Introduction of Nano Science and Tech

From Macro Down to Micro and Nano
- Scaling Issues

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Course Website: **www.nanoHUB.org**
Compass.uiuc.edu

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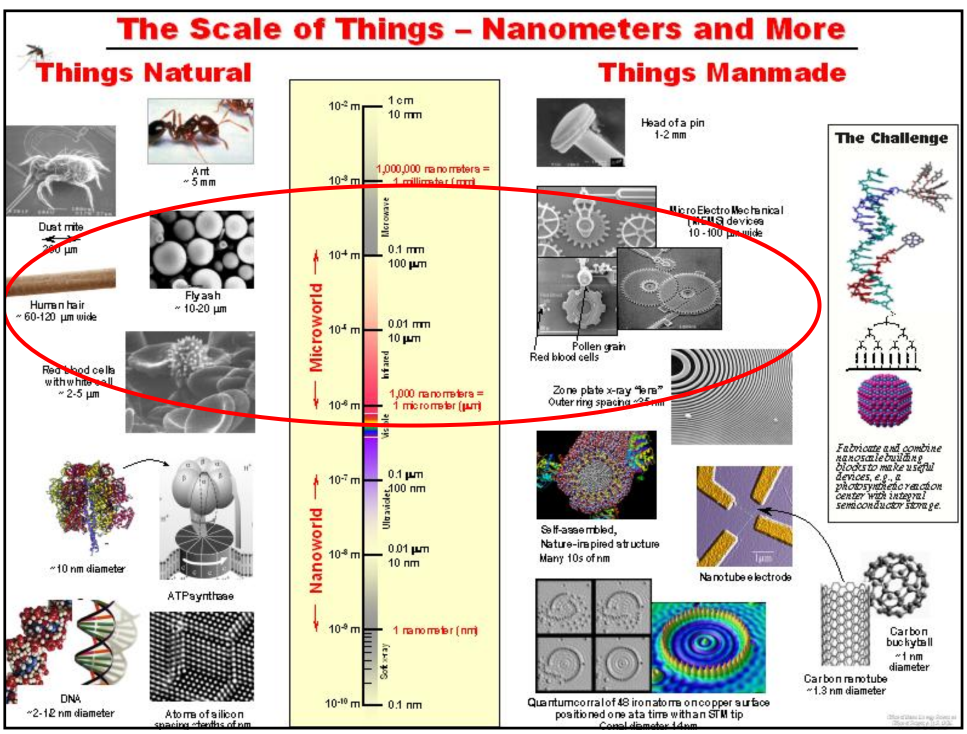

How things behave in small scale?

- E.g. Ants vs. Human

Lift 10 times of its own weight	(Barely 1 times)
Do not injure when falling	(easily injured)
Some can fly	(we wish)
(Dead near fire)	We cook
(Dry cleaning)	We take showers
(? If they are smart?)	We use tools

AntZ

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Problem of Direct Scaling

- We don't have a good intuition of microscale physics. Our common sense in the macro-world often get flawed in the microworld
- What shall we do?
  - Dimensional analysis could reveal "How things behave" in the microscopic world without full scale analysis.
  - In fact, you might have exercised these in many thermal/fluidic problems!

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Scaling at Micro/Nanoscale

- **Classical cases**

- Departure from continuum

Scaling Physics

- Most physical quantities (force, torque, power, etc) scales differently with dimension L

- Assuming geometric similarity
  - – Surface force $\propto$ L (size)
  - – Heat transfer, Diffusion $\propto$ L^2 (area)
  - – compression/elongation stiffness $\propto$ L^2 (area)
  - - Mass $\propto$ L^3 (volume)

Why dimensional analysis?

- Intuition tends to perceive the micro/nanoscale objects as geometrically similar counterparts of those in macroworld

  → hinders conceptual level designs

- As scale reduces, some physical quantities become dominant while others become negligible

Classical Mechanical Scaling Laws

Quantity	Scaling Law
Acceleration	L^{-1}
Energy density	L^{0}
Characteristic time	L
Friction force	L^{2}
Mechanical power	L^{2}
Torque	L^{3}
Energy/work	L^{3}

A flying bee

Standing

– The diameter of a tall tree must increase as 3/2 of its height ($L^{3/2}$)

– Young and small trees appear slender, and old and tall ones appear squat or stunted

Also applies to weight lifting and even to production of nails

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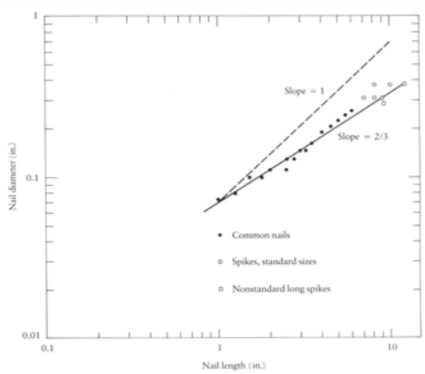
Scaling of Nails

Common nails arranged by size from 60 penny (6 inches) to 2 penny (1 inch).



Nail diameter vs. nail length on a log-log plot, showing the allometric formula d = 0.07/2/3. A broken line of slope 1.0, representing strict isometry, is also shown.

(*On Size and Life* by McMahon and Bonner)

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Jump Height Record

Jumping:

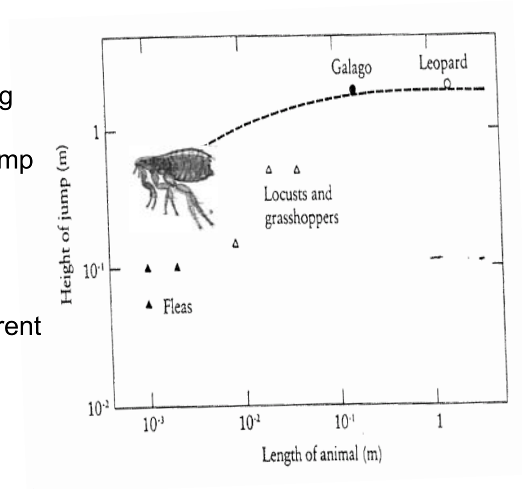
- The work done in leaping up is proportional to the mass and height of the jump

W/M ~ gh

-Power density is almost invariant with size in different animals:

W/ M~L^0

-Thus h tends to be constant irrespective of the animal



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Flying and Rotating

- Flying (birds fly from 10.8 to 20.7 m/sec):
  - Wing length ~ l ~ M $^{1/3}$ and wing area ~ l^2 ~ M $^{2/3}$
  - The characteristic speed for flying varies as l $^{1/2}$ or M $^{0.17}$
  - Drag/lift forces are given by L = 1/2 C_L $\rho A v^2$. This expression has an order of 2 + 2v
  - Of key importance is the lift-to weight ratio (divide by l^3) which is of the order 2v-1
  - Since the lift-to-weight ratio should be invariant with scale to achieve flight a zero order scaling law is needed thus v must be 1/2 to achieve sufficient lift (same result as above)
- Rotation:
  - Mass moment of inertia (I) = $\int r^2 dm$. This is a fifth order expression
  - A small motor will reach top speed in a fraction of a second, large motors may require seconds to reach full speed

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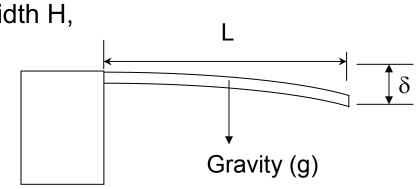
Relative Significance of Forces

e.g. Absolute deflection of a beam (width H, length L) due to its own weight



$$\delta = \frac{3}{2}\frac{\rho g}{E}\left(\frac{L}{H}\right)^2 L^2$$

Percent deflection (Keep L/H the same)

$$\frac{\delta}{L} = \frac{3}{2}\frac{\rho g}{E}\left(\frac{L}{H}\right)^2 L \propto L$$

The relative deformation of 2 um long beam would be 1/1000 of the 2mm beam!

- **In microscale, structures appear to be stiffer against inertia forces**

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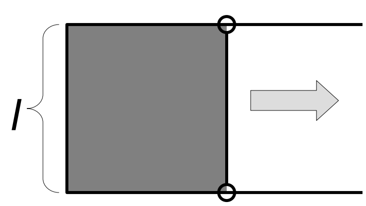
Surface Force

- Adhesion, surface tension

A thermodynamic property



$F = 2\gamma l$

- $dG = \gamma dA$
- Unit : J/m^2 or N/m

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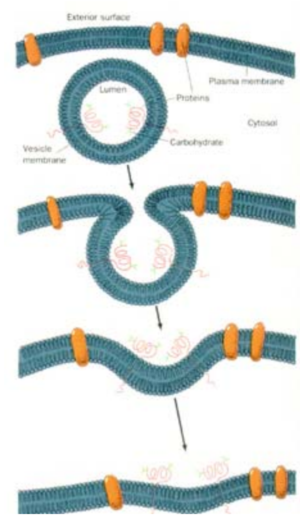
Pressure in Small Bubbles/Droplets

Laplace-Young Equation relates the pressure drop to the surface tension and the dimension of the bubble/droplets:

$$\Delta P = \frac{2\gamma}{r} \quad \text{droplets}$$

$$\Delta P = \frac{4\gamma}{r} \quad \text{bubbles}$$

The relative pressure inside a 50nm vesicle could exceed 400kPa (~4 atm)!

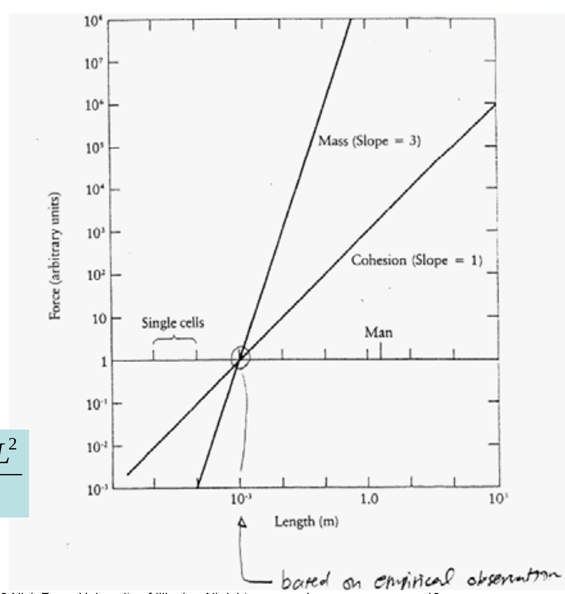


Body Force vs. Surface Force

A water strider floating on a pond without worrying gravity

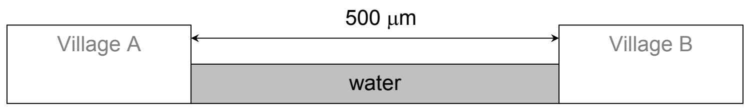
Bond Number

$$Bo = \frac{gravity}{viscous} = \frac{\rho g L^3}{\gamma L} = \frac{\rho g L^2}{\gamma}$$

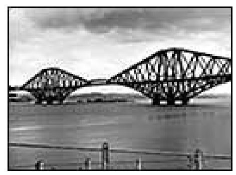


Quiz: a bridge for cells

Asssuming we need to construct a 1 μm thick silicon bridge for the cells to travel between two of their villages, 500μm apart, on top of a drop of water between the two villages. We want to construct a strong bridge that would not collapse easily. Which type of bridge among the followings is the most suitable?



(A) Beam bridge, aspect ratio =~10



(B) Truss bridge, aspect ratio =~100

(C) Suspended bridge, aspect ratio > 100

http://www.pbs.org/wgbh/buildingbig/bridge/basics.html

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Scaling in Fluidics

Reynolds Number:

$$\mathrm{Re} = \frac{\rho \dfrac{U^2}{L} \quad \text{(inertia)}}{\mu \dfrac{U}{L^2} \quad \text{(viscous)}} = \frac{\rho U L}{\mu}$$

In typical cases of micro/nanofluidics

$$\rho = 1g/cm^3, L = 10^{-6} m$$

$$U = 0.01 m/s, \mu = 10^{-3} Pa.s$$

$$\mathrm{Re} = 0.01$$

The flow is highly laminar!

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Consequence of Microfluidics

Incredibly High Flow Resistance $R = \frac{8\mu L}{\pi r^4}$ $\propto$ **L^-3**

Viscous Drag $C_d \sim 1/Re$

Poor Mixing

No turbulent mixing

Slow diffusion at L-L interface only

Branebjerg *et al. MicroTAS Proceedings*, 141 (1995)

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Stopping Distance at Low Re

- Can a Fly Catch a fish like Pelican?

- A viscous drag would prevent that!

$$(3\pi\mu D v)l = \frac{1}{6}(\rho - \rho_0)\pi D^3 v^2$$

- Stopping Distance ~(Re/18)D

A fly could only dive in a few centimeters;

A cell would only slide at a fraction of the diameter (<1um)

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Swimming in Low Re Flow

- It's much like swimming in a slurry
  - Your body move forward on the thrust stroke but go backward at equal distance at return stroke: effect of **reversibility**

Without diffusion, a vesicle released by the cell would be taken back by an reverse motion of the membrane!

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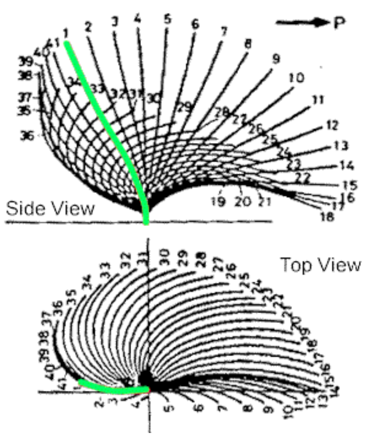
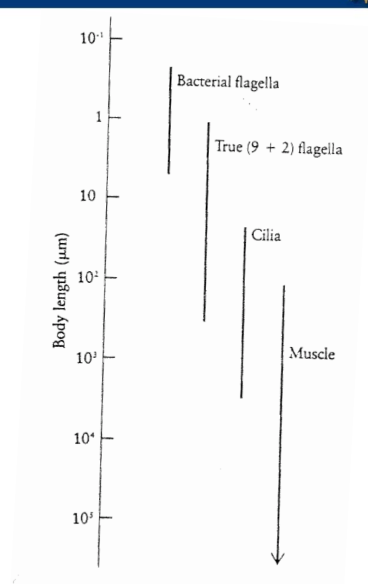
Ciliary Propulsion

A flexible propeller has to be used





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Thermal scaling (Classical)

- Heat capacity ~L^3
  - assuming that the specific heat capacity is to remain constant
- Heat dissipation (by conduction, convection and radiation) ~L^2
- **Biot Number representing the heat resistance:**

$$B = \frac{hL}{\kappa}$$

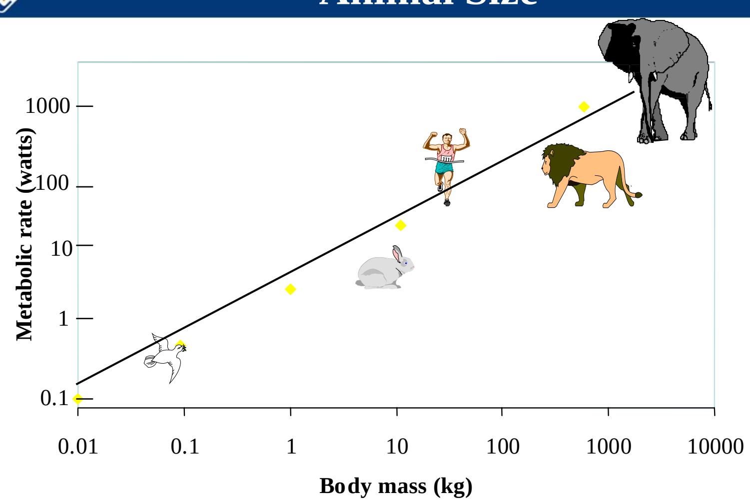
This implies that micro/nanoscale object can be heated faster and cooled faster

- **Peclet Number** showing the ratio between convection to conduction:

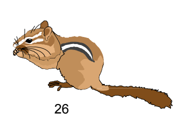
$$Pe = \frac{\rho C_p v L}{\kappa}$$

This implies that convective heat transfer become negligible compared to conduction

Animal Size



Small mammals must keep on eating to stay warm ((heat loss ~ L^2 and heat generation (through eating) is ~ L^3))---insects avoid this problem by staying cold blooded.

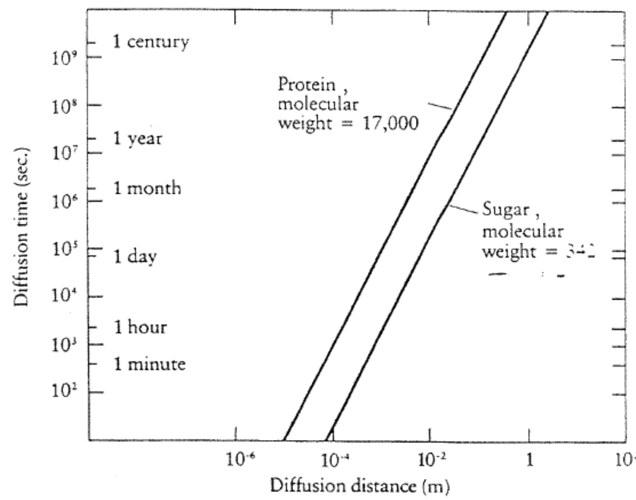


Mass Transport

Diffusion time

~ L^2

- Bacteria has the fastest response to chemical gradient!



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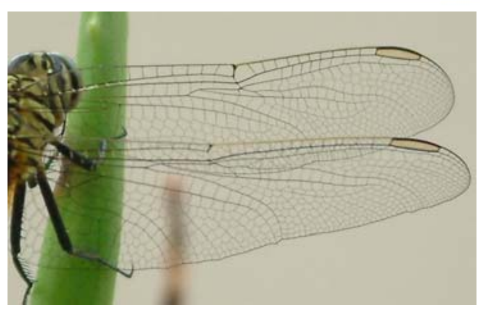
Another note on mass transport

- Dragonfly uses air filled tubes instead of blood to transport oxygen to muscles



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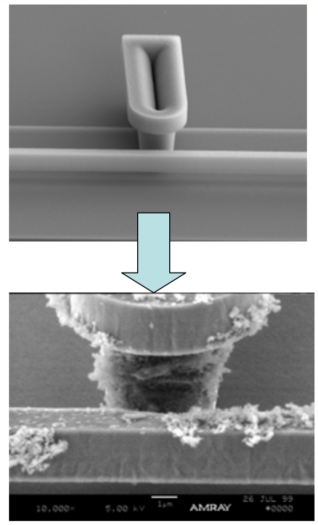
Classical Wear Life

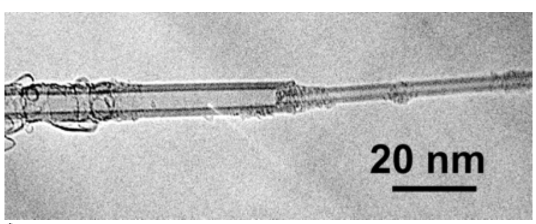
- Unlubricated system
  - Constant interfacial stress
  - Constant speed
  - → Erosion rate = constant
- Wear life $\propto$ thickness/erosion rate $\propto$ L
- 1-cm part → 10 years
  1-nm part → 30 seconds !

Fortunately, that never happened



Michael T. Dugger, Sandia National Lab

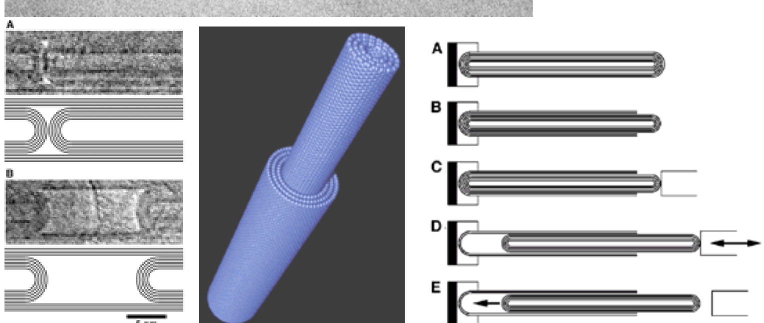
Nanoscale Friction



•Telescoping nanotube segments

•No wear or fatigue

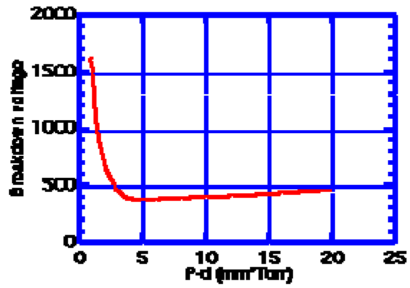
•van der Waals energy-based retraction force

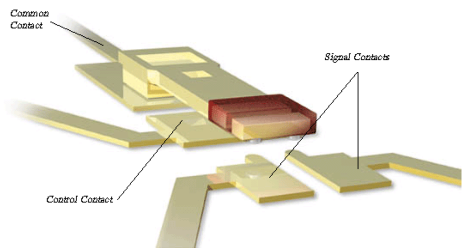


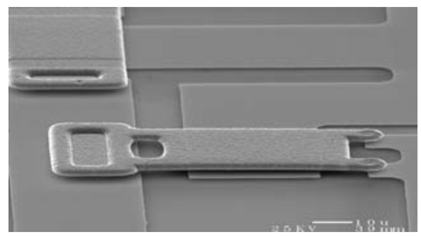
John Cumings, A. Zettl, *Science* 2000

Breakdown of Continuum Theory

- Because of non-linear effects electrostatic devices can be operated in air without breaking down (operation on the left side of the Paschen curve).
- New physics and chemistries to be explored.







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Additional Reading

- Madou Chapter 9, “Scaling, Actuators, and Power in Miniaturized Systems”
- Dimensionless Numbers, Wikipedia “http://en.wikipedia.org/wiki/Category:Dimensionless_numbers”

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