ECE 656: Fall 2009
Lecture 3 Homework SOLUTIONS
(Revised 8/29/09)

1) According to our general model,

$$G = \frac{2q}{h}\int \gamma(E)\pi\frac{D(E)}{2}\left(-\frac{\partial f_0}{\partial E}\right)dE .$$

As will discussed in Lecture 4, at low temperatures, $-\partial f_0/\partial E = \delta(E_F)$, so

$$G = \frac{2q^2}{h}\gamma(E_F)\pi\frac{D(E_F)}{2} \qquad (1)$$

The broadening is given by

$$\gamma(E_F) = \frac{\hbar}{\tau(E_F)}$$

where $\tau(E_F)$ is the transit time for charge to cross the device.

1a) Show that for ballistic transport in a 1D system at low temperature, (1) becomes

$$G = \frac{2q^2}{h}M(E_F)$$

where

$M(E_F) = \frac{h}{2}\upsilon(E_F)\left[D(E_F)/2L\right]$ where $D(E_F)/2L$ is the density of states per unit length per spin. This exercise shows that the number of modes is proportional to velocity times density of states.

1b) Show that for diffusive transport, (1) becomes

$$G = q^2\left[D(E_F)/L\right]D_n(E)\frac{1}{L} = \sigma_{1D}\frac{1}{L}$$

where

$$\sigma_{1D} = q^2\left[D\left(E_F\right)/L\right]D_n\left(E\right) \qquad (2)$$

Equation (2) is a standard, well-known result for diffusive transport that is usually derived by solving the Boltzmann Transport Equation.

2) The ballistic conductance is often derived from a k-space treatment, which writes the current from left to right as

$$I^+ = \frac{1}{L}\sum_{k>0} q v_x f_0(E_{F1})$$

and the current from right to left as

$$I^- = \frac{1}{L}\sum_{k<0} q v_x f_0(E_{F2})$$

The net current is the difference between the two.

2a) Work out the expression for current in 1D assuming T = 0K, and show that the resulting conductance is ($2q^2/h$), as expected.

2b) Work out the expression for the current in 2D assuming T = 0K, and show that the result is $G = \left(2q^2/h\right)M\left(E_F\right)$. You may assume parabolic energy bands.

HW3 Solution

1a) $$G = \frac{2q^2}{h} \cdot \gamma(E_F)\,\pi\,\frac{D(E_F)}{2} = \frac{2q^2}{h}\,\frac{\hbar}{(L/v(E))}\,\pi\,\frac{D(E_F)}{2}$$

$$G = \frac{2q^2}{h}\left[\frac{h}{2}\,v(E_F)\,\frac{D(E_F)}{2L}\right] \qquad v(E_F) = \sqrt{2E_F/m^*}$$

$$= \frac{2q^2}{h}\,M(E_F) \checkmark$$

1b) $$M(E) = \gamma(E_F)\,\pi\,D(E_F)/2$$

$$= \frac{\hbar}{\langle \tau \rangle}\,\pi\,\frac{D(E_F)}{2} \qquad \langle \tau \rangle = L^2/2D_n(E)$$

$$= \frac{h}{2\cancel{\pi}}\,2D_n(E_F)\,\cancel{\pi}\,\frac{D(E_F)}{2L}\,\frac{1}{L}$$

$$G = \frac{\cancel{2}q^2}{\cancel{h}} \cdot \frac{\cancel{h}}{\cancel{2}}\,\cancel{2}D_n(E_F)\,\frac{D(E_F)}{\cancel{2}L}\,\frac{1}{L}$$

$$G = \underbrace{\left[q^2\,\frac{D(E_F)}{L} \cdot D_n(E_F)\right]}_{\sigma_{1D}\ \checkmark} \cdot \frac{1}{L}$$

2a)

$$I^+ = \frac{1}{L} \cdot \frac{L}{2\pi} \cdot 2 \int_0^{\infty} dk \, q v_x f_0(E_{F_1})$$

$$= \frac{1}{\pi} \int_0^{\infty} \frac{d(\hbar k)}{dE} \, dE \, q v_x f_0(E_F)$$

$$= \frac{q}{\pi} \cdot \frac{1}{\hbar} \int_0^{E_{F_1}} 1 \, dE = \frac{2q}{h} E_{F_1}$$

similarly $I^- = \frac{2q}{h} E_{F2}$

$$I = I^+ - I^- = \frac{2q}{h} \cdot (E_{F_1} - E_{F_2}) = \frac{2q^2}{h} \cdot V \quad \checkmark$$

2b)

$$\underset{(A/cm)}{J^+} = \frac{1}{A} \cdot \frac{A}{(2\pi)^2} \cdot 2 \int_0^{\infty} \int_{-\pi/2}^{\pi/2} k \, dk \, d\theta \, q v \cos\theta \, f_0(E_{F_1})$$

$$= \frac{q}{2\pi^2} \cdot \int_0^{\infty} k \, dk \underbrace{\int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta}_{2} \cdot v f_0(E_{F_1})$$

$$I^+ = W J^+$$

2b)

$$I^+ = \frac{qW}{\pi^2}\int k\,dk\,\frac{\hbar k}{m^*}\,f_0(k)$$

$$= \frac{q\hbar W}{\pi^2 m^*}\int_0^{k_F} k^2\,dk = \frac{q\hbar W}{\pi^2 m^*}\,\frac{k_F^3}{3}$$

$$\frac{\hbar^2 k_F^2}{2m^*} = E_F, \quad k_F = \frac{\sqrt{2m^* E_F}}{\hbar} \quad k_F^3 = \frac{(2m^* E_F)^{3/2}}{\hbar^3}$$

$$I^+ = q\,\frac{W\,(2m^* E_F)^{3/2}}{3\pi^2\hbar^2 m^*}$$

$$I^- = I^+ - \frac{\partial I^+}{\partial E_F}\Delta E_F \qquad I = I^+ - I^- = \frac{\partial I^+}{\partial E_F}\cdot(qV)$$

$$\frac{\partial I^+}{\partial E_F} = q\,\frac{W k_F}{\pi\hbar} = \frac{2q}{h}\cdot\frac{W k_F}{\pi}$$

$$I = \frac{2q^2}{h}\underbrace{\left(\frac{W k_F}{\pi}\right)}_{M(E)} V$$

$$= \frac{W}{(\lambda(E_F)/2)}$$

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