# ECE 656: Fall 2009
## Lecture 5 Homework SOLUTIONS
(Revised 9/3/09)

1) Work out the 1D ballistic conductance for finite temperature.

1a) Show that $\int_{\varepsilon_1}^{\infty} f_0(E)dE = \int_{\varepsilon_1}^{\infty} \frac{1}{1+e^{(E-E_F)/k_BT}} dE = k_B T \mathcal{F}_0(\eta_F)$

where $\mathcal{F}_0(\eta_F) = \ln(1+e^{\eta_F})$

1b) Use the results of problem 1a) so show what $G_{1D} = \frac{2q^2}{h}\mathcal{F}_{-1}(\eta_F)$

1c) Show that $\mathcal{F}_{-1}(\eta_F) = e^{\eta_F}/(1+e^{\eta_F})$

2) For single subband conduction, the diffusive conductance in 1D is

$$G_{1D} = \frac{2q^2}{h}\frac{\langle\lambda(E)\rangle}{L}\mathcal{F}_{-1}(\eta_F)$$

where

$$\langle\lambda(E)\rangle = \frac{\int_{\varepsilon_1}^{+\infty} \lambda(E)\left(-\frac{\partial f_0}{\partial E}\right)dE}{\int_{\varepsilon_1}^{+\infty}\left(-\frac{\partial f_0}{\partial E}\right)dE}$$

Assume power law scattering, $\lambda(E) = \lambda_0\left[(E-\varepsilon_1)/k_BT\right]^r$ where *r* is a characteristic exponent that depends on the scattering physics. Work out an expression for $\langle\lambda(E)\rangle$ in terms of $\lambda_0$ and the characteristic exponent, *r*.

HINT: For the above two problems, you may find the following notes useful.

"Notes on Fermi-Dirac Integrals," 3rd Ed., by R. Kim and M. Lundstrom)
https://www.nanohub.org/resources/5475

3) Consider the conductance in the diffusive limit and under non-degenerate conditions. Assume that the energy dependent mean free path and mean scattering time can be written in power law form as

$$\lambda(E) = \lambda_0\left[(E-\varepsilon_1)/k_BT\right]^r$$

Derive an expression for the mobility in terms of the pre-factor and the characteristic exponent.

HW5

1a) $$\int_{\epsilon_1}^{\infty} \frac{dE}{1+e^{(E-\epsilon_1+\epsilon_1-E_F)/k_BT}}$$

$$\eta = (E-\epsilon_1)/k_BT \quad \eta_F = (E_F-\epsilon_1)/k_BT$$

$$dE = k_BT\,d\eta$$

$$k_BT\int_0^{\infty} \frac{d\eta}{1+e^{\eta-\eta_F}} \equiv \mathcal{F}_0(\eta_F) \times k_BT$$

$$= \ln\left(1+e^{\eta_F}\right)$$

(see notes)

1b) $$G = \frac{2q^2}{h} \int dE \cdot 1 \times \left(-\frac{\partial f_0}{\partial E}\right)$$

$$= \frac{2q^2}{h} \cdot \frac{\partial}{\partial E_F} \int f_0(E)\,dE$$

$$= \frac{2q^2}{h} \frac{\partial}{\partial E_F} \cdot k_BT\,\mathcal{F}_0(\eta_F)$$

$$= \frac{2q^2}{h} \frac{\partial}{\partial \eta_F} \mathcal{F}_0(\eta_F) = \frac{2q^2}{h} \mathcal{F}_{-1}(\eta_F) \checkmark$$

1)

HW 5

1c)

$$\mathcal{F}_{-1}(n_F) = \frac{\partial}{\partial n_F} \ln(1+e^{n_F})$$

$$= \frac{1}{1+e^{n_F}} \times e^{n_F} \quad \checkmark$$

2)

$$\langle \lambda(E) \rangle = \frac{\int_{\epsilon_1}^{\infty} \lambda_0 (E/k_BT)^r (-\partial f_0/\partial E)\, dE}{\mathcal{F}_{-1}(n_F)}$$

numerator:

$$NUM = \lambda_0 \frac{\partial}{\partial E_F} \int_{\epsilon_1}^{\infty} [(E-\epsilon_1)/(k_BT)]^r \frac{1}{1+e^{(E-E_F)/k_BT}}\, dE$$

$n = (E-\epsilon_1)/k_BT \quad n_F = (E_F-\epsilon_1)/k_BT$

$$NUM = \lambda_0 \frac{\partial}{\partial E_F} \int_0^{\infty} \frac{n^r}{1+e^{n-n_F}} k_BT\, dn$$

$$= \lambda_0 \frac{\partial}{\partial n_F} \int \frac{n^r\, dn}{1+e^{n-n_F}} = \lambda_0 \frac{\partial}{\partial n_F} \Gamma(r+1) \times \mathcal{F}_r(n_F)$$

2)

$$NUM = \lambda_0 \Gamma(r+1) \mathcal{F}_{r-1}(\eta_F)$$

$$\langle \lambda(E) \rangle = \lambda_0 \Gamma(r+1) \frac{\mathcal{F}_{r-1}(\eta_F)}{\mathcal{F}_{-1}(\eta_F)} \quad \checkmark$$

3) $$G_{1D} = \frac{2q^2}{h} \cdot \frac{\langle \lambda(E) \rangle}{L} \cdot \mathcal{F}_{-1}(\eta_F) = n_L q \mu_n \frac{1}{L}$$

$$\mu_n = \frac{2q}{h} \frac{\lambda_0 \Gamma(r+1) \mathcal{F}_{r-1}(\eta_F)}{n_L}$$

can work out $n_L$:

$$n_L = \frac{\sqrt{2mk_BT}}{\hbar\sqrt{\pi}} \mathcal{F}_{-1/2}(\eta_F)$$

$$\mu_n = \frac{\hbar 2q}{h} \frac{\lambda_0 \Gamma(r+1) \sqrt{\pi}}{\sqrt{2mk_BT}} \cancel{\frac{\mathcal{F}_{r-1}}{\mathcal{F}_{-1/2}}} \quad \text{| non-deg.}$$

$$= q \frac{\lambda_0}{\sqrt{2mk_BT\pi}} \Gamma(r+1)$$

$\sqrt{\frac{2kT}{\pi m}}$

$$= \frac{q}{k_BT} \frac{\lambda_0 k_BT}{\sqrt{2mk_BT\pi}} \Gamma(r+1) = \left(\frac{q}{k_BT}\right) \frac{\lambda_0}{2} \sqrt{\frac{2k_BT}{\pi m^*}} \Gamma(r+1)$$

3)

3) cont.

$$\mu_n = \left(\frac{1}{k_BT/q}\right) \frac{\lambda_0 \Gamma(r+1)}{2} v_T$$

$$= \frac{D_n}{k_BT/q} \qquad D_n = \frac{(\lambda_0 \Gamma(r+1))v_T}{2}$$

$v_T = 2 = v_R$ "Richardson velocity"