ECE-656: Fall 2009

Lecture 5: 1D Resistors

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outline

1) **Review**

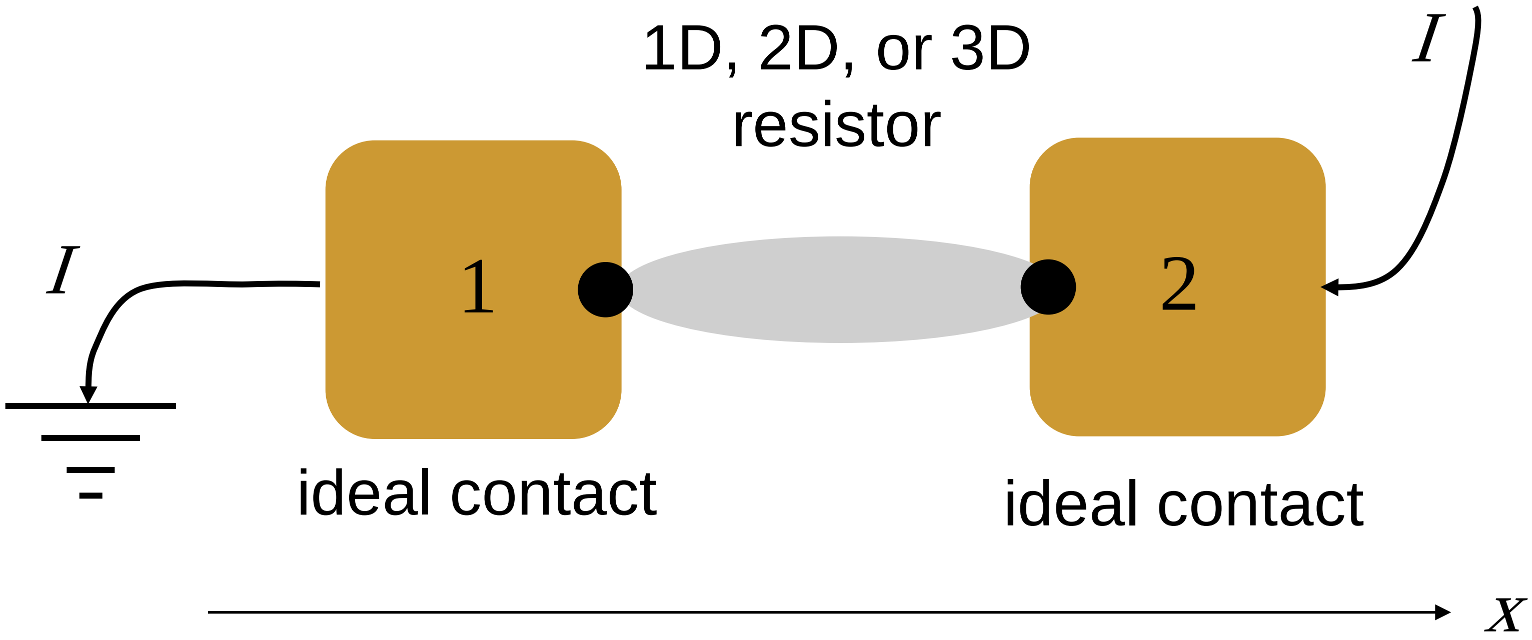
2) 1D ballistic resistors

3) 1D diffusive resistors

4) Discussion

5) Summary

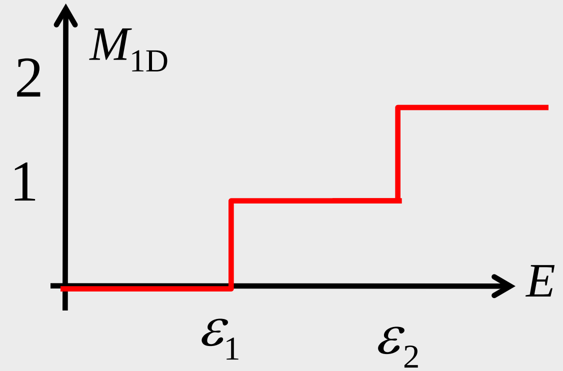
Landauer picture



Current is positive when it flows **into** contact 2 (i.e. positive current flows in the -*x* direction).

$$I = \frac{2q^2}{h}\left(\int T(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE\right)V$$

modes



$$M_{1D}(E) = \sum_i \Theta(E - \varepsilon_i)$$

independent
of
bandstructure

transmission

$$T(E) = \frac{\lambda(E)}{\lambda(E) + L}$$

λ is the "mean-free-path for backscattering" and is determined by the physics of carrier scattering from phonons, ionized impurities, etc.

$$\lambda(E) \propto \upsilon(E)\tau(E)$$

The <u>backscattering</u> mean-free-path is proportional to $\upsilon(E)\tau(E)$. The constants depend on dimensionality.

transport regimes

1) Ballistic: $\lambda >> L,\ T \approx 1$

2) Quasi-Ballistic: $\lambda \approx L,\ T < 1$

$$T(E) = \frac{\lambda(E)}{\lambda(E) + L}$$

3) Diffusive: $\lambda << L,\ T << 1$

driving forces for transport

$$I = \frac{2q}{h}\int T(E)M(E)\left(f_1 - f_2\right)dE$$

Differences in occupation, *f*, produce current.

$$\left(f_1 - f_2\right) \approx f_1 - \left(f_1 + \frac{\partial f_1}{\partial E_F}\Delta E_F\right) = -\frac{\partial f_1}{\partial E}\Delta E_F$$

Assumes $T_1 = T_2$.

but differences in temperature also produce differences in *f* and can, therefore, drive current (thermoelectric effects).

$$\Delta E_F = -q\Delta V = -q\left(V_2 - V_1\right)$$

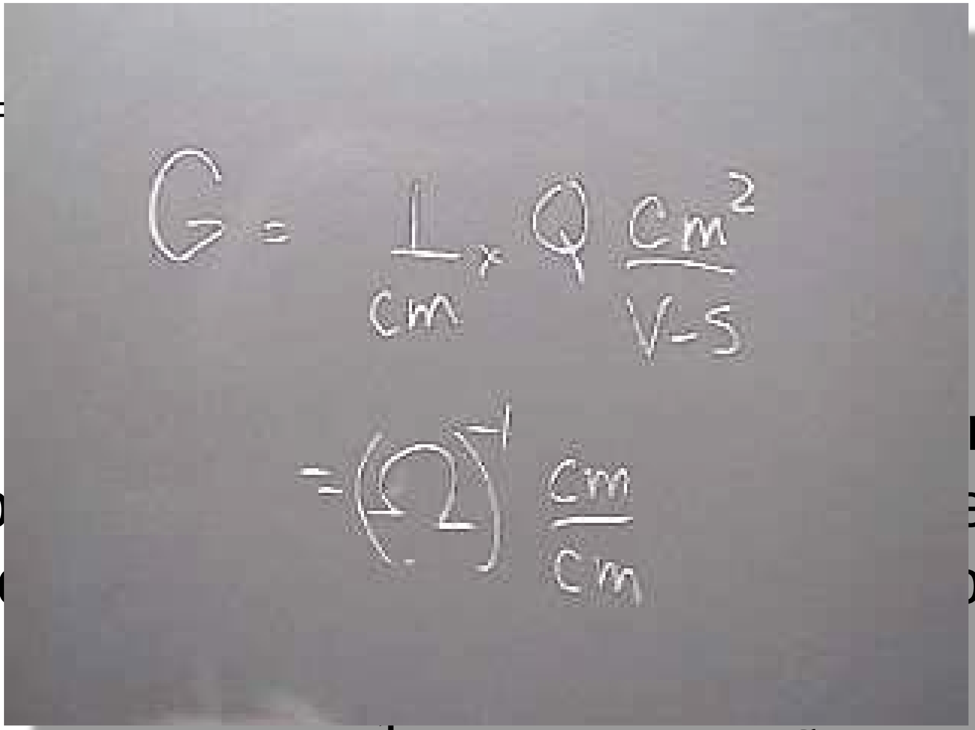
$$\Delta T = T_2 - T_1$$

←this lecture

goals for this lecture

$$G = \frac{I}{V} = \quad\quad (T_1 = T_2)$$

To evaluate



We will see resistance that is indep e the result to the expecte onductor theory:

$$G_{1D} = n_L q \mu_n \frac{1}{L} \qquad \mu_n = \frac{q\tau_0}{m^*}$$

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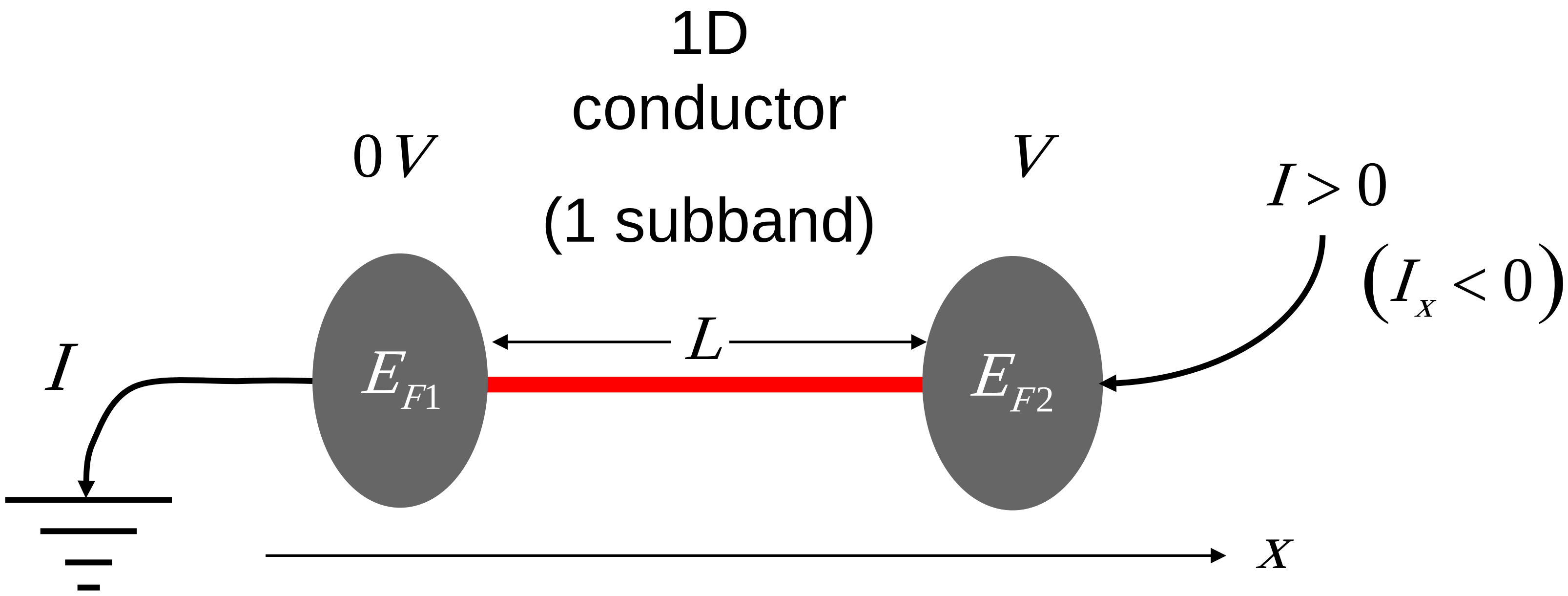
1D resistors



1) T = 0K ballistic

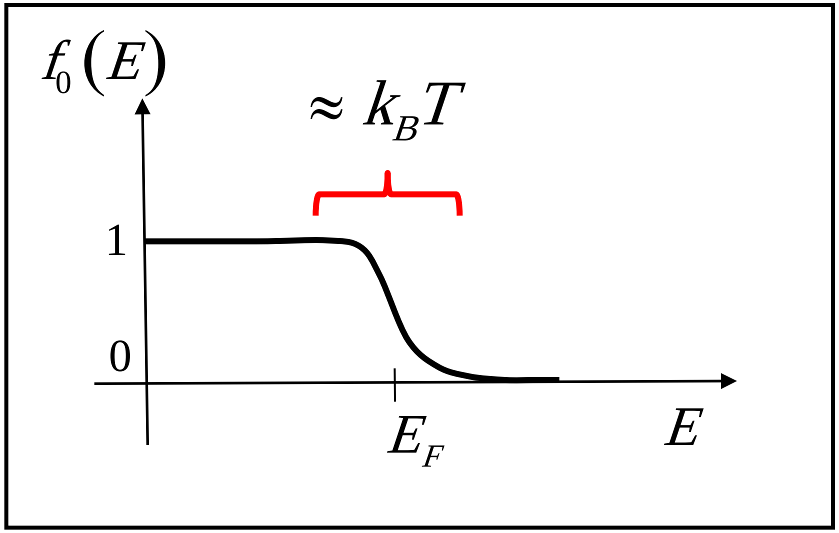
2) T > 0K general

computing G

$$G = \frac{2q^2}{h} \int_{-\infty}^{+\infty} T(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$f_0(E) = \frac{1}{1 + e^{(E - E_F)/k_B T}}$$

$$M_{1D}(E) = \Theta(E - \varepsilon_1)$$



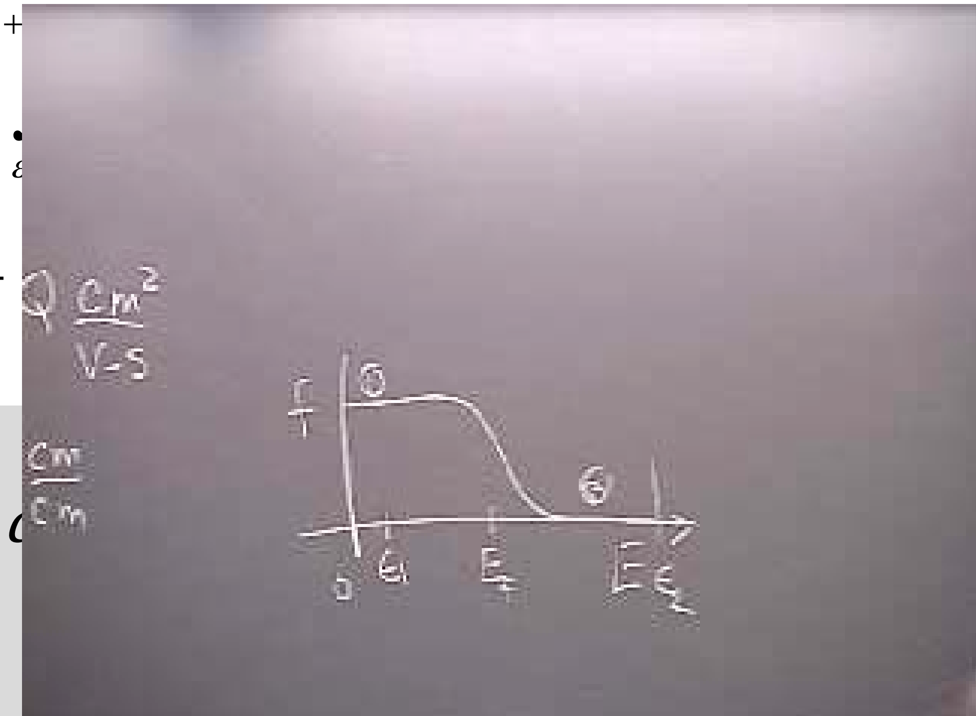
$$T(E) = 1$$ (ballistic)

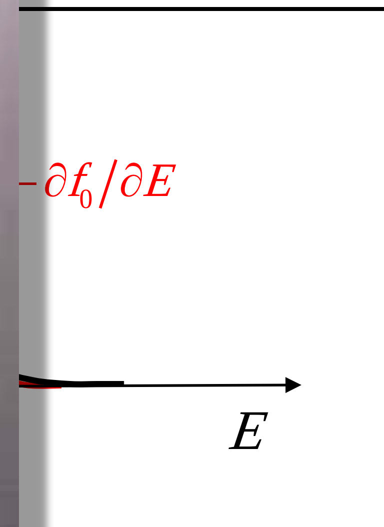
$$G_{1D} = \frac{2q^2}{h} \int_{\varepsilon_1}^{+\infty} \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$T = 0\text{K}$

$$G_{1D} = \frac{2q^2}{h} \int_{\varepsilon_1}^{+\infty} \ldots$$

$$G_{1D} = \frac{2q^2}{h} \ldots$$





$$\int_{\varepsilon_1}^{+\infty} \left(-\frac{\partial f_0}{\partial E}\right) d\ldots$$

$$= -(0-1)$$

$$= 1$$

$$\left(-\frac{\partial f_0}{\partial E}\right) \approx \delta(E_F)$$

T = 0K ballistic conductance

$$G_{1D} = \frac{2q^2}{h}$$

For M subbands:

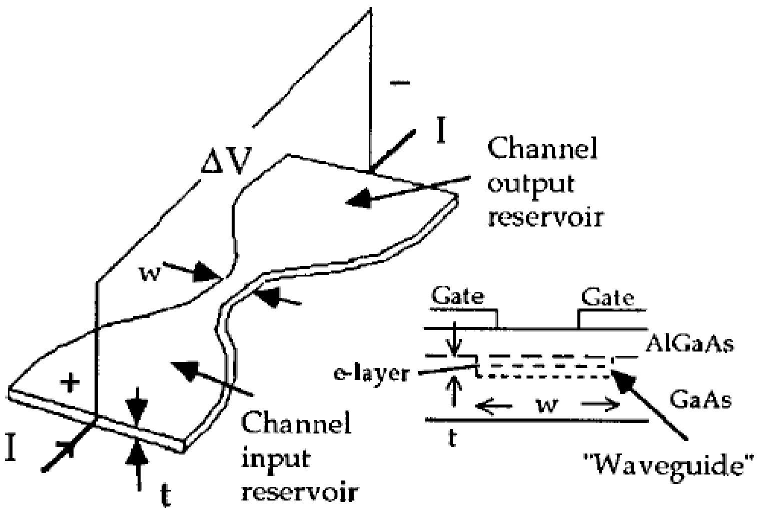
$$G_{1D} = \frac{2q^2}{h} M\left(E_F\right)$$

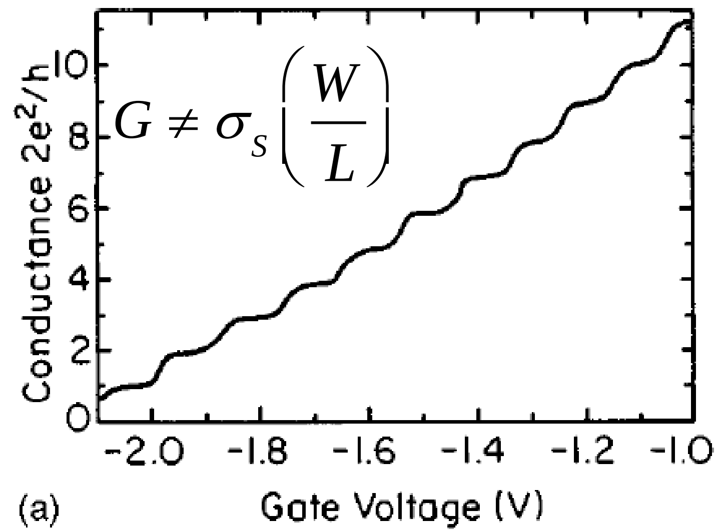
$$G_{1D} = \frac{2q^2}{h} M$$ "quantized conductance"

$$R_{1D} = \frac{1}{M}\frac{h}{2q^2} = \frac{12.8\,\text{kž}}{M}$$

Resistance is independent of length, L, **and** electron density.

quantized conductance





W -->

Conductance is quantized in units of $2q^2/h$ - it is not proportional to *W*.

The conductance is finite as L --> 0.

B. J. van Wees, H. van Houten, C. W. J. Beenakker, J. G. Williamson, L. P. Kouwenhoven, D. van der Marel, and C. T. Foxon, ‘‘Quantized conductance of point contacts in a two-dimensional electron gas,’’ *Phys. Rev. Lett.* **60**, 848–851,1988.

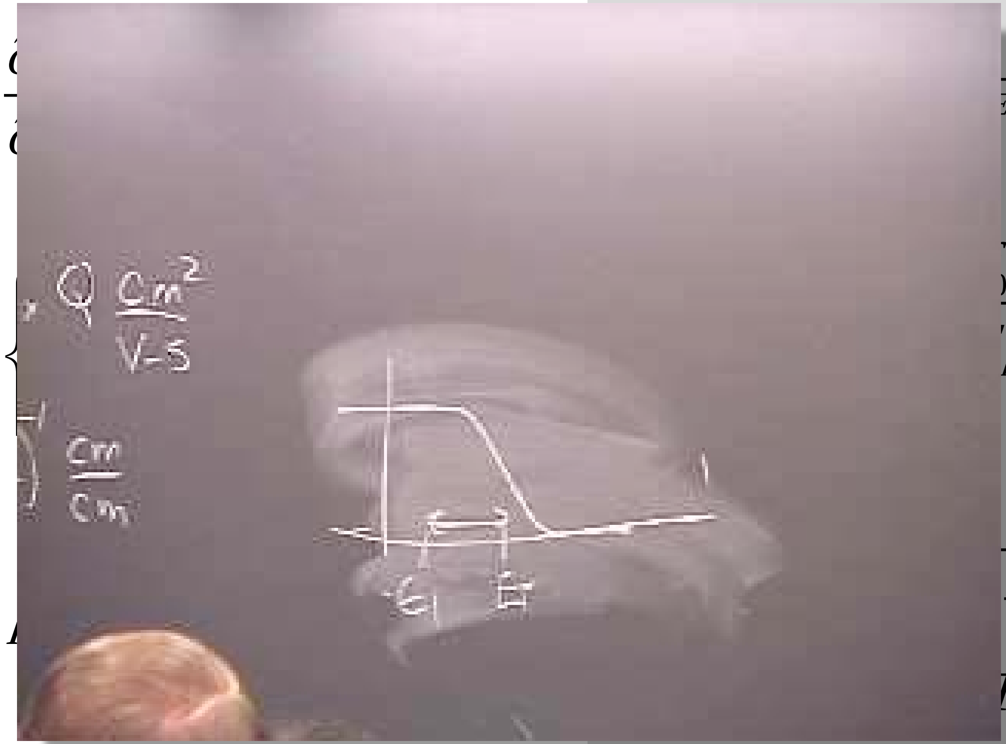
1D resistors: $T > 0$

$$G_{1D} = \frac{2q^2}{h}\int_{\varepsilon_1}^{\infty}\left(-\frac{\partial}{\partial} \cdots \right.$$

$$G_{1D} = \frac{2q^2}{h}\frac{\partial}{\partial E_F}\left\{ \cdots \right.$$

$$G_{1D} = \frac{2q^2}{h}\frac{\partial}{\partial E_F} \cdots$$

$$G_{1D} = \frac{2q^2}{h}\mathcal{F}_{-1}\left(\eta_F\right)$$



$$\cdots \frac{1}{1 + e^{(E-E_F)/k_BT}}\,dE$$

$$\cdots T\mathcal{F}_0\left(\eta_F\right)$$

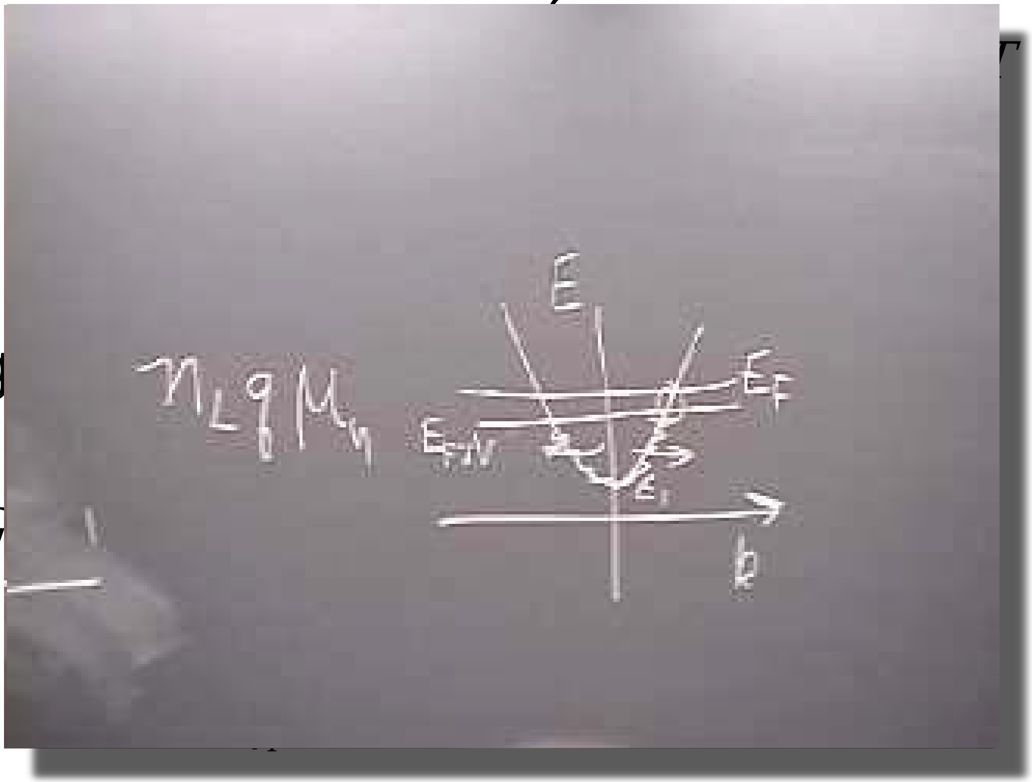
$$\eta_F = \left(E_F - \varepsilon_1\right)/k_BT$$

$$\frac{\partial \mathcal{F}_0\left(\eta_F\right)}{\partial \eta_F} = \mathcal{F}_{-1}\left(\eta_F\right)$$

1D resistors: $T > 0$

1) non-deg

G



2) degene

$$G_{1D} = \frac{2q^2}{h}$$

independent of n_L

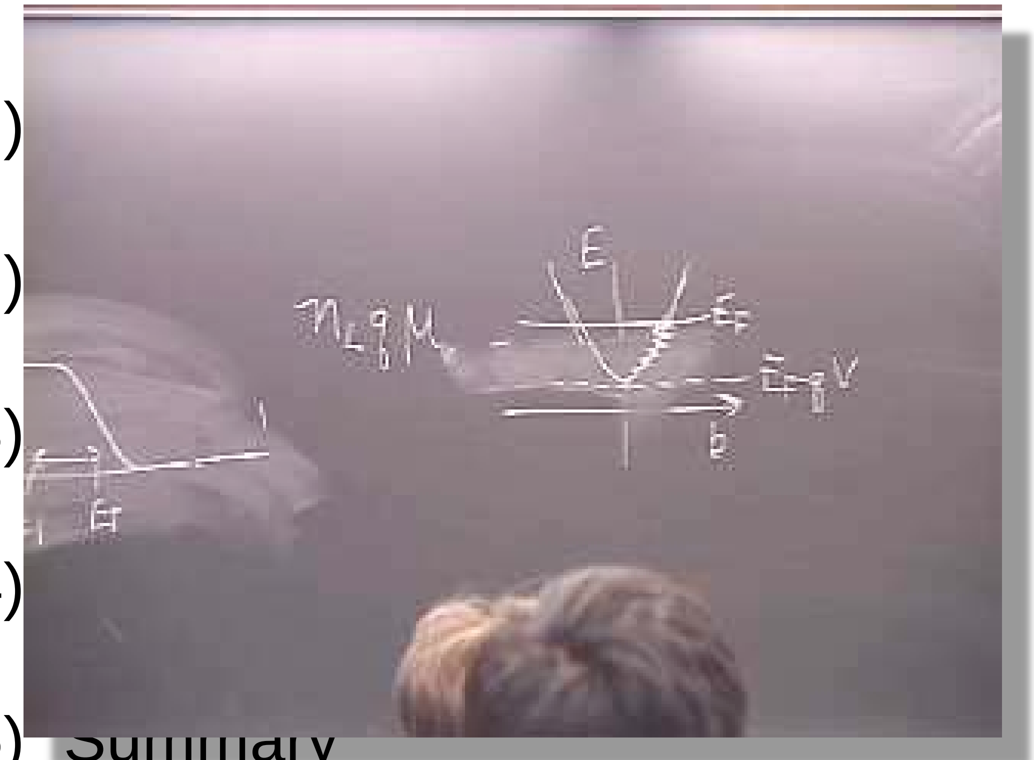
outline

1)

2)

3)

4)



5) Summary

transport in the presence of scattering

$$G = \frac{2q^2}{h} \int_{-\infty}^{+\infty} T(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$M_{1D}(E) = \Theta(E - \varepsilon_1)$$

$$G_{1D} = \frac{2q^2}{h} \int_{\varepsilon_1}^{+\infty} T(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

general:

$$T(E) = \lambda / (\lambda + L)$$

ballistic:

$$\lambda >> L \quad T(E) \rightarrow 1$$

diffusive:

$$\lambda << L \quad T(E) \rightarrow \frac{\lambda}{L} << 1$$

quasi-ballistic:

$$\lambda \approx L \quad 0 < T(E) < 1$$

diffusive transport (T = 0K)

$$G_{1D} = \frac{2q^2}{h} \int_{\varepsilon_1}^{+\infty} T(E)\left(-\frac{\partial f_0}{\partial E}\right) dE$$

$$\left(-\frac{\partial f_0}{\partial E}\right) \approx \delta(E_F)$$

$$G_{1D} = \frac{2q^2}{h} T(E_F)$$

$$\lambda << L \quad T(E) \to \frac{\lambda}{L} << 1$$

diffusive transport

$$G_{1D} = \frac{2q^2}{h} \frac{\lambda(E_F)}{L}$$

$$G_{1D} \propto \frac{1}{L}$$

diffusive transport (T > 0K)

$$\lambda << L \quad T(E) \rightarrow \frac{\lambda}{L} << 1$$

$$G_{1D} = \frac{2q^2}{h} \int_{\varepsilon_1}^{+\infty} T(E)\left(-\frac{\partial f_0}{\partial E}\right) dE$$

$$G_{1D} = \frac{2q^2}{h} \frac{\int_{\varepsilon_1}^{+\infty} \frac{\lambda(E)}{L}\left(-\frac{\partial f_0}{\partial E}\right) dE}{\int_{\varepsilon_1}^{+\infty} \left(-\frac{\partial f_0}{\partial E}\right) dE} \int_{\varepsilon_1}^{+\infty} \left(-\frac{\partial f_0}{\partial E}\right) dE$$

$$G_{1D} = \frac{2q^2}{h} \frac{\langle\langle \lambda(E) \rangle\rangle}{L} \mathcal{F}_{-1}(\eta_F)$$

$$\langle\langle \lambda(E) \rangle\rangle = \frac{\int_{\varepsilon_1}^{+\infty} \lambda(E)\left(-\frac{\partial f_0}{\partial E}\right) dE}{\int_{\varepsilon_1}^{+\infty} \left(-\frac{\partial f_0}{\partial E}\right) dE}$$

$$G = G_B \frac{\langle\langle \lambda(E) \rangle\rangle}{L}$$

$$G << G_B$$

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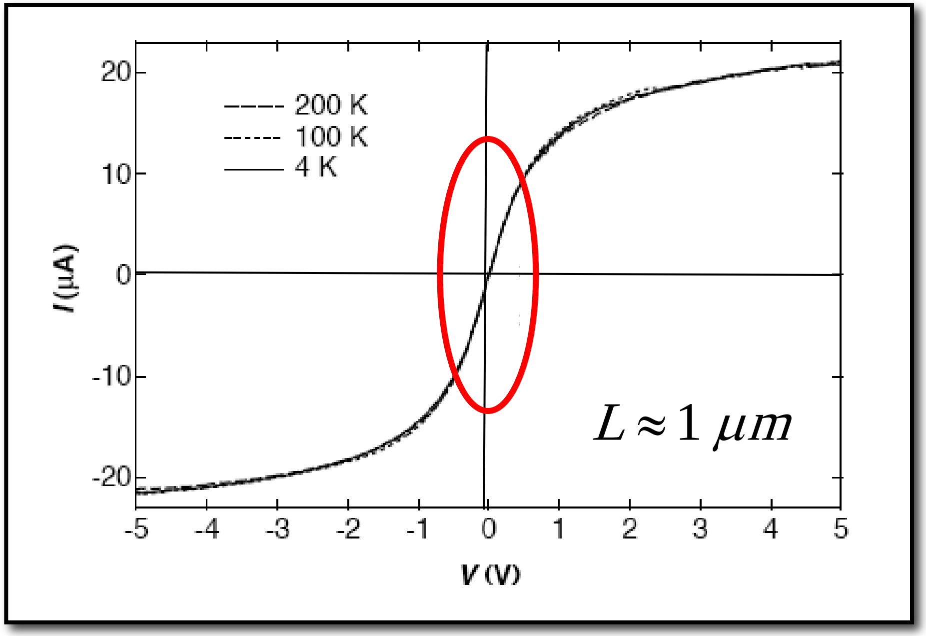
example: low-field transport in metallic CNTs

$$G_{1D} = \frac{\Delta I}{\Delta V} = \frac{22\ \mu A}{1.0\ V} = 22\ \mu S$$

$$G_B = \frac{4q^2}{h} = 154\ \mu S \quad (g_V = 2)$$

$$G_{1D} = \frac{4q^2}{h}\frac{\lambda_0}{\lambda_0 + L}$$

$$\lambda_0 \approx 167\ \text{nm} << L$$



Zhen Yao, Charles L. Kane, and Cees Dekker, "High-Field Electrical Transport in Single-Wall Carbon Nanotubes," *Phys. Rev. Lett*., **84**, 2941-2944, 2000.

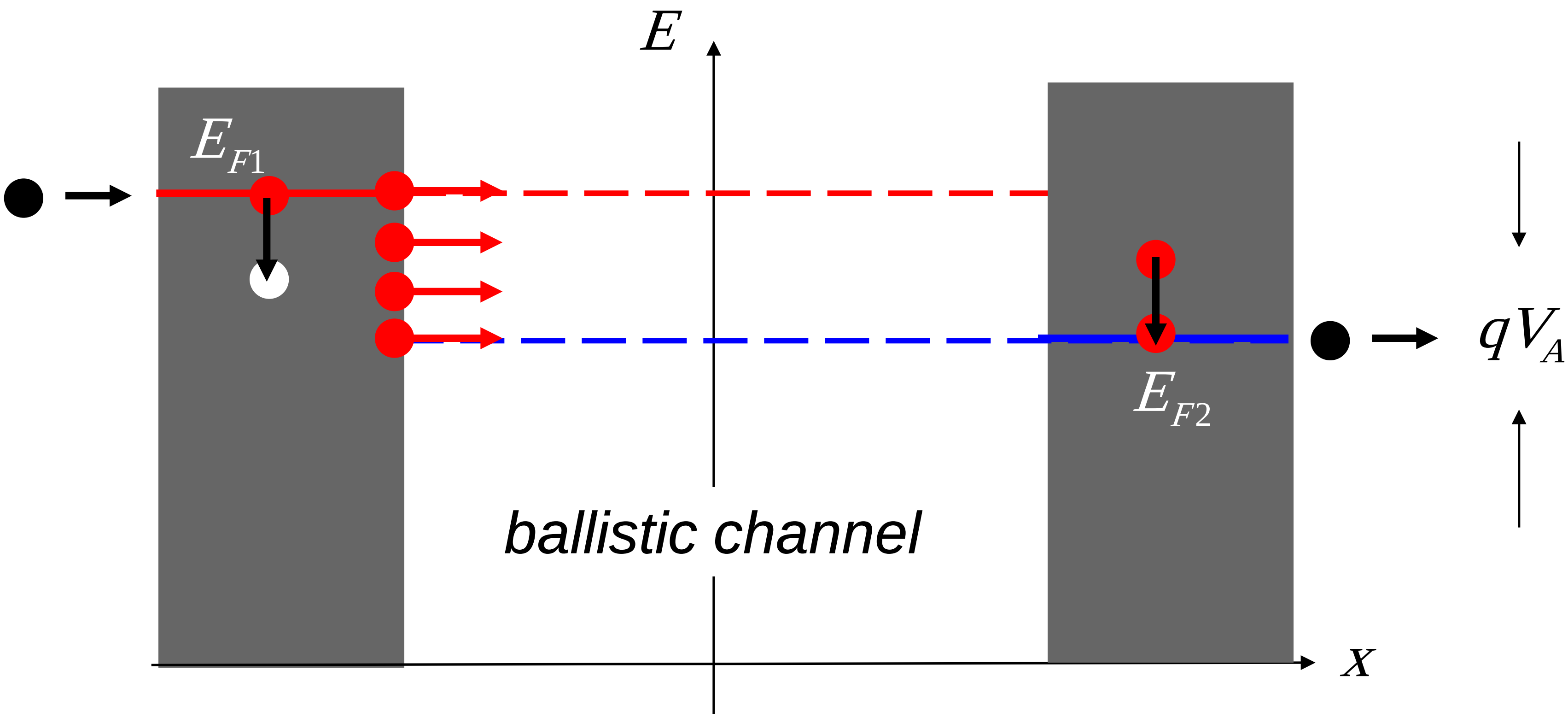
power dissipation in a ballistic resistor

$$P_D = IV = GV^2 = V^2 / R$$

Where is the power dissipated in a ballistic resistor?

Answer: In the two contacts.

power dissipation in a ballistic resistor



dissipation occurs in the contacts

relation to conventional expression

Conventionally: $$G_{1D} = n_L q \mu_n \frac{1}{L}$$

Question: Under what conditions is conductance proportional to carrier density (and how is the mobility related to the mfp)?

discussion

Conventionally: $G_{1D} = n_L q \mu_n \frac{1}{L}$

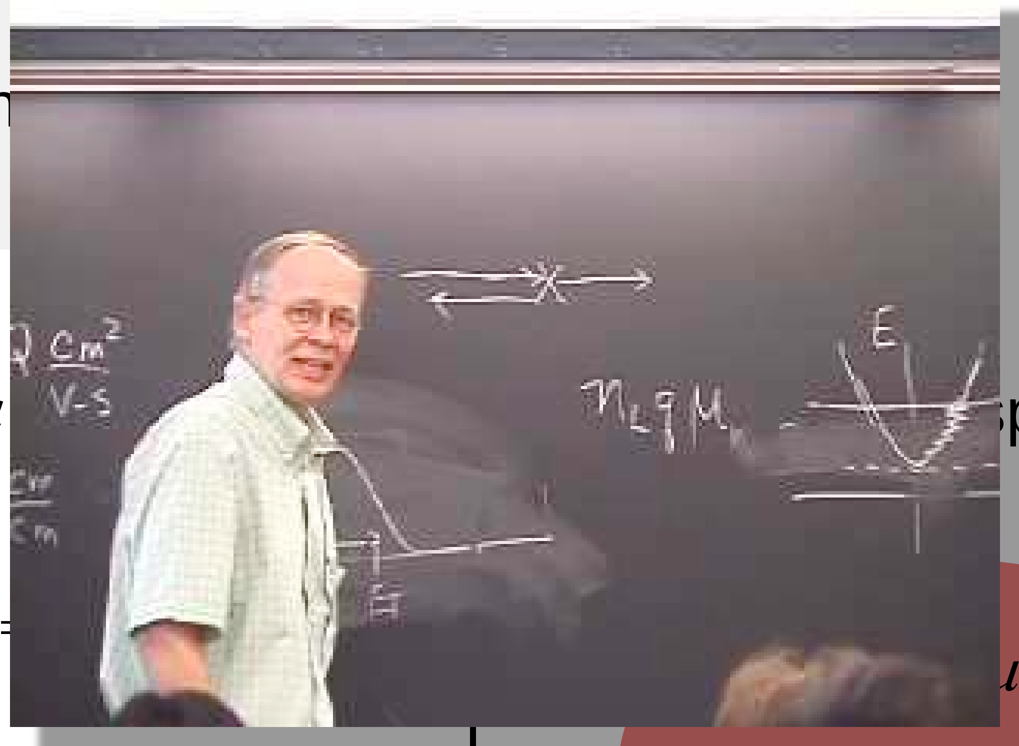
$T = 0\text{K}:\quad G = \frac{2q^2}{h}\frac{\lambda(E_F)}{L}$ diffusive transport

$T > 0\text{K}:\quad G = \frac{2q^2}{h}\frac{\lambda_0}{L}\mathcal{F}_{-1}(\eta_F)$ diffusive, constant mfp

$G = \frac{2q^2}{h}\frac{\lambda_0}{L}e^{\eta_F}$ diffusive, non-degenerate

T = 0K

Convention



$T = 0\text{K}: \quad G$ ... sport

$$n_L = \frac{2k_F}{\pi} = \ldots$$

$$\ldots \mu_n \frac{1}{L}$$

$$\lambda(E_F) = 2\upsilon(E_F)\tau(E_F)$$

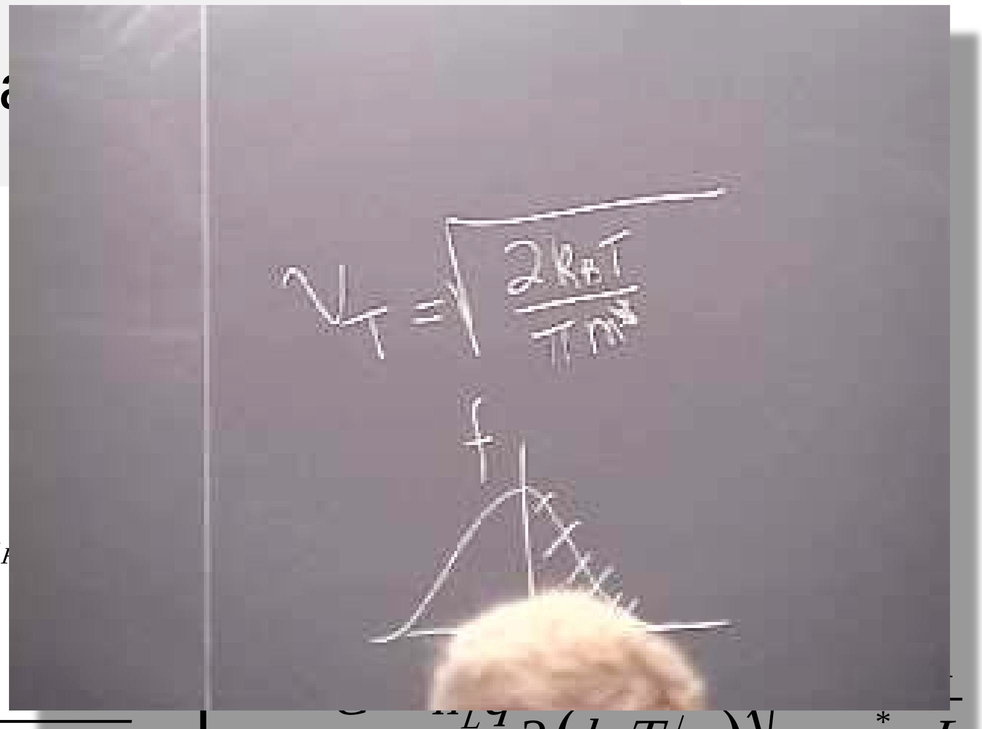
$$\upsilon(E_F) = \sqrt{\frac{2(E_F - \varepsilon_1)}{m^*}}$$

$$\mu_n = \frac{q\tau(E_F)}{m^*}$$

T > 0K, non-degenerate

Conventiona

Landauer:



$$n_L = N_{1D} e^{\eta_F}$$

$$2\left(k_B T/q\right)\sqrt{\pi m^*}\ L$$

$$N_{1D} = \sqrt{\frac{2m^* k_B T}{\pi \hbar^2}}$$

T > 0K, non-degenerate

Conventionally: $$G_{1D} = n_L q \mu_n \frac{1}{L}$$

$$G = n_L q \frac{\lambda_0}{2\left(k_B T/q\right)} \sqrt{\frac{2 k_B T}{\pi m^*}} \frac{1}{L}$$

$$G = n_L q \mu_n \frac{1}{L}$$ ✓

- diffusive
- non-degenerate
- constant mfp

$$\upsilon_T = \sqrt{\frac{2 k_B T}{\pi m^*}}$$

$$D_n = \frac{\lambda_0 \upsilon_T}{2}$$

$$\frac{D_n}{\mu_n} = \frac{k_B T}{q}$$

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Landauer formula

$$G = \frac{2q^2}{h}\left(\int T(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right) dE\right)$$

- G_B is independent of L and represents an upper limit to G (lower limit to R)

- $G_{diff} \sim 1/L$ because $T_{diff} \sim 1/L$

Landauer formula

$$G = \frac{2q^2}{h}\left(\int T(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right) dE \right)$$

$$T = 0\text{K}: \quad G = \frac{2q^2}{h} T(E_F)M(E_F)$$

$$T > 0\text{K}: \quad G = \frac{2q^2}{h} \langle T(E)M(E) \rangle$$

Landauer and conventional

$$G_{1D} = n_L q \mu_n \frac{1}{L}$$

Conductance is proportional to carrier density and inversely proportional to the length of the conductor for:

i) diffusive transport
ii) Boltzmann (non-degenerate) statistics

references

1) Supriyo Datta, *Electronic Transport in Mesoscopic Systems*, Cambridge Univ. Press, 1995. (See chapter 2)

2) D.F. Holcomb, “Quantum Transport in Samples of Limited Dimensions,” *Am. J. Phys*., **67** (4), pp. 278-297, April, 1999.

questions

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