ECE 656: Fall 2009
Lecture 8 Homework SOLUTIONS

1) Work out the Seebeck coefficient for a 3D semiconductor assuming that the mfp, λ_0 is independent of energy and show that the result is:

$$S_{3D} = \left(\frac{k_B}{-q}\right)\left(\frac{2\mathcal{F}_1(\eta_F)}{\mathcal{F}_0(\eta_F)} - \eta_F\right)$$

HW8 Solution

$$S = \left(\frac{k_B}{-q}\right)\frac{I_1}{I_0} \qquad I_j = \int \left(\frac{E-E_F}{k_BT}\right)^j \cdot \frac{\lambda_0}{L} \cdot \frac{m^*(E-E_c)A}{2\pi\hbar^2} \times \left(-\frac{\partial f_0}{\partial E}\right) dE$$

$$I_0 = \underbrace{\frac{\lambda_0 A}{L}\frac{m^*}{2\pi\hbar^2}}_{C} \int (E-E_c)\left(-\frac{\partial f_0}{\partial E}\right) dE$$

$$I_0 = C\left(+\frac{\partial}{\partial E_F}\right)\int \frac{(E-E_c)\, dE}{1+e^{(E-E_F)/k_BT}} \qquad \eta_F = (E_F-E_c)/k_BT$$

$$\eta = (E-E_c)/k_BT$$

$$I_0 = C\left(\frac{\partial}{\partial E_F}\right)\int \frac{k_BT\eta\,(k_BT\,d\eta)}{1+e^{\eta-\eta_F}}$$

$$I_0 = C k_BT \cdot \frac{\partial}{\partial \eta_F}\int \frac{\eta\, d\eta}{1+e^{\eta_F}} = C k_BT \frac{\partial}{\partial \eta_F}\mathcal{F}_1(\eta_F)$$

$$\underline{I_0 = C k_BT\, \mathcal{F}_0(\eta_F)} \; *$$

$$I_1 = \int \frac{(E-E_F)}{k_BT} \cdot \frac{\lambda_B}{L}\frac{m^*}{2\pi\hbar^2}(E-E_c)A\left(-\frac{\partial f_0}{\partial E}\right) dE$$

$$= C\int \frac{(E-E_F)}{k_BT}(E-E_c)\left(-\frac{\partial f_0}{\partial E}\right) dE$$

$$I_1 = C \int (\eta - \eta_F) \cdot k_B T \eta \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$= C k_B T \cdot \int (\eta - \eta_F) \eta \left(+ \frac{\partial f_0}{\partial E_F} \right) k_B T d\eta$$

$$= C k_B T \int (\eta - \eta_F) \eta \left(\frac{\partial f_0}{\partial \eta_F} \right) d\eta \qquad (A)$$

$$(\eta - \eta_F) \eta \left(\frac{\partial f_0}{\partial \eta} \right) = \frac{\partial}{\partial \eta_F} \left((\eta - \eta_F) \eta f_0 \right) + \eta f_0$$

(A) becomes

(B) $$I_1 = C k_B T \times \left[\frac{\partial}{\partial \eta_F} \underbrace{\int (\eta^2 - \eta \eta_F) f_0 d\eta} + \underbrace{\int \eta f_0 d\eta}_{\mathcal{F}_1(\eta_F)} \right]$$

$$= \int \eta^2 f_0 d\eta - \eta_F \int \eta f_0 d\eta$$

$$= 2 \mathcal{F}_2(\eta_F) - \eta_F \mathcal{F}_1$$

2)

So (B) becomes

$$I_1 = Ck_BT\left[\frac{\partial}{\partial n_F}\left(2\mathcal{F}_2(n_F) - n_F\mathcal{F}_1(n_F)\right) + \mathcal{F}_1(n_F)\right]$$

$$= Ck_BT\left[2\mathcal{F}_1(n_F) - \cancel{\mathcal{F}_1(n_F)} - n_F\mathcal{F}_0(n_F) + \cancel{\mathcal{F}_1(n_F)}\right]$$

$$\underline{I_1 = Ck_BT\left(2\mathcal{F}_1(n_F) - n_F\mathcal{F}_0(n_F)\right)} \quad **$$

$$S = \left(\frac{k_B}{-q}\right)\frac{I_1}{I_0} \quad \text{use } (*) \text{ and } (**)$$

$$S = \left(-\frac{k_B}{q}\right)\left(2\frac{\mathcal{F}_1(n_F)}{\mathcal{F}_0(n_F)} - n_F\right) \checkmark$$

3)