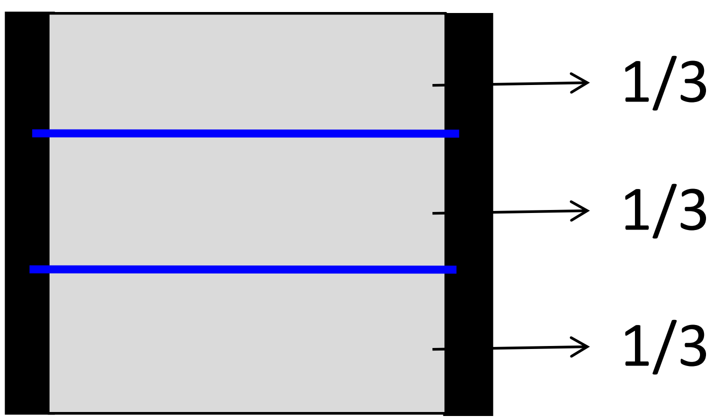
finite widths and end of Ohm's law

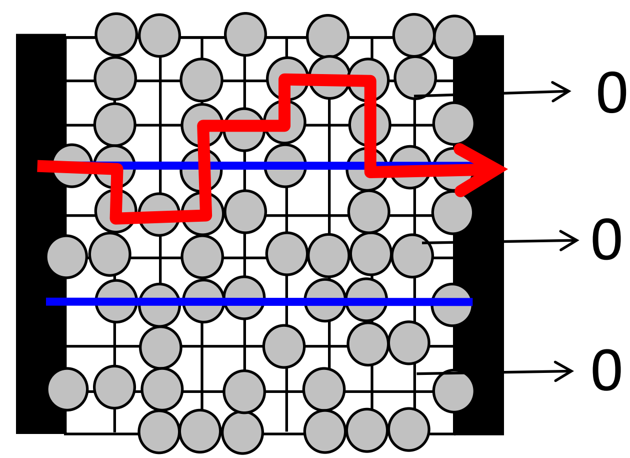
Ohm's law says …

$$G \sim \sigma_0 \frac{W}{L}$$

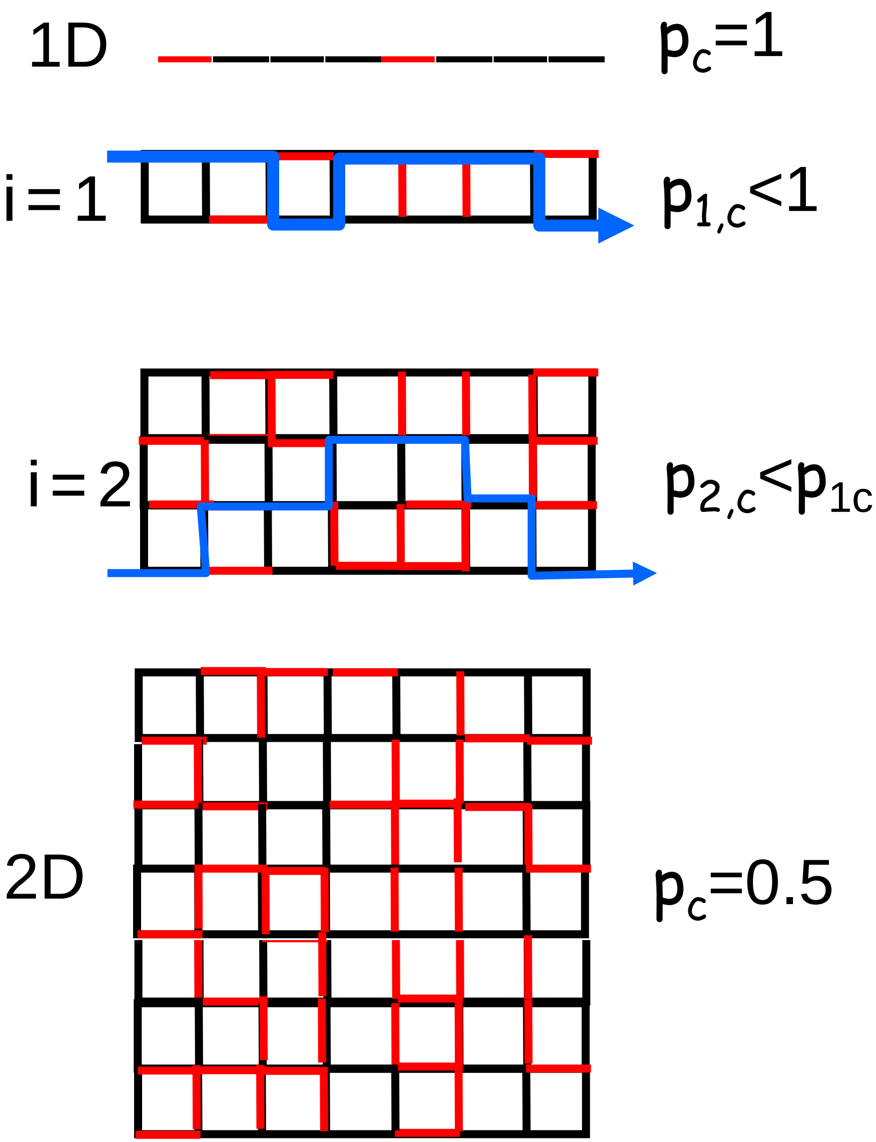


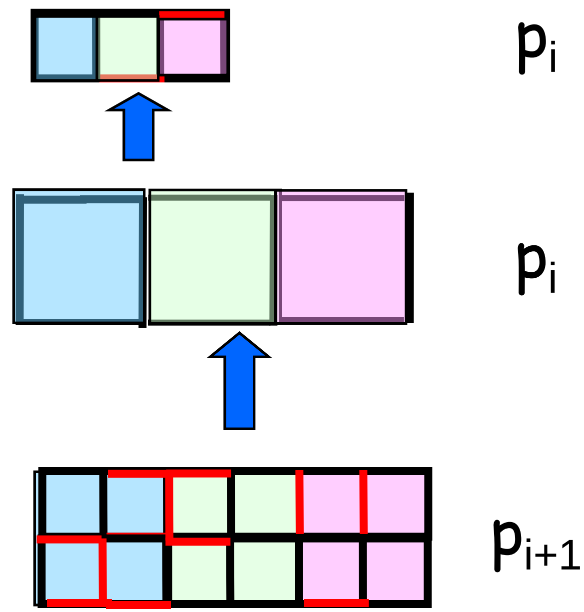
… but in real systems

$$G \neq \sigma_0 \frac{W}{L}$$



finite thickness effect (1D to 2D transition)



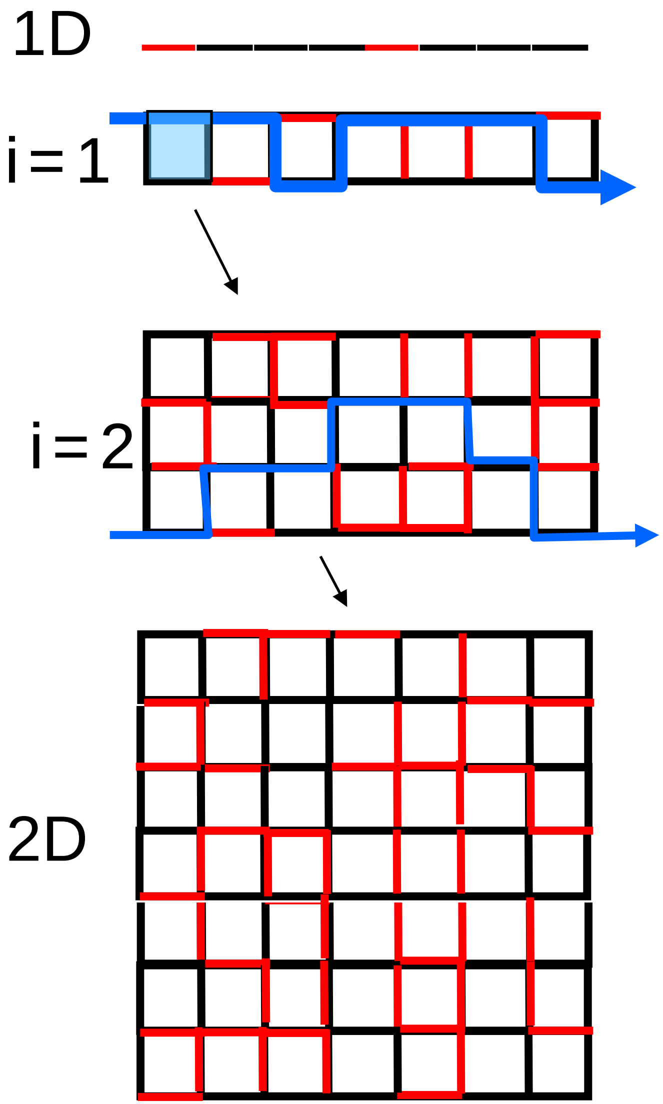


Percolation threshold …

$$p_i = 2p_{i+1}^5 - 5p_{i+1}^4 + 2p_{i+1}^3 + 2p_{i+1}^2$$

$$p_1 \to p_2 \to p_3 \cdots \to p_\alpha$$

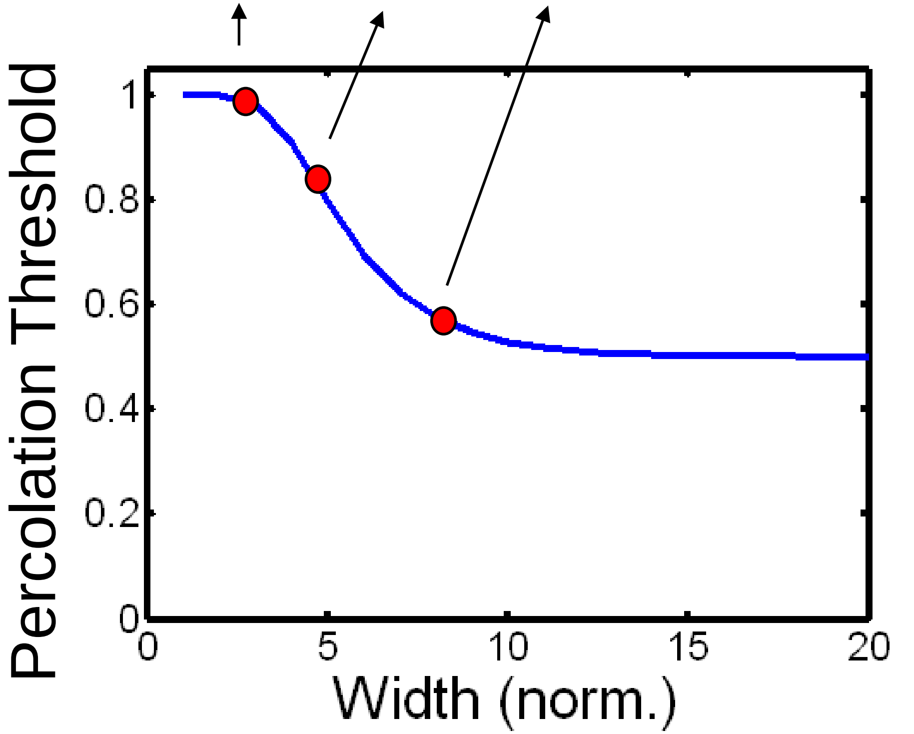
finite thickness effect (1D to 2D transition)



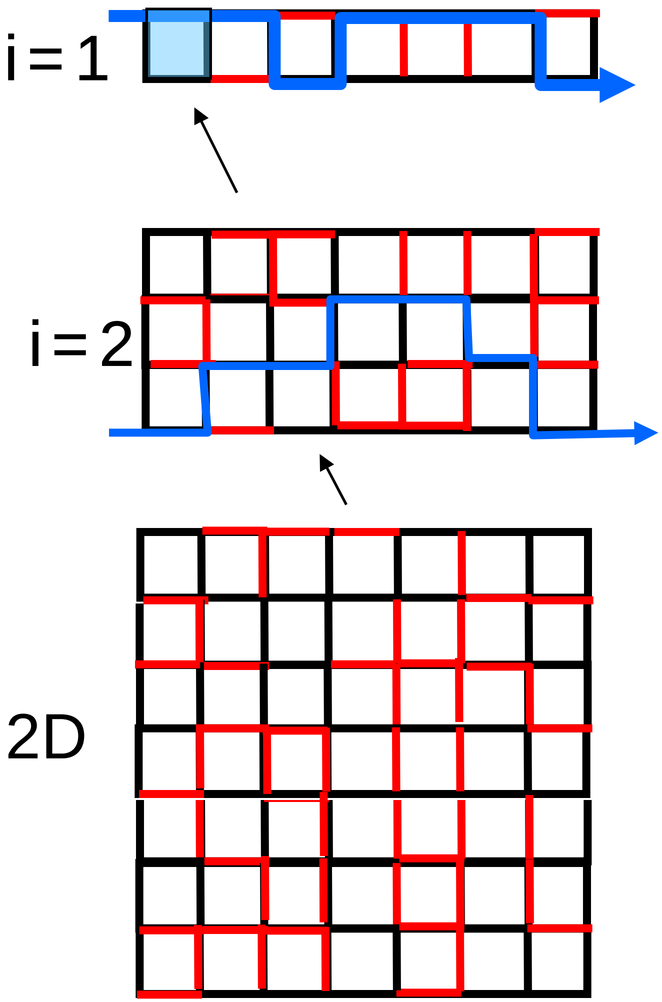
Percolation threshold ...

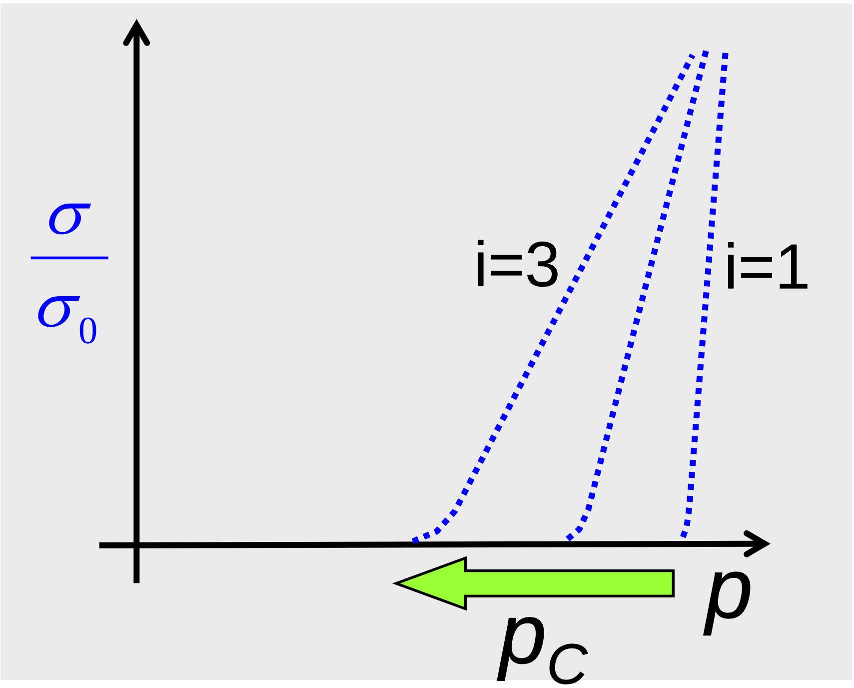
$$p_i = 2p_{i+1}^5 - 5p_{i+1}^4 + 2p_{i+1}^3 + 2p_{i+1}^2$$

$$p_1 \rightarrow p_2 \rightarrow p_3 \cdots \rightarrow p_\alpha$$



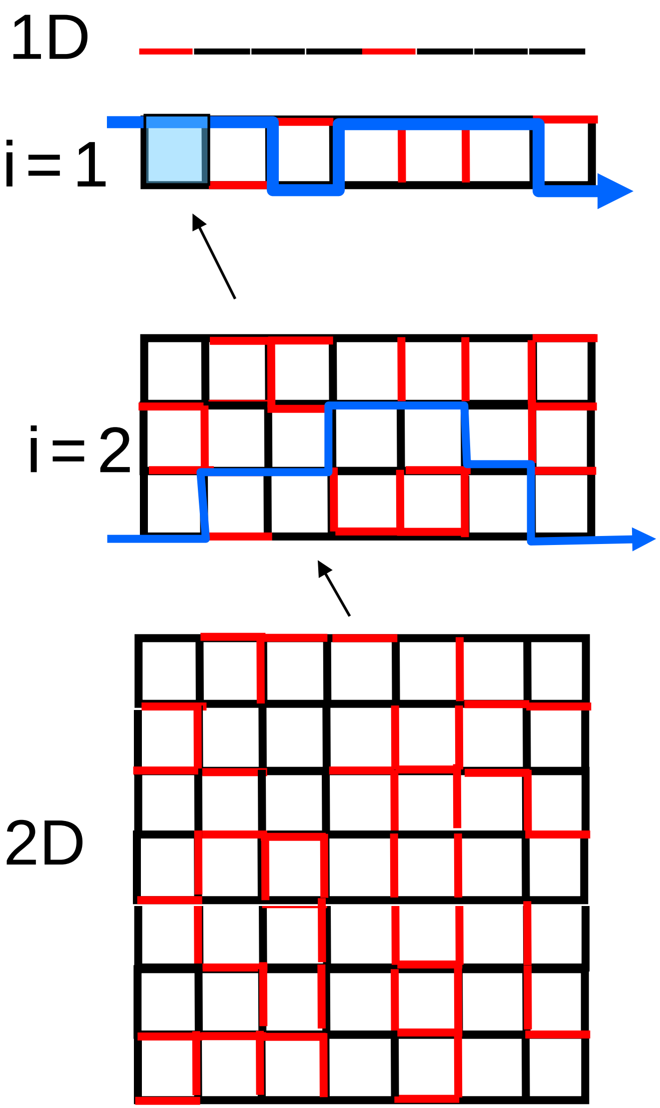
shift in percolation threshold

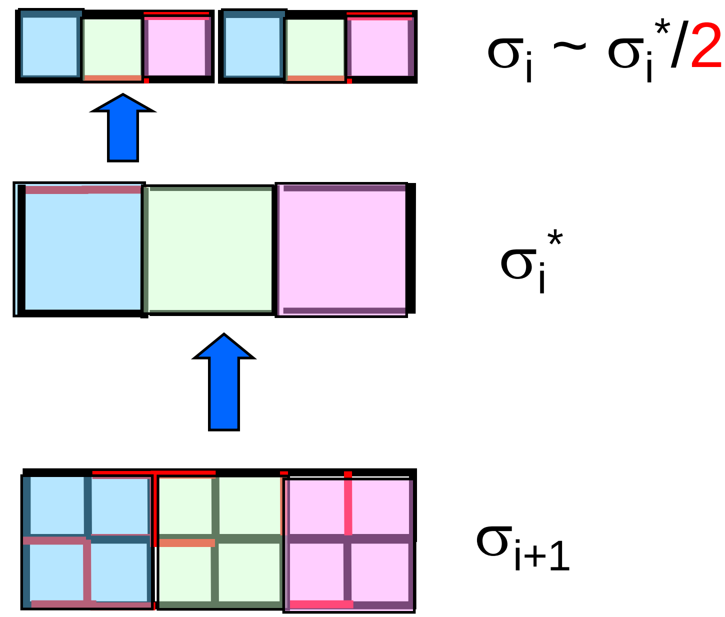




Striping allows shifting of percolation threshold

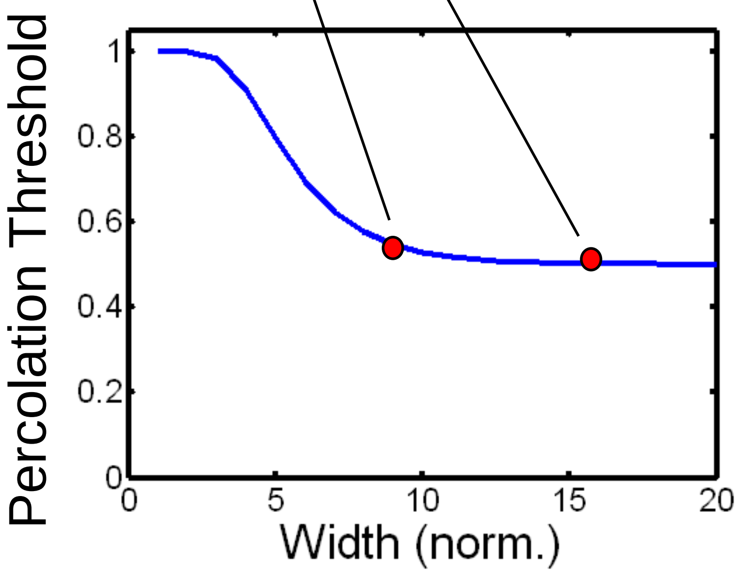
conductance of finite stripes

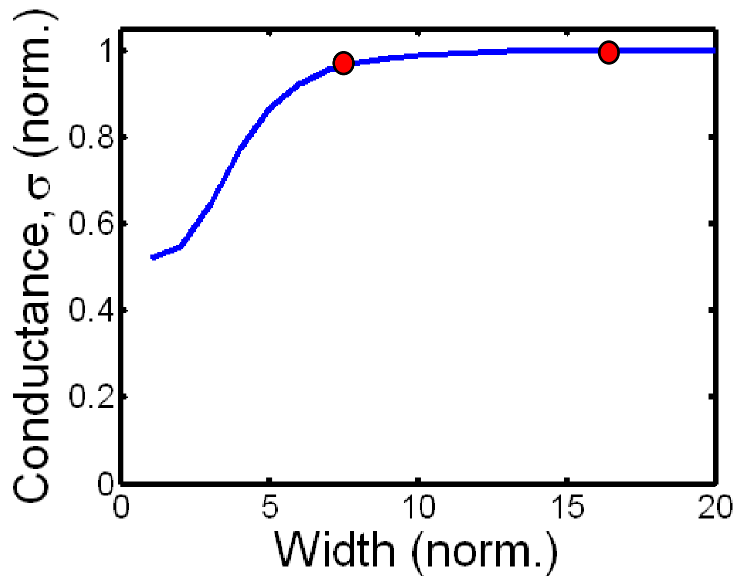




conductance of finite size stripes

$$\frac{\sigma_{i+1}}{2\sigma_i} = \frac{1}{p_i}\left[p_{i+1}^5 + \frac{23}{3} p_{i+1}^4 (1 - p_{i+1}) + 18\, p_{i+1}^3 (1 - p_{i+1})^2 + 4\, p_{i+1}^2 (1 - p_{i+1})^3 \right]$$





Ion/W changes by a factor of ~2

conclusions

- Non-ohmic conduction is a feature of percolative transport. It arises from "length-dependent" effective width in which additional islands can join the percolation network as the path length is shortened.
- The nonlinearly in the short and the long-channel limits are distinct. Given that many problems in device physics involve short channel transistors, one should be careful in using the appropriate formula.
- Quasi-2D percolating network allows tailoring of percolation threshold without affecting the on-current significantly. As we see later, this has remarkable implications for flexible electronics.

Notes and References

This lecture is mostly based on my unpublished results.

I follow D. Stauffer and A. Ahrony, Introduction to Percolation Theory, Revised 2nd Edition, 2003 for the scaling arguments and generalize is appropriately for our specific discussion.

Width dependence and the physics of striping has extensive experimental support in the following publication: N. Pimparkar et al; Nano Research, 2009.

The figures in Slide 14 and 20 are inspired by related figures in “The Physics of Amorphous Solids” by Richard Zallen, 1983.