ECE 656: Fall 2009
Lecture 2 Homework SOLUTIONS
(revised 8/28/09)

1) Assume $T = 0\text{K}$ and work out the electron density per unit area for two cases:

   i) A 2D semiconductor with parabolic energy bands and an effective mass of m^* and
   ii) Graphene, where we consider $E > 0$ to be the conduction band. ($E = 0$ is where the bands cross, the so-called Dirac point.)

   1a) Express your two answers in terms of the Fermi wavevector and show that they are the same.

   1ba) Express your two answers in terms of the Fermi energy, and show that they are different.

2) Assume a finite temperature and work out the sheet carrier densities for a 2D parabolic band semiconductor and for electrons in the conduction band ($E > 0$) of graphene.

3) Assume $T = 0\text{K}$ and work out the average +x-directed velocity for electrons in:

   i) A 2D emiconductor with parabolic energy bands and
   ii) In the conduction ($E > 0$) of graphene.

   Your answer should be in terms of the Fermi energy, E_F.

HW2 Solutions

1a)

$$n_s = \frac{\pi k_F^2}{(2\pi)^2} \times 2 = \frac{k_F^2}{2\pi}$$ independent of $E(k)$

1b)

$$E = \frac{\hbar^2 k^2}{2m^*}$$

$$E_F = \hbar^2 k_F^2 / 2m^*$$

$$k_F = \sqrt{2m^* E_F} / \hbar$$

$$n_s = \frac{m^*}{\pi \hbar^2} E_F$$

$$|E| = \hbar v_F k$$

$$E_F = \hbar v_F k_F$$

$$k_F = E_F / \hbar v_F$$

$$n_s = \frac{E_F^2}{2\pi \hbar^2 v_F^2} \times 2$$

valley degen.

2)

parabolic bands:

$$n_s = \int_0^{\infty} \frac{m^*}{\pi\hbar^2} \cdot \frac{dE}{1+e^{(E-E_F)/k_BT}}$$

$\eta_F = E_F/k_BT$
$\eta = E/k_BT$
$dE = k_BT\,d\eta$

$$= \frac{m^*}{\pi\hbar^2} \cdot k_BT \underbrace{\int_0^{\infty} \frac{d\eta}{1+e^{\eta-\eta_F}}}_{\mathcal{F}_0(\eta_F) = \ln(1+e^{\eta_F})}$$

$$n_s = \frac{m^* k_BT}{\pi\hbar^2} \mathcal{F}_0(\eta_F) \checkmark$$

graphene: $$n_s = \int_0^{\infty} \frac{2E}{\pi\hbar^2 v_F^2} \cdot \frac{dE}{1+e^{(E-E_F)/k_BT}}$$

$$n_s = \frac{2}{\pi\hbar^2 v_F^2} \cdot (k_BT)^2 \cdot \underbrace{\int_0^{\infty} \frac{\eta\,d\eta}{1+e^{\eta-\eta_F}}}_{\mathcal{F}_1(\eta_F)}$$

$$n_s = \frac{2}{\pi}\left(\frac{k_BT}{\hbar v_F}\right)^2 \mathcal{F}_1(\eta_F) \checkmark$$

3)

i)

$$\langle v(E)\rangle = \frac{\int_0^{E_F} \cdot \sqrt{2E/m^*}\, dE}{E_F} = \frac{\sqrt{\frac{2}{m^*}} \cdot \frac{2}{3} E_F^{3/2} \Big|_0^{E_F}}{E_F}$$

$$\langle v(E)\rangle = \sqrt{\frac{2E_F}{m^*}} \cdot \frac{2}{3} = \frac{2}{3} v_F$$

v_F = Fermi velocity

but we also need to ave over θ

$$\langle v(\theta)\rangle = \frac{\int_{-\pi/2}^{\pi/2} v \cos\theta \, d\theta}{\pi} = \frac{2}{\pi} v$$

so $\overline{v} = \frac{4}{3\pi} v_F$ ✓

ave over energy <u>and</u> angle

ii)

$v(E) = v_F$ independent of E

$\overline{v} = \frac{2}{\pi} v_F$ ✓

3)