ECE-656: Fall 2009

Lecture 6: Discussion

Professor Mark Lundstrom
Electrical and Computer Engineering
Purdue University, West Lafayette, IN USA

miscellaneous topics

1) Quantum confinement and effective mass

2) Bulk 1D transport and mfp

3) Periodic vs. Box boundary conditions

4) Thermal velocities

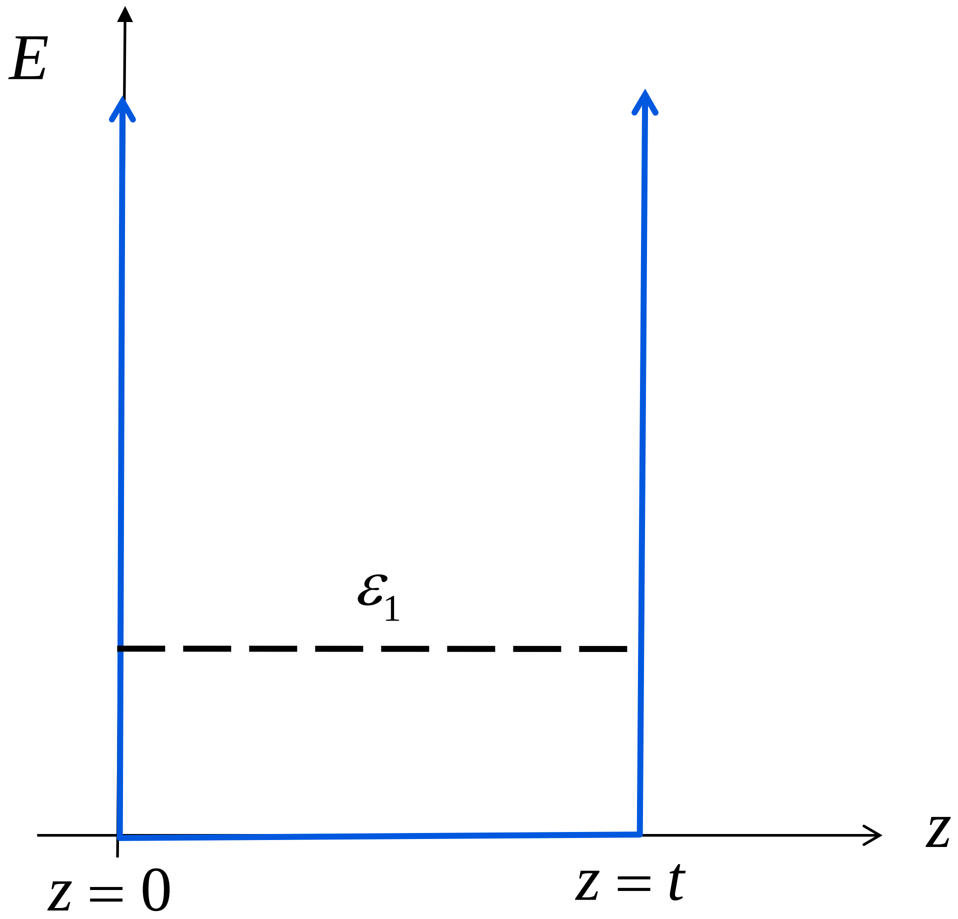
5) "Ballistic mobility"

quantum confinement and effective mass

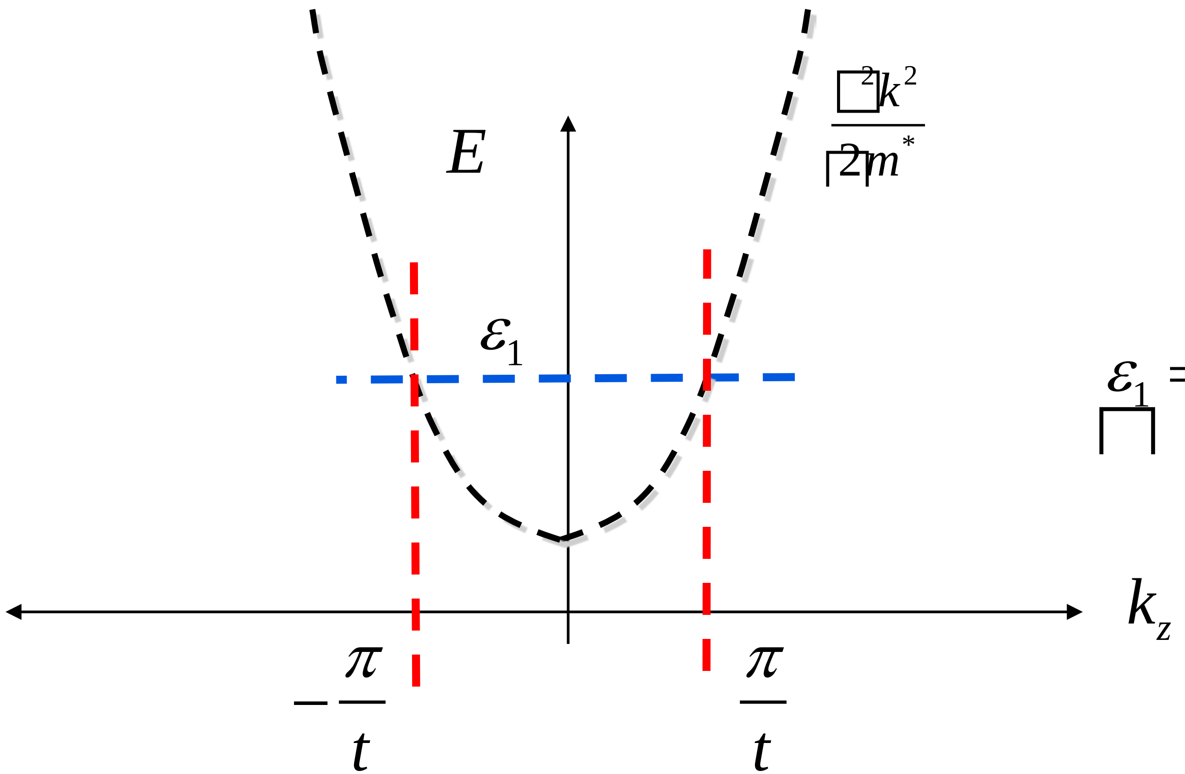
$$\psi(z) = \sin k_z z$$

$$k_z t = n\pi$$

$$\varepsilon_n = \frac{\hbar^2 k_n^2}{2m^*} = \frac{\hbar^2 n^2 \pi^2}{2m^* t^2}$$

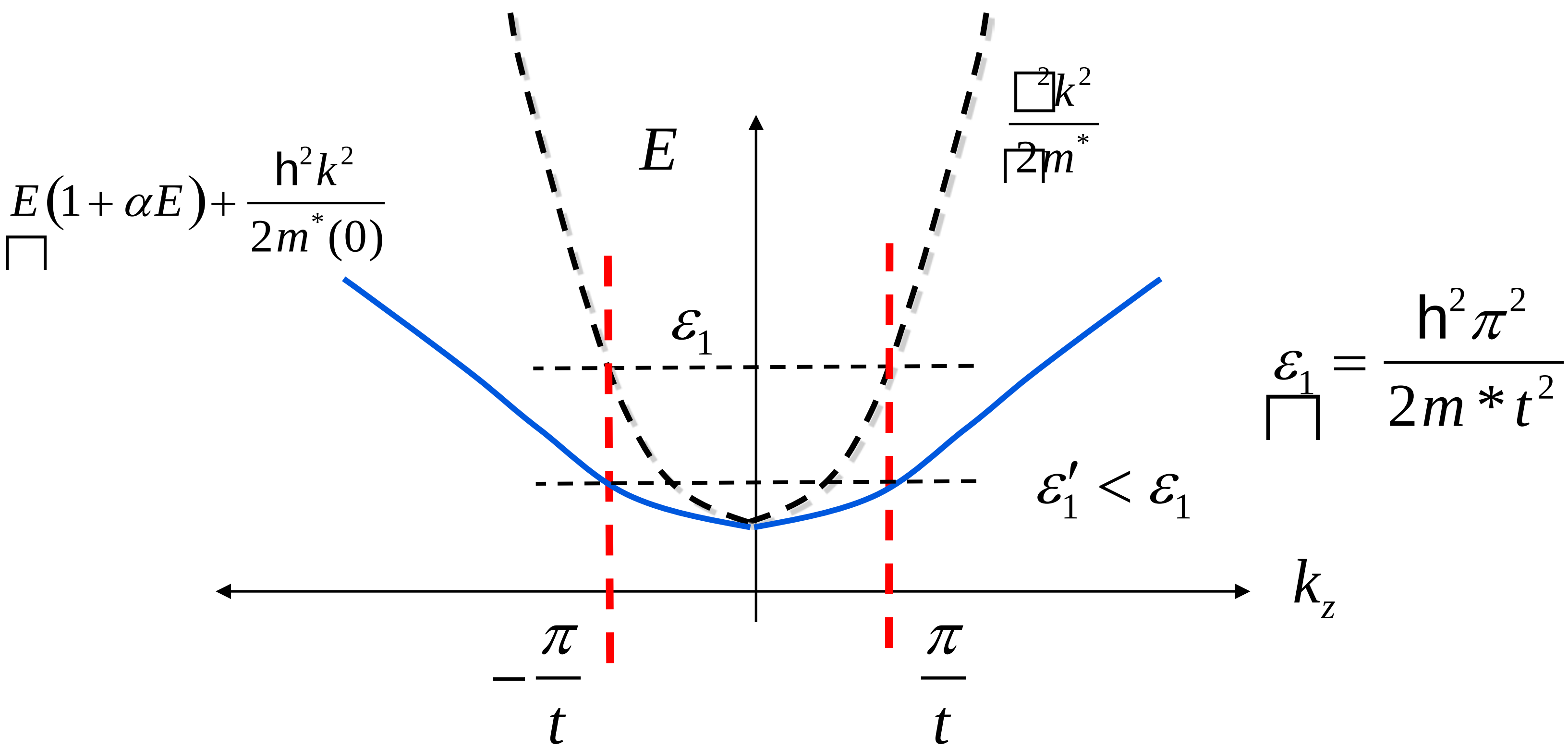


quantum confinement and effective mass



$$\varepsilon_1 = \frac{\hbar^2 \pi^2}{2m^* t^2}$$

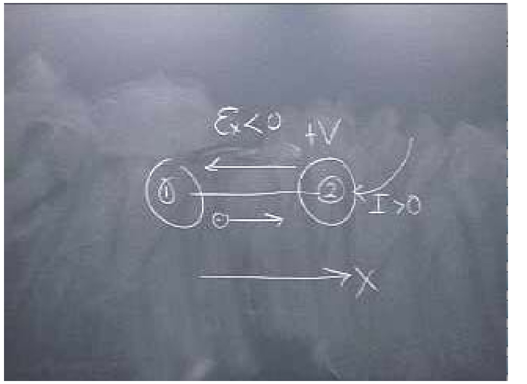
quantum confinement and effective mass



confinement mass > bulk mass

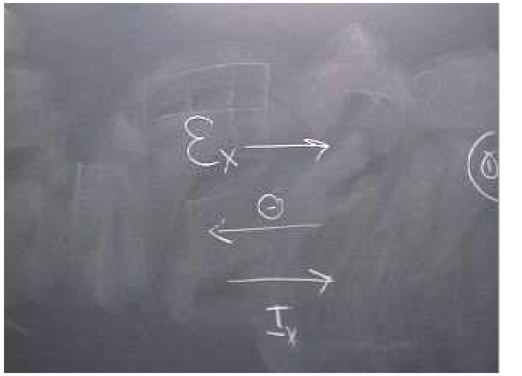
1D diffusive transport: “bulk”

$$I_x = -\frac{2q^2}{h}\left\{\int_{\varepsilon_1}^{\infty} T(E)\left(-\frac{\partial f_0}{\partial E}\right) dE\right\} V_D \approx -\frac{2q^2}{h} T\left(E_F\right) V$$



$$T\left(E_F\right) \approx \frac{\lambda(E_F)}{\lambda(E_F) + L} \approx \frac{\lambda(E_F)}{L}$$

(diffusive)



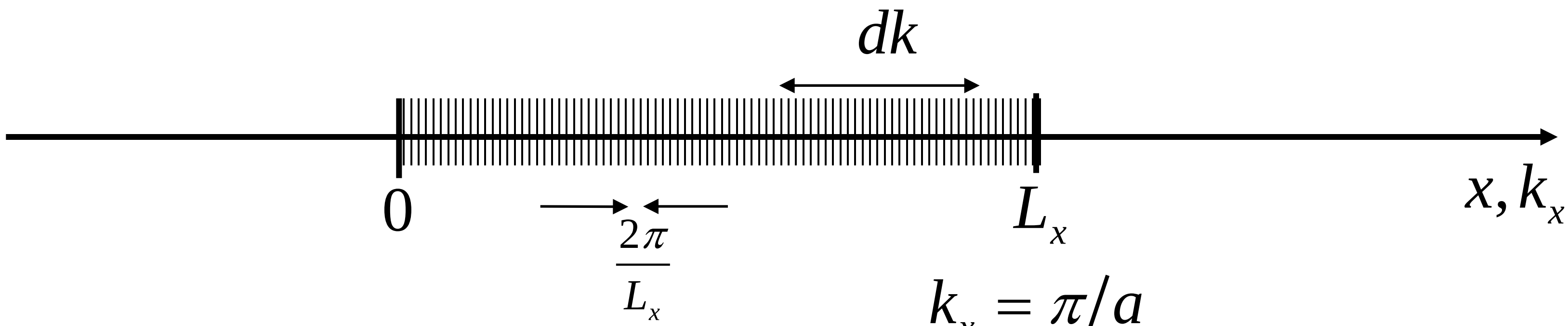
$$I_x \approx \frac{2q^2 \lambda\left(E_F\right)}{h}\left(-\frac{V}{L}\right)$$

$$I_x \approx \frac{2q^2}{h}\left[\lambda\left(E_F\right)\mathcal{E}_x\right]$$

In 1D, the diffusive current is the quantum conductance times the volt drop across one mean-free-path.

periodic vs. box boundary conditions

periodic boundary conditions



$$k_x = \pi / a$$

$$\psi(x) = u_k(x) e^{i k_x x}$$

$$\psi(0) = \psi(L_x) \rightarrow e^{i k_x L_x} = 1$$

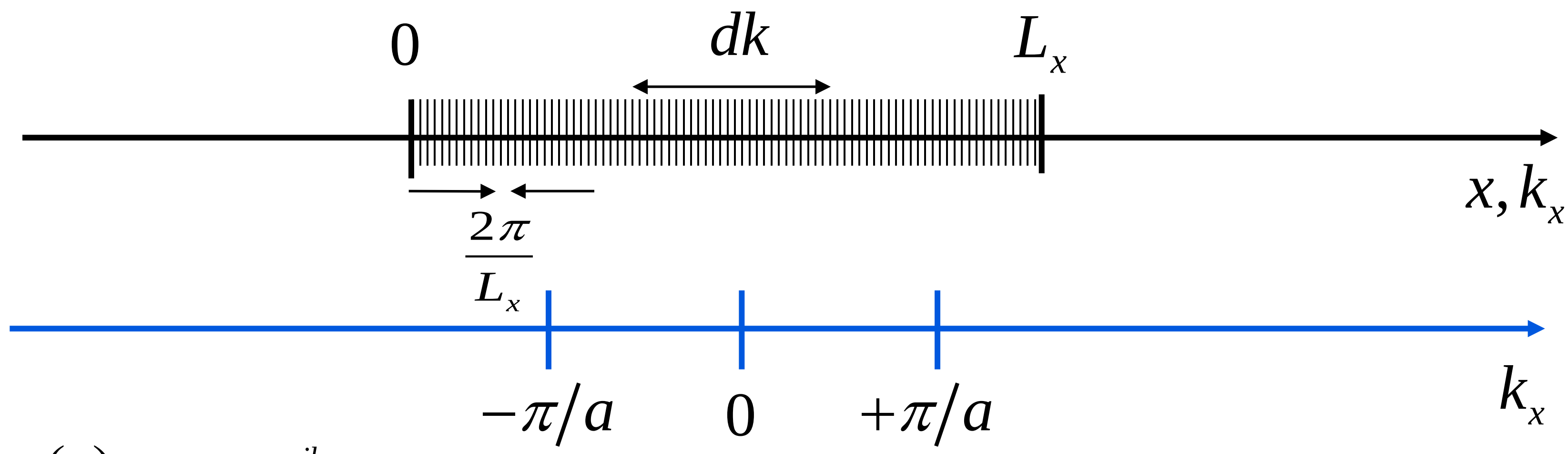
$$k_x L_x = 2\pi j \quad j = 1, 2, 3, \ldots$$

$$k_x = \frac{2\pi}{L_x} j$$

$$\#\text{ of states} = \frac{dk_x}{\left(2\pi / L_x\right)} \times 2 = N_k dk$$

$$N_k = \frac{L_x}{\pi} = \text{density of states in } k\text{-space}$$

periodic boundary conditions



$$\psi(x) = u_k(x)e^{ik_x x}$$

$$\psi(0) = \psi(L_x) \rightarrow e^{ik_x L_x} = 1$$

$$k_x L_x = 2\pi j \quad j = 1, 2, 3, \ldots$$

$$k_x = \frac{2\pi}{L_x} j$$

$$L_x = N_A a$$

$$k_x = \frac{2\pi}{L_x} j = \frac{2\pi}{a} \frac{j}{N_A}$$

$$\theta = ik_x x = i\left(2\pi \frac{j}{N_A}\right)\frac{x}{a}$$

$$j_{\max} = N_A \qquad k_{\max} = \frac{2\pi}{a}$$

thermal velocity

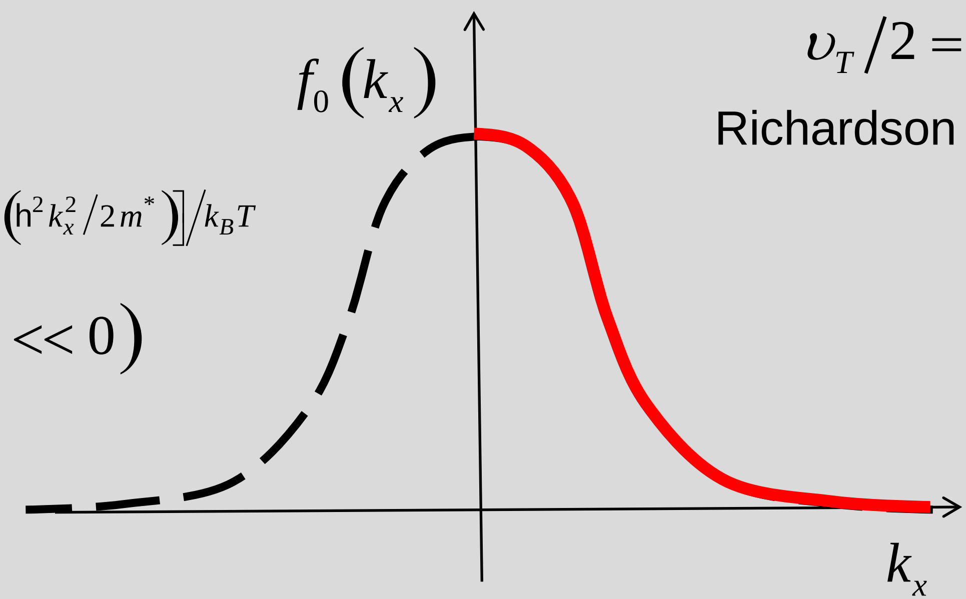
$$\left\langle \upsilon^{+} \right\rangle = \frac{\frac{1}{\Omega}\sum\limits_{k_x>0,k_y,k_z} \upsilon_k f_0\left(E_k\right)}{\frac{1}{\Omega}\sum\limits_{k_x>0,k_y,k_z} f_0\left(E_k\right)} \quad \text{cm/s} = \upsilon_T$$

$$\upsilon_T = \sqrt{\frac{2k_BT}{\pi m^*}}$$

$$\left(\eta_F << 0\right)$$

$$f_0 \approx e^{\left[E_F - \left(\hbar^2 k_x^2/2m^*\right)\right]/k_BT}$$

$$\left(\eta_F << 0\right)$$



$$\upsilon_T/2 = \upsilon_R$$

Richardson velocity

rms thermal velocity

$$\left\langle E - E_C \right\rangle = \frac{3}{2} k_B T \qquad E - E_C = \frac{1}{2} m^* \upsilon^2 (E)$$

$$\frac{1}{2} m^* \left\langle \upsilon^2 \right\rangle = \frac{3}{2} k_B T \qquad \left\langle E - E_C \right\rangle = \frac{1}{2} m^* \left\langle \upsilon^2 \right\rangle$$

$$\left\langle \upsilon^2 \right\rangle^{1/2} = \upsilon_{rms} = \sqrt{\frac{3k_B T}{m^*}} \neq \upsilon_T = \sqrt{\frac{2k_B T}{\pi m^*}}$$

“ballistic mobility”

$$G_{1D} = n_L\, q\, \mu_n \frac{1}{L} - = \sigma_{1D} \frac{1}{L}$$

$$\mu_n = \frac{\sigma_{1D}}{n_L\, q}$$

$$G_{1D} = \frac{2q^2}{h} \frac{\lambda}{\lambda + L} = \frac{2q^2}{h} \left(\frac{1}{\frac{1}{L} + \frac{1}{\lambda}} \right) \frac{1}{L}$$

$$\sigma_{1D} = \frac{2q^2}{h} \left(\frac{1}{L} + \frac{1}{\lambda} \right)^{-1}$$

$$\frac{\sigma_{1D}}{n_L q} = \frac{2q}{h n_L} \left(\frac{1}{L} + \frac{1}{\lambda} \right)^{-1}$$

"ballistic mobility"

$$\mu_{eff} = \frac{2q}{hn_L}\left(\frac{1}{L}+\frac{1}{\lambda}\right)^{-1}$$

$$\mu_{eff} = \frac{q}{2m^*\upsilon_F}\left(\frac{1}{L}+\frac{1}{\lambda}\right)^{-1}$$

$$n_L = \frac{2k_F}{2\pi}\times 2 = \frac{2k_F}{\pi}$$

$$\frac{1}{\mu_{eff}} = \frac{2m^*\upsilon_F}{q}\left(\frac{1}{L}+\frac{1}{\lambda}\right)$$

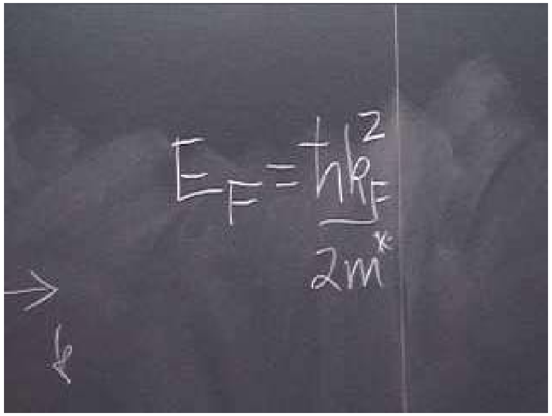
$$\upsilon_F = \frac{h k_F}{m^*}$$

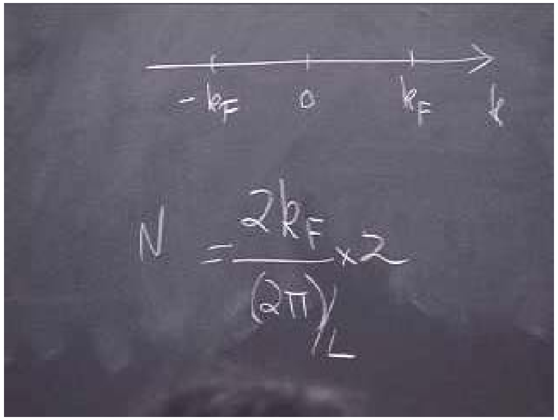
$$k_F = \frac{m^*\upsilon_F}{h}$$

$$\frac{1}{\mu_{eff}} = \frac{2m^*\upsilon_F}{qL} + \frac{2m^*\upsilon_F}{q\lambda}$$

$$n_L = \frac{2m^*\upsilon_F}{\pi h} = \frac{4m^*\upsilon_F}{h}$$

$$\frac{1}{\mu_{eff}} = \frac{m^*}{q\left(L/2\upsilon_F\right)} + \frac{m^*}{q\left(\lambda/2\upsilon_F\right)}$$





“ballistic mobility”

$$\frac{1}{\mu_{eff}} = \frac{m^*}{q(L/2\upsilon_F)} + \frac{m^*}{q(\lambda/2\upsilon_F)}$$

$$\frac{1}{\mu_{eff}} = \frac{1}{\mu_B} + \frac{1}{\mu_n}$$

$$\mu_B = \frac{q(L/2\upsilon_F)}{m^*}$$

$$\mu_n = \frac{q(\lambda/2\upsilon_F)}{m^*}$$

$$\frac{1}{\mu_{eff}} = \frac{1}{\mu_B} + \frac{1}{\mu_n}$$

$$\mu_n = \frac{q\tau}{m^*} \qquad \lambda = 2\upsilon_F\tau$$

$$\mu_B = \frac{q(L/2\upsilon_F)}{m^*}$$

(M.S. Shur, IEED EDL, Sept. 2002)

Example Problem: InGaAs Transistor

Please see video of this lecture for chalkboard details/footage.